6) Statistical distributions

6.1) Probability distributions

Chapter CONTENTS

Worked example	Your turn
Let X = number of tails when a fair coin is tossed 4 times. Write a list of all the possible outcomes.	Let X = number of tails when a fair coin is tossed 3 times. Write a list of all the possible outcomes. HHH THH HTH HHT TTH THT HTT TTT

Worked example	Your turn

Worked example

Let X = number of tails when a fair coin is tossed 4 times.

Describe the probability distribution of X:

- Using a table

Your turn

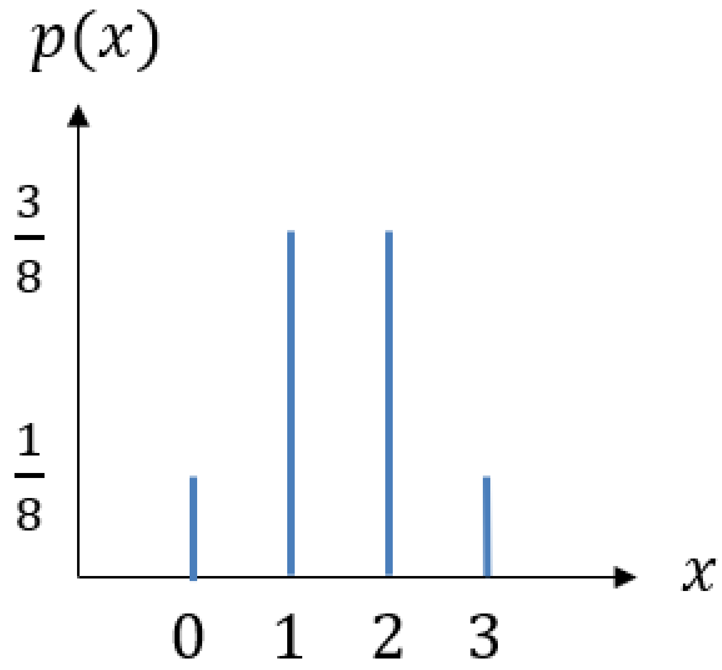
Let X = number of tails when a fair coin is tossed 3 times.

Describe the probability distribution of X:

- Using a table

x	0	1	2	3
$P(X = x)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

Worked example	Your turn
Let X = number of tails when a fair coin is tossed 4 times. Describe the probability distribution of X: • Using a diagram	Let X = number of tails when a fair coin is tossed 3 times. Describe the probability distribution of X: • Using a diagram



Worked example	Your turn
Let X = number of tails when a fair coin is tossed 4 times. Describe the probability distribution of X: • As a probability mass function	Let X = number of tails when a fair coin is tossed 3 times. Describe the probability distribution of X: • As a probability mass function $P(X = x) = \begin{cases} \frac{1}{8} & x = 0, 3 \\ \frac{3}{8} & x = 1, 2 \\ 0 & otherwise \end{cases}$

Worked example

A biased six-sided dice with faces numbered $1, 2, 3, 4, 5$ and 6 is rolled. The number on the bottom-most face is modelled as a random variable X.

Given that $P(X = x) = \frac{k}{x}$,

a) Find the value of k

b) Give the probability distribution of X in table form

c) Find the probability that:

i) $X \geq 2$

ii) $1 \leq X < 4$

iii) $X < 1$

iv) $2X + 1 > 11$

Your turn

A biased four-sided dice with faces numbered $1, 2, 3$ and 4 is rolled.. The number on the bottom-most face is modelled as a random variable X.

Given that $P(X = x) = \frac{k}{x}$,

a) Find the value of k

b) Give the probability distribution of X in table form

c) Find the probability that:

i) $X > 2$

ii) $1 \leq X < 4$

iii) $X \leq 4$

iv) $3X - 5 < 0$

a) $k = \frac{12}{25}$

b)

x	1	2	3	4
$P(X = x)$	$\frac{12}{25}$	$\frac{6}{25}$	$\frac{4}{25}$	$\frac{3}{25}$

c)

i) $\frac{7}{25}$

ii) $\frac{22}{25}$

iii) 1

iv) $\frac{12}{25}$

Worked example

The random variable X has a probability function

$$P(X = x) = \frac{k}{x^3}, \qquad x = 1, 2, 3, 4$$

Find the value of k

Your turn

The random variable X has a probability function

$$P(X = x) = \frac{k}{x^2}, \qquad x = 1, 2, 3, 5$$

Find the value of k

$$k = \frac{900}{1261}$$

Worked example	Your turn
The random variable X has a probability function $$P(X = x) = \begin{cases} kx & x = 1, 3 \\ k(x-2) & x = 2, 4 \end{cases}$$ a) Find the value of k b) Find $P(X > 1)$	The random variable X has a probability function $$P(X = x) = \begin{cases} kx & x = 1, 2 \\ k(x-3) & x = 3, 4 \end{cases}$$ a) Find the value of k b) Find $P(X < 4)$ a) $k = \frac{1}{4}$ b) $\frac{3}{4}$

Worked example	Your turn
The random variable X has a probability function $$P(X=x)=\begin{cases}k(2-x)^2 & x=-2,-1,0,1,2,3\\ 0 & otherwise\end{cases}$$ Find the value of k	The random variable X has a probability function $$P(X=x)=\begin{cases}k(1-x)^2 & x=-1,0,1,2\\ 0 & otherwise\end{cases}$$ Find the value of k $$k=\frac{1}{6}$$

Worked example	Your turn
A spinner has six equally-sized sections. Four contain the letter G. 2 contain the letter Y. The spinner is spun until it lands on Y or has been spun five times in total. Find the probability distribution of the random variable S, the number of times the spinner is spun.	A spinner has five equally-sized sections. Three contain the letter B. 2 contain the letter R. The spinner is spun until it lands on R or has been spun four times in total. Find the probability distribution of the random variable S, the number of times the spinner is spun.

s	1	2	3	4
$P(S = s)$	$\frac{2}{5}$	$\frac{6}{25}$	$\frac{18}{125}$	$\frac{27}{125}$

Worked example	Your turn
The random variable X can taken any integer value from 1 to 30. Given that X has a discrete uniform distribution, find: a) $P(X = 5)$ b) $P(X \geq 20)$ c) $P(12 < X < 21)$	The random variable X can taken any integer value from 1 to 40. Given that X has a discrete uniform distribution, find: a) $P(X = 3)$ b) $P(X \geq 21)$ c) $P(13 < X < 31)$ a) $\frac{1}{40}$ b) $\frac{1}{2}$ c) $\frac{17}{40}$

Worked example	Your turn

Worked example

A discrete random variable has a probability distribution as shown in the table. Find the value of a

$\boldsymbol{x}$	0	1	2	3
$\boldsymbol{P(X=x)}$	a	$a-\frac{1}{4}$	$a+\frac{1}{3}$	$3a$

Your turn

A discrete random variable has a probability distribution as shown in the table. Find the value of a

$\boldsymbol{x}$	1	2	3	4
$\boldsymbol{P(X=x)}$	$2a$	$a-\frac{1}{3}$	$a+\frac{1}{4}$	$5a$

$$a=\frac{13}{108}$$

6.2) The binomial distribution

Worked example	Your turn
The probability of a lightbulb being faulty is 0.12. A random sample of 34 lightbulbs is taken from the production line. a) Define a suitable distribution to model the number of faulty lightbulbs in this sample. b) Find the probability that the sample contains fewer than 3 faulty lightbulbs.	The probability of a bolt being faulty is 0.21. A random sample of 43 bolts is taken from the production line. a) Define a suitable distribution to model the number of faulty bolts in this sample. b) Find the probability that the sample contains fewer than 2 faulty bolts. a) Let X = number of faulty bolts. $X \sim B(43, 0.21)$ b) 0.000493 (3 sf)

Worked example

The random variable $X \sim B\left(8, \frac{1}{10}\right)$. Find:

a) $P(X = 2)$

b) $P(X = 5)$

c) $P(X \leq 1)$

d) $P(X \geq 7)$

Your turn

The random variable $X \sim B\left(12, \frac{1}{6}\right)$. Find:

a) $P(X = 2)$

b) $P(X = 9)$

c) $P(X \leq 1)$

d) $P(X \geq 11)$

a) 0.2961 (4 dp)

b) 0.0000126 (3 sf)

c) 0.3813 (4 dp)

d) 0.000000280 (3 sf)

Worked example	Your turn
A company claims that a third of the lightbulbs sent to them are faulty. To test this claim the number of faulty lightbulbs in a random sample of 100 is recorded. Give two reasons why a binomial distribution may be a suitable model for the number of faulty lightbulbs in the sample.	A company claims that a quarter of the bolts sent to them are faulty. To test this claim the number of faulty bolts in a random sample of 50 is recorded. Give two reasons why a binomial distribution may be a suitable model for the number of faulty bolts in the sample. • Two possible outcomes (bolt faulty or not faulty) • Constant probability of bolt being faulty $(p = \frac{1}{4})$ • A bolt being faulty is independent of other bolts being faulty (assuming they do not influence each other) • 50 bolts in the sample (fixed number of trials)

6.3) Cumulative probabilities

Worked example	Your turn
Using your calculator, if $X \sim B(40, 0.2)$ find, to 4 dp, a) $P(X = 3)$ b) $P(X \leq 5)$ c) $P(X < 5)$ d) $P(X \geq 7)$ e) $P(X > 7)$ f) $P(4 < X < 9)$ g) $P(4 \leq X \leq 9)$ h) $P(4 \leq X < 9)$ i) $P(4 < X \leq 9)$	Using your calculator, if $X \sim B(20, 0.4)$ find, to 4 dp, a) $P(X = 4)$ b) $P(X \leq 6)$ c) $P(X < 6)$ d) $P(X \geq 8)$ e) $P(X > 8)$ f) $P(5 < X < 10)$ g) $P(5 \leq X \leq 10)$ h) $P(5 \leq X < 10)$ i) $P(5 < X \leq 10)$ a) 0.0345 b) 0.2500 c) 0.1256 d) 0.5841 e) 0.4044 f) 0.6297 g) 0.8215 h) 0.7044 i) 0.7469

Worked example	Your turn
Using your calculator, if $X \sim B(40, 0.2)$ find, to 4 dp, $P(X = 3)$	Using your calculator, if $X \sim B(20, 0.4)$ find, to 4 dp, $P(X = 4)$ 0.0345

Worked example	Your turn
Using your calculator, if $X \sim B(40, 0.2)$ find, to 4 dp, $P(X \leq 5)$	Using your calculator, if $X \sim B(20, 0.4)$ find, to 4 dp, $P(X \leq 6)$ 0.2500

Worked example	Your turn
Using your calculator, if $X \sim B(40, 0.2)$ find, to 4 dp, $P(X$ is at most $5)$	Using your calculator, if $X \sim B(20, 0.4)$ find, to 4 dp, $P(X$ is at most $6)$ 0.2500

Worked example	Your turn
Using your calculator, if $X \sim B(40, 0.2)$ find, to 4 dp, $P(X < 5)$	Using your calculator, if $X \sim B(20, 0.4)$ find, to 4 dp, $P(X < 6)$ 0.1256

Worked example	Your turn
Using your calculator, if $X \sim B(40, 0.2)$ find, to 4 dp, $P(X \geq 7)$	Using your calculator, if $X \sim B(20, 0.4)$ find, to 4 dp, $P(X \geq 8)$ 0.5841

Worked example	Your turn
Using your calculator, if $X \sim B(40, 0.2)$ find, to 4 dp, $P(X$ is at least $7)$	Using your calculator, if $X \sim B(20, 0.4)$ find, to 4 dp, $P(X$ is at least $8)$ 0.5841

Worked example	Your turn
Using your calculator, if $X \sim B(40, 0.2)$ find, to 4 dp, $P(X > 7)$	Using your calculator, if $X \sim B(20, 0.4)$ find, to 4 dp, $P(X > 8)$ 0.4044

Worked example	Your turn
Using your calculator, if $X \sim B(40, 0.2)$ find, to 4 dp, $P(4 < X < 9)$	Using your calculator, if $X \sim B(20, 0.4)$ find, to 4 dp, $P(5 < X < 10)$ 0.6297

Worked example	Your turn
Using your calculator, if $X \sim B(40, 0.2)$ find, to 4 dp, $P(4 \leq X \leq 9)$	Using your calculator, if $X \sim B(20, 0.4)$ find, to 4 dp, $P(5 \leq X \leq 10)$ 0.8215

Worked example	Your turn
Using your calculator, if $X \sim B(40, 0.2)$ find, to 4 dp, $P(4 \leq X < 9)$	Using your calculator, if $X \sim B(20, 0.4)$ find, to 4 dp, $P(5 \leq X < 10)$ 0.7044

Worked example	Your turn
Using your calculator, if $X \sim B(40, 0.2)$ find, to 4 dp, $P(4 < X \leq 9)$	Using your calculator, if $X \sim B(20, 0.4)$ find, to 4 dp, $P(5 < X \leq 10)$ 0.7469

Worked example	Your turn
A spinner is designed so that probability it lands on red is 0.2. Jane decides to use this spinner for a class competition. She wants the probability of winning a prize to be less than 0.03. Each member of the class will have 15 spins and the number of reds will be recorded. Find how many reds are needed to win the prize.	A spinner is designed so that probability it lands on red is 0.3. Jane decides to use this spinner for a class competition. She wants the probability of winning a prize to be less than 0.05. Each member of the class will have 12 spins and the number of reds will be recorded. Find how many reds are needed to win the prize. 7 or more

Worked example	Your turn
At a university, students have 10 exams at the end of the year. All students pass each individual exam with probability 0.55. Students are only allowed to continue into the next year if they pass some minimum of exams out of the 10. What do the university administrators need to set this minimum number such that the probability of continuing to next year is at least 80%?	At a university, students have 20 exams at the end of the year. All students pass each individual exam with probability 0.45. Students are only allowed to continue into the next year if they pass some minimum of exams out of the 20. What do the university administrators need to set this minimum number such that the probability of continuing to next year is at least 90%? 6

Worked example	Your turn
The random variable $X \sim B(40, 0.3)$. Find: a) The largest value of p such that $P(X \leq p) < 0.05$ b) The largest value of r such that $P(X < r) < 0.1$ c) The smallest value of s such that $P(X \geq s) < 0.15$ d) The smallest value of t such that $P(X > t) < 0.2$	The random variable $X \sim B(30, 0.4)$. Find: a) The largest value of p such that $P(X \leq p) < 0.2$ b) The largest value of r such that $P(X < r) < 0.15$ c) The smallest value of s such that $P(X \geq s) < 0.1$ d) The smallest value of t such that $P(X > t) < 0.05$ a) $p = 9$ b) $r = 9$ c) $s = 16$ d) $t = 16$

Worked example	Your turn
The random variable $X \sim B(40, 0.3)$. Find the largest value of p such that $P(X \leq p) < 0.05$	The random variable $X \sim B(30, 0.4)$. Find the largest value of p such that $P(X \leq p) < 0.2$ $p = 9$

Worked example	Your turn
The random variable $X{\sim}B(40, 0.3)$. Find the largest value of r such that $P(X < r) < 0.1$	The random variable $X{\sim}B(30, 0.4)$. Find the largest value of r such that $P(X < r) < 0.15$ $r = 9$

Worked example	Your turn
The random variable $X \sim B(40, 0.3)$. Find the smallest value of s such that $P(X \geq s) < 0.15$	The random variable $X \sim B(30, 0.4)$. Find the smallest value of s such that $P(X \geq s) < 0.1$ $s = 16$

Worked example	Your turn
The random variable $X \sim B(40, 0.3)$. Find the smallest value of t such that $P(X > t) < 0.2$	The random variable $X \sim B(30, 0.4)$. Find the smallest value of t such that $P(X > t) < 0.05$ $t = 16$

Worked example	Your turn
Each day a person plays 10 games of chess. The probability that they win each game is 0.7. They consider it a successful day if they win at least 8 games. Calculate the probability that in a seven-day week, they have at least five successful days.	Each day a person plays 20 games of chess. The probability that they win each game is 0.6. They consider it a successful day if they win at least 13 games. Calculate the probability that in January they have at least sixteen successful days. 0.1708 (4 dp)