6) Circles

Worked example	Your turn
Find the midpoint of the line segment between: $$(2, 4) \text{ and } (8, 8)$$ $$(-2, 4) \text{ and } (-9, 9)$$	Find the midpoint of the line segment between: $$(2, -4) \text{ and } (11, 8)$$ $(6.5, 2)$

Worked example	Your turn
Find the perpendicular bisector of $A\ (2, 5)$ and $B\ (6, 7)$	Find the perpendicular bisector of $A\ (2, 5)$ and $B\ (6, 7)$ $$y = -2x + 14$$

Worked example	Your turn
Find the perpendicular bisector of A $(3, 8)$ and $B(1, -2)$	Find the perpendicular bisector of A $(3, 8)$ and $B(1, -4)$ $$y = -\frac{1}{2}x + 7$$

Worked example	Your turn
A line segment AB is the diameter of a circle with centre $(4, -5)$. If A has coordinates $(2, -1)$, what are the coordinates of B?	A line segment AB is the diameter of a circle with centre $(5, -4)$. If A has coordinates $(1, -2)$, what are the coordinates of B? <center>$(9, -6)$</center>

6.2) Equation of a circle

Worked example	Your turn
Write down the equation of the circle with centre $(6, 7)$ and radius 2	Write down the equation of the circle with centre $(4, 5)$ and radius 9
	$$(x - 4)^2 + (y - 5)^2 = 81$$
Write down the equation of the circle with centre $(-6, 7)$ and radius 3	Write down the equation of the circle with centre $(-4, 5)$ and radius 8
	$$(x + 4)^2 + (y - 5)^2 = 64$$
Write down the equation of the circle with centre $(-6, -7)$ and radius 4	Write down the equation of the circle with centre $(-4, -5)$ and radius 7
	$$(x + 4)^2 + (y + 5)^2 = 49$$
Write down the equation of the circle with centre $(6, -7)$ and radius 5	Write down the equation of the circle with centre $(4, -5)$ and radius 6
	$$(x - 4)^2 + (y + 5)^2 = 36$$

Worked example	Your turn
Write down the equation of the circle with centre $(0, 0)$ and radius 3	Write down the equation of the circle with centre $(0, 0)$ and radius 5 $$x^2 + y^2 = 25$$
Write down the equation of the circle with centre $(0, 3)$ and radius 2	Write down the equation of the circle with centre $(0, -5)$ and radius 4 $$x^2 + (y + 5)^2 = 16$$
Write down the equation of the circle with centre $(-2, 0)$ and radius $2\sqrt{3}$	Write down the equation of the circle with centre $(4, 0)$ and radius $4\sqrt{5}$ $$(x - 4)^2 + y^2 = 80$$

Worked example	Your turn
A circle has equation $(x + 5)^2 + (y - 2)^2 = 80$. a) Write down the centre and radius b) Show that the circle passes through $(-1, 10)$	A circle has equation $(x - 3)^2 + (y + 4)^2 = 20$. a) Write down the centre and radius b) Show that the circle passes through $(5, -8)$ a) Centre $(3, -4)$, radius $2\sqrt{5}$ b) Shown via substitution

Worked example	Your turn
A line segment AB is the diameter of a circle, where A and B have coordinates $(5,8)$ and $(-7,4)$ respectively. Determine the equation of the circle.	A line segment AB is the diameter of a circle, where A and B have coordinates $(4,7)$ and $(-8,3)$ respectively. Determine the equation of the circle.

$$(x+2)^2 + (y-5)^2 = 40$$

Worked example	Your turn
Find the centre and radius of the circle with equation $$x^2 + y^2 + 8x - 6y - 2 = 0$$	Find the centre and radius of the circle with equation $$x^2 + y^2 - 14x + 16y - 12 = 0$$ Centre $(7, -8)$, radius $5\sqrt{5}$

Worked example	Your turn
A circle has equation $x^2 + y^2 + 6x + 4y = k$, where k is a constant. State the range of possible values of k	A circle has equation $x^2 + y^2 + 10x + 8y = k$, where k is a constant. State the range of possible values of k $$k > -41$$

Worked example	Your turn
The circle with equation $x^2 + (y - k)^2 = 41$ passes through the point $(5, 6)$. Find the two possible values of k	The circle with equation $(x - k)^2 + y^2 = 45$ passes through the point $(4, 3)$. Find the two possible values of k $k = -2, k = 10$

6.3) Intersections of straight lines and circles

Worked example	Your turn
Find the coordinates of the points where the line $y = x + 4$ meets the circle $x^2 + (y-3)^2 = 61$	Find the coordinates of the points where the line $y = x + 5$ meets the circle $x^2 + (y-2)^2 = 29$
	$(-5, 0)$ and $(2, 7)$

Worked example	Your turn
Show that the line $y = x + 4$ does not meet the circle $(x - 3)^2 + y^2 = 22$	Show that the line $y = x - 7$ does not meet the circle $(x + 2)^2 + y^2 = 33$ Shown using discriminant.

Worked example

The line with equation $y = kx$ intersects the circle with equation $x^2 - 8x + y^2 + 12y + 32 = 0$ at two distinct points.

Find the range of possible values of k to 2 dp

Your turn

The line with equation $y = kx$ intersects the circle with equation $x^2 - 4x + y^2 + 10y + 23 = 0$ at two distinct points.

Find the range of possible values of k to 2 dp

$$k = -0.87, k = 10.87 \text{ (2 dp)}$$

Worked example	Your turn
Using an algebraic method, determine k such that the line $y = x + k$ touches the circle with equation $x^2 + y^2 = 9$	Using an algebraic method, determine k such that the line $y = x + k$ touches the circle with equation $x^2 + y^2 = 1$
	$k = \pm\sqrt{2}$

Worked example	Your turn
The line with equation $y = 5x + 2$ meets the circle with equation $x^2 + kx + y^2 = 6$ at exactly one point. Find the two possible values of k	The line with equation $y = 4x + 3$ meets the circle with equation $x^2 + kx + y^2 = 7$ at exactly one point. Find the two possible values of k $$k = -24 \pm 4\sqrt{6}$$

Worked example

The line with equation $y = 4x - 3$ does not intersect the circle with equation $x^2 + 2x + y^2 = k$.

Find the range of possible values of k.

Your turn

The line with equation $y = 3x - 2$ does not intersect the circle with equation $x^2 + 4x + y^2 = k$.

Find the range of possible values of k.

$$k < \frac{12}{5}$$

6.4) Use tangent and chord properties

Worked example	Your turn
Find the equation of the tangent to $x^2 + y^2 = 25$ at the point $(3, 4)$	Find the equation of the tangent to $x^2 + y^2 = 25$ at the point $(4, 3)$ $$y = -\frac{4}{3}x + \frac{25}{3}$$

Worked example	Your turn
Find the equation of the tangent to $x^2 + y^2 = 169$ at the point $(5, 12)$	Find the equation of the tangent to $x^2 + y^2 = 169$ at the point $(-5, 12)$ $$y = \frac{5}{12}x + \frac{169}{12}$$

Worked example	Your turn
The circle C has equation $$(x-3)^2 + (y-7)^2 = 100$$ a) Verify the point $P(11,1)$ lies on C. b) Find an equation of the tangent to C at the point P, giving your answer in the form $ax + by + c = 0$	The circle C has equation $$(x-2)^2 + (y-6)^2 = 100$$ a) Verify the point $P(10,0)$ lies on C. b) Find an equation of the tangent to C at the point P, giving your answer in the form $ax + by + c = 0$ a) Verified using substitution b) $4x - 3y = 40 = 0$

| Worked example	Your turn
A circle C has equation $$(x - 4)^2 + (y + 4)^2 = 10$$ The line l is a tangent to the circle and has gradient -3. Find two possible equations for l, giving your answers in the form $y = mx + c$.	A circle C has equation $$(x - 5)^2 + (y + 3)^2 = 10$$ The line l is a tangent to the circle and has gradient -3. Find two possible equations for l, giving your answers in the form $y = mx + c$. $y = -3x + 2$ and $y = -3x + 22$

Worked example

The point P has coordinates $(-8, -2)$ and the point Q has coordinates $(2, -6)$.

M is the midpoint of the line segment PQ

a) Find an equation for l.

b) Given that the y-coordinate of C is -9:

   i) show that the x-coordinate of C is -5.

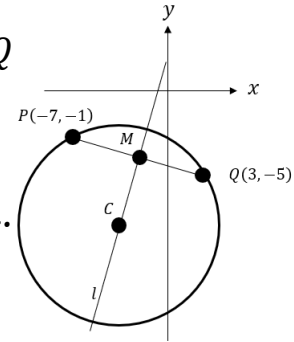   ii) find an equation of the circle.
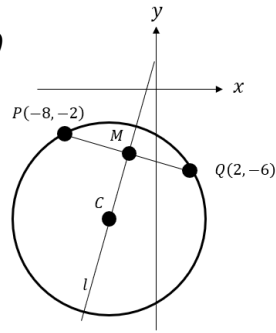
Your turn

The point P has coordinates $(-7, -1)$ and the point Q has coordinates $(3, -5)$.

M is the midpoint of the line segment PQ

a) Find an equation for l.

b) Given that the y-coordinate of C is -8:

   i) show that the x-coordinate of C is -4.

   ii) find an equation of the circle.

a) $y = \dfrac{5}{2}x + 2$

b) i) Shown

   ii) $(x + 4)^2 + (y + 8)^2 = 58$

Worked example	Your turn
The line with equation $4x + y - 5 = 0$ is a tangent to the circle with equation $(x - 3)^2 + (y - p)^2 = 2$. Find the two possible values of p	The line with equation $4x + y - 3 = 0$ is a tangent to the circle with equation $(x - 2)^2 + (y - p)^2 = 5$. Find the two possible values of p $$p = 3 \pm \sqrt{19}$$

Worked example	Your turn
A circle has centre $C(5,3)$, and passes through the point $P(2,6)$. Find the equation of the tangent of the circle at the point P, giving your equation in the form $ax + by + c = 0$ where a, b, c are integers..	A circle has centre $C(3,5)$, and passes through the point $P(6,9)$. Find the equation of the tangent of the circle at the point P, giving your equation in the form $ax + by + c = 0$ where a, b, c are integers.. $$3x + 4y - 54 = 0$$

Worked example	Your turn
A circle passes through the points $A(0,0)$ and $B(2,8)$. The centre of the circle has x value -2. Determine the equation of the circle.	A circle passes through the points $A(0,0)$ and $B(4,2)$. The centre of the circle has x value -1. Determine the equation of the circle. $$(x+1)^2 + (y-7)^2 = 50$$

6.5) Circles and triangles

Worked example	Your turn
The points $A(-1,8), B(-5,-4), C(-9,4)$ lie on a circle. a) Show that AB is a diameter of the circle. b) Hence find the equation of the circle	The points $A(-8,1), B(4,5), C(-4,9)$ lie on a circle. a) Show that AB is a diameter of the circle. b) Hence find the equation of the circle a) Shown b) $(x+2)^2 + (y-3)^2 = 40$

Worked example	Your turn
The points $A(0,2), B(2,0), C(8,18)$ lie on the circumference of a circle. Determine the equation of the circle.	The points $A(3,16), B(11,12), C(-7,6)$ lie on the circumference of a circle. Determine the equation of the circle. $(x-3)^2 + (y-6)^2 = 100$