Tangents and Normals

Remember how you found the gradient given equations in parametric form.

$$x = \cos t\,,\ \ y = \sin t$$

$$\frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} = \frac{\cos t}{-\sin t} = -\cot t$$

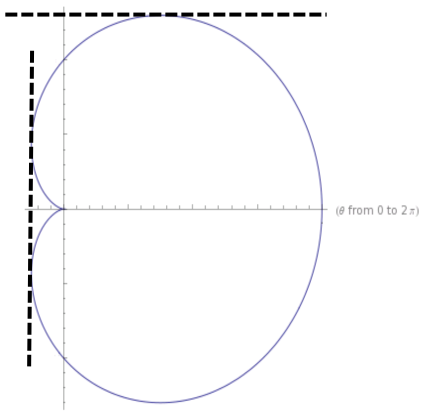
In the same way for polar coordinates:

$$x = r\cos\theta \qquad y = r\sin\theta$$

$$\frac{dy}{dx} = \frac{\left(\frac{dy}{d\theta}\right)}{\left(\frac{dx}{d\theta}\right)}$$

We can find the gradient at any point by differentiating parametrically.

- If $\frac{dy}{d\theta} = 0$ **the tangent is parallel to the initial line**
- If $\frac{dx}{d\theta} = 0$ **the tangent is perpendicular to the initial line.**



Eaxmple

Find the coordinates of the points on $r = a(1 + \cos\theta)$ where the tangents are parallel to the initial line $\theta = 0$.

Test Your Understanding

The curve C has polar equation

$$r = 1 + 2\cos\theta\,, \quad 0 \leq \theta \leq \frac{\pi}{2}$$

At the point P on C, the tangent to C is parallel to the initial line.

Given that O is the pole, find the exact length of the line OP.

Example

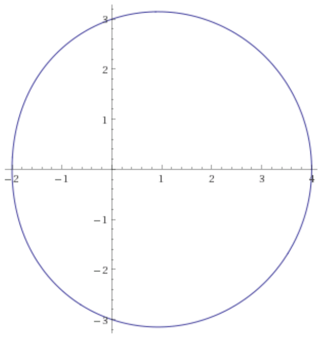
Find the equations and the points of contact of the tangents to the curve

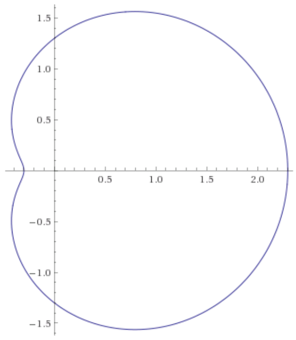
$$r = a\sin 2\theta,\ 0 \leq \theta \leq \frac{\pi}{2}$$

that are (a) parallel to the initial line and (b) perpendicular to the initial line.

Proof of dimple vs egg

Prove that for $r = p + q\cos\theta$ we have a 'dimple' if $p < 2q$.





Ex 5D pg 115