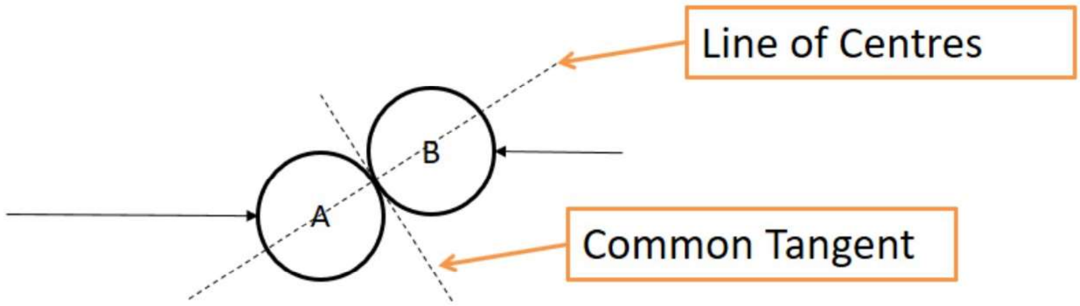
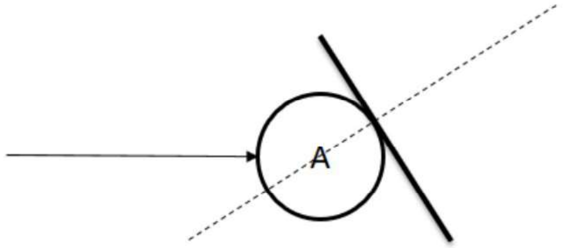
Oblique Impact of Smooth Spheres





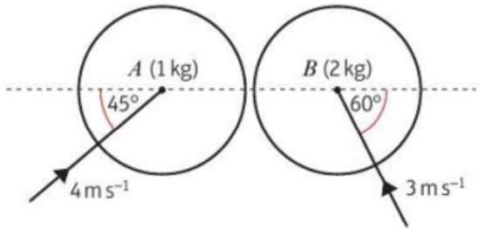
Key idea 1:
As with the smooth wall, the *component of velocity parallel to the common tangent* doesn't change.

Key idea 2:
The *components of velocities parallel to the line of centres* are treated exactly like they were in Chapter 4 :
- Use PCLM
- Consider NLR

Example 7

A smooth sphere A, of mass 2 kg and moving with speed $6\,\text{m}\,\text{s}^{-1}$ collides obliquely with a smooth sphere B of mass 4 kg. Just before the impact B is stationary and the velocity of A makes an angle of 60° with the lines of centres of the two spheres. The coefficient of restitution between the spheres is $\frac{1}{4}$. Find the magnitudes and directions of the velocities of A and B immediately after the impact.

Example 8



A small smooth sphere A of mass 1 kg collides with a small smooth sphere B of mass 2 kg. Just before the impact A is moving with a speed of $4\,\text{m s}^{-1}$ in a direction at 45° to the line of centres and B is moving with speed $3\,\text{m s}^{-1}$ at 60° to the line of centres, as shown in the diagram. The coefficient of restitution between the spheres is $\frac{3}{4}$. Find:

a the kinetic energy lost in the impact

b the magnitude of the impulse exerted by A on B.

Example 9

A smooth sphere A of mass 5 kg is moving on a smooth horizontal surface with velocity $(2\mathbf{i} + 3\mathbf{j})\,\text{m s}^{-1}$. Another smooth sphere B of mass 3 kg and the same radius as A is moving on the same surface with velocity $(4\mathbf{i} - 2\mathbf{j})\,\text{m s}^{-1}$. The spheres collide when their line of centres is parallel to $\mathbf{j}$. The coefficient of restitution between the spheres is $\frac{3}{5}$. Find the velocities of both spheres after the impact.

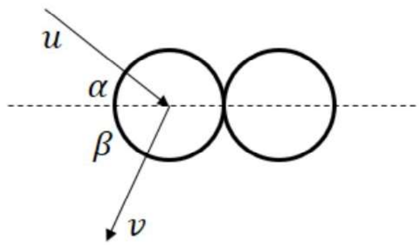
Example 10

Two small smooth spheres A and B have equal radii. The mass of A is $2m$ kg and the mass of B is $3m$ kg. The spheres are moving on a smooth horizontal plane and they collide. Immediately before the collision the velocity of A is $5\mathbf{j}$ m s^{-1} and the velocity of B is $(3\mathbf{i} - \mathbf{j})$ m s^{-1}. Immediately after the collision the velocity of A is $(3\mathbf{i} + 2\mathbf{j})$ m s^{-1}. Find:

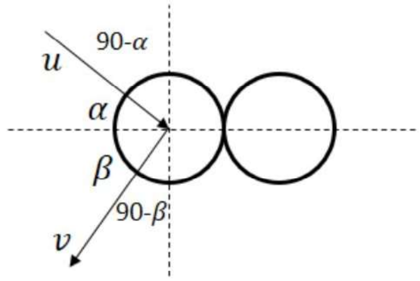
a the speed of B immediately after the collision

b a unit vector parallel to the line of centres of the spheres at the instant of the collision.

Angle of deflection – a common source of errors

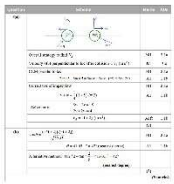


The angle of deflection is NOT $\alpha + \beta$.



The angle of deflection IS $(90 - \alpha) + (90 - \beta) = \mathbf{180 - \alpha - \beta}$.

6. [In this question **i** and **j** are perpendicular unit vectors in a horizontal plane.]



A smooth uniform sphere A has mass $2m$ kg and another smooth uniform sphere B, with the same radius as A, has mass $3m$ kg.

The spheres are moving on a smooth horizontal plane when they collide obliquely.

Immediately before the collision the velocity of A is $(3\mathbf{i} + 3\mathbf{j})\,\text{m s}^{-1}$ and the velocity of B is $(-5\mathbf{i} + 2\mathbf{j})\,\text{m s}^{-1}$.

At the instant of collision, the line joining the centres of the spheres is parallel to **i**.

The coefficient of restitution between the spheres is $\frac{1}{4}$

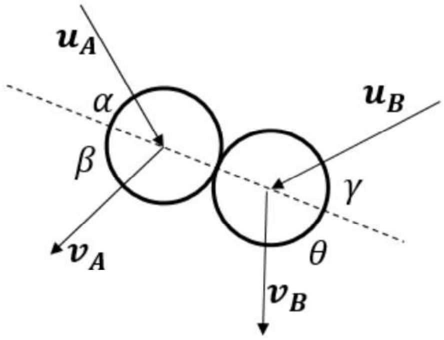
(a) Find the velocity of B immediately after the collision.

(7)

(b) Find, to the nearest degree, the size of the angle through which the direction of motion of B is deflected as a result of the collision.

(2)

Using the Scalar Product -
when the line of centres is not in the 'i' or 'j' directions



✏ $-e(\boldsymbol{u_A} - \boldsymbol{u_B}).\boldsymbol{I} = (\boldsymbol{v_A} - \boldsymbol{v_B}).\boldsymbol{I}$

Note: Like the similar method introduced earlier for balls and walls, this is not explicitly covered in the textbook but is a good way to simplify some questions.
Note: There is only one qu in the textbook where this method is helpful: Ex 5C qu 14 but the answer is an impossible $e = -\frac{1}{7}$

Two small smooth spheres A and B have equal radii. The mass of A is $2m$kg and the mass of B is $20m$kg. The spheres are moving on a smooth horizontal plane and they collide. Immediately before the collision the velocity of A is $(2\boldsymbol{i} + \boldsymbol{j})ms^{-1}$ and B is stationary. Immediately after the collision the velocity of A is $2\boldsymbol{j}\ ms^{-1}$. Find:

a) The velocity of B after the collision
b) The coefficient of restitution between the two spheres

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A smooth uniform sphere S, of mass m, is moving on a smooth horizontal plane when it collides obliquely with another smooth uniform sphere T, of the same radius as S but of mass $2m$, which is at rest on the plane. Immediately before the collision the velocity of S makes an angle α, where $\tan \alpha = \frac{3}{4}$, with the line joining the centres of the spheres.

Immediately after the collision the speed of T is V. The coefficient of restitution between the two spheres is $\frac{3}{4}$.

(a) Find, in terms of V, the **speed** of S

(i) immediately before the collision,

(ii) immediately after the collision.

(9)

(b) Find the angle through which the direction of motion of S is deflected as a result of the collision.

(4)

Challenging Questions:
Review Exercise 2 qu 31, 36, 38
(Review Exercise 2 qu 19 – 39 are all good questions but these three combine several skills in an unusual way)