**11** A small smooth ball of mass 2 kg is moving in the $xy$-plane and collides with a smooth fixed vertical wall which contains the line $y = x$. The velocity of the ball just before impact is $(4\mathbf{i} + 2\mathbf{j})\text{ m s}^{-1}$. The coefficient of restitution between the sphere and the wall is $\frac{1}{3}$. Find:

**a** the velocity of the ball immediately after the impact

**b** the proportion of the original kinetic energy lost as a result of the impact

**c** the angle of deflection of the ball.

# Successive oblique impacts

Sometimes you may be asked to consider two successive impacts.
1)Find the speed and direction of motion after the first impact.
2)Use angle properties to calculate the angle of approach for the second collision using the direction of motion after the first impact.
3)Look at the second impact starting by drawing a new diagram.

There are no new concepts needed to address this type of question.

**Example 4**

Two vertical walls meet at right angles. A smooth sphere slides across a smooth, horizontal floor, bouncing off each wall in turn. Just before the first impact the sphere is moving with speed $4\text{ m s}^{-1}$ at an angle of 30° to the wall. The coefficient of restitution between the sphere and both walls is $\frac{3}{4}$
Find:

**a** the direction of motion and speed of the sphere after the first collision

**b** the direction of motion and speed of the sphere after the second collision.

## Example 5

Two cushions of a snooker table $W_1$ and $W_2$ meet at right angles. A snooker ball travels across the table and collides with $W_1$ and then $W_2$. The cushions are modelled as smooth.
Just before the first impact the ball is moving with speed $u\,\text{m s}^{-1}$ at an angle of 20° to $W_1$.
The coefficients of restitution between the ball and the cushions $W_1$ and $W_2$ are $\frac{1}{2}$ and $\frac{2}{5}$ respectively.

**a** Find the percentage of the ball's original kinetic energy that is lost in the collisions.

**b** In reality the cushions may not be smooth. What effect will the model have had on the calculation of the percentage of kinetic energy remaining?

## Example 6

Two smooth vertical walls stand on a smooth horizontal surface and intersect at an angle of 60°. A smooth sphere is projected across the surface with speed $1\text{ m s}^{-1}$ at an angle of 20° to one of the walls and towards the intersection of the walls. The coefficient of restitution between the sphere and the walls is 0.4. Work out the speed and direction of motion of the sphere after:

**a** the first collision **b** the second collision

Ex 5B odd

## Edexcel M4 June 2015 Q7a

Figure 4 represents the plan view of part of a smooth horizontal floor, where AB and BC are smooth vertical walls. The angle between AB and BC is 120°. A ball is projected along the floor towards AB with speed u m s–1 on a path at an angle of 60° to AB. The ball hits AB and then hits BC. The ball is modelled as a particle. The coefficient of restitution between the ball and each wall is $\frac{1}{2}$.

(a) Show that the speed of the ball immediately after it has hit AB is $\frac{\sqrt{7}}{4}u$. (6)

The speed of the ball immediately after it has hit *BC* is *w* m s$^{-1}$.

(*b*) Find *w* in terms of *u*. (7)

| |
|---|
| Angle of approach with $BC = 19.1°$ |
| $v\cos 19.1 = w\cos\phi$ |
| $\frac{1}{2}v\sin 19.1 = w\sin\phi$ |
| |
| |
| Form equation in $v$ and $\phi$ |
| $w^2 = v^2\left(\frac{1}{4}\sin^2 19.1 + \cos^2 19.1\right)$ |
| $0.634u$ |

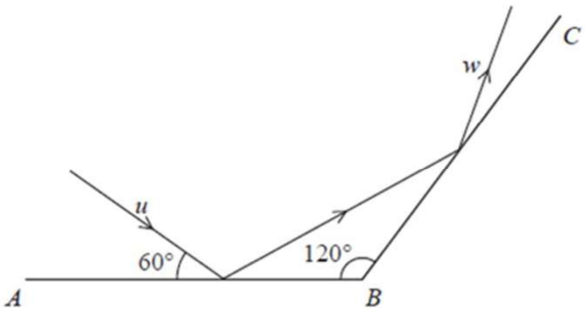


**Figure 4**

**4.**

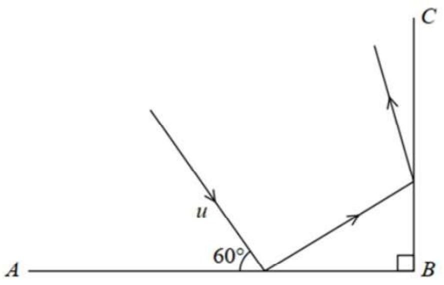


**Figure 1**

Figure 1 represents the plan view of part of a horizontal floor, where $AB$ and $BC$ are perpendicular vertical walls.

The floor and the walls are modelled as smooth.

A ball is projected along the floor towards $AB$ with speed $u\,\text{m}\,\text{s}^{-1}$ on a path at an angle of $60°$ to $AB$. The ball hits $AB$ and then hits $BC$.

The ball is modelled as a particle.

The coefficient of restitution between the ball and wall $AB$ is $\frac{1}{\sqrt{3}}$

The coefficient of restitution between the ball and wall $BC$ is $\sqrt{\frac{2}{5}}$

(a) Show that, using this model, the final kinetic energy of the ball is 35% of the initial kinetic energy of the ball.

**(8)**

(b) In reality the floor and the walls may not be smooth. What effect will the model have had on the calculation of the percentage of kinetic energy remaining?

**(1)**

**5.** [*In this question* **i** *and* **j** *are perpendicular unit vectors in a horizontal plane*]

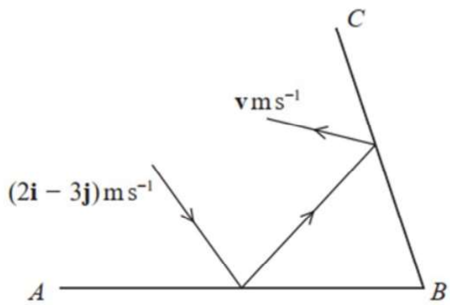


**Figure 3**

Figure 3 represents the plan view of part of a horizontal floor, where *AB* and *BC* represent fixed vertical walls. The direction of $\overrightarrow{AB}$ is in the direction of the vector **i** and the direction of $\overrightarrow{BC}$ is in the direction of the vector $(-\mathbf{i} + 3\mathbf{j})$.

A small ball is projected along the floor towards wall *AB* so that, immediately before hitting wall *AB*, the velocity of the ball is $(2\mathbf{i} - 3\mathbf{j})\,\text{m s}^{-1}$.

The ball hits wall *AB* and then hits wall *BC*.

The coefficient of restitution between the ball and wall *AB* is $\frac{1}{2}$

The coefficient of restitution between the ball and wall *BC* is $\frac{1}{3}$

The velocity of the ball immediately after hitting wall *BC* is $\mathbf{v}\,\text{m s}^{-1}$.

The floor and the walls are modelled as being smooth. The ball is modelled as a particle.

Show that $\mathbf{v} = \left(-\mathbf{i} + \frac{1}{2}\mathbf{j}\right)$.

**(12)**