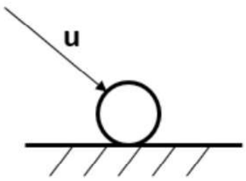
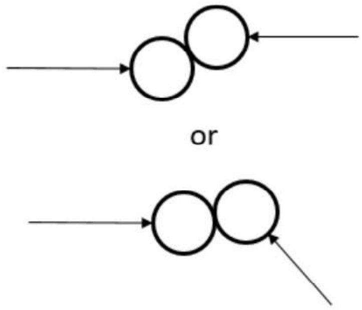
# Elastic Collisions in Two Dimensions (Chapter 5)

## Oblique impact with a fixed smooth surface

In this chapter we will consider:




What happens when a sphere hits a wall at an angle other than 90°?

What happens when two spheres that are not travelling along the same straight line collide?

The key to all the questions in this chapter is that **all** the spheres and **all** the surfaces are **ALWAYS SMOOTH.**

A smooth surface cannot apply a frictional force.
It can only apply a normal force.

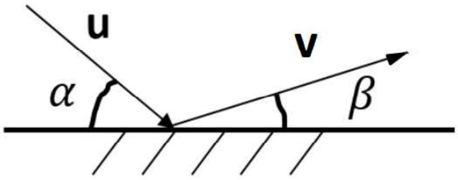
The impulse is always normal to the surface, so momentum (and velocity) is **only changed perpendicular to the surface**.

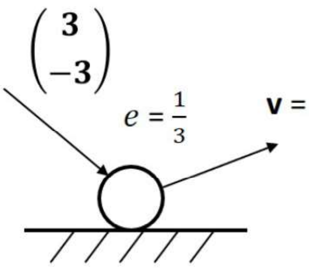
This means momentum (and velocity) **parallel to the surface remains unchanged**.

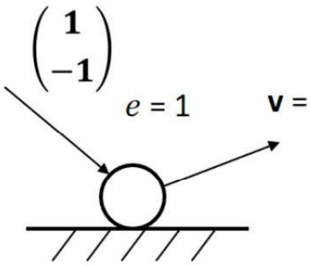
✏ The component of velocity **parallel** to the surfaces in contact is **unchanged**
✏ The component of velocity **perpendicular** to the surfaces in contact depends on **the coefficient of restitution (e).**

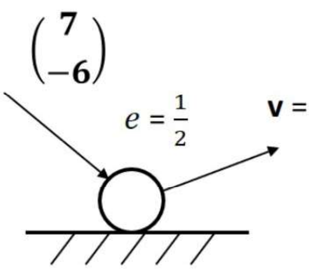
## The theory

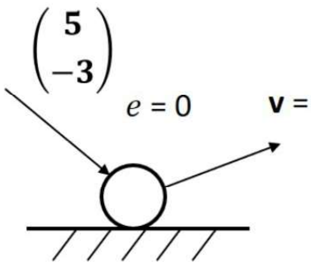
The angle of deflection is the total angle through which the path of the sphere changes.
With this diagram it is α + β



## Example 1

A smooth sphere $S$ is moving on a smooth horizontal plane with speed $u$ when it collides with a smooth fixed vertical wall. At the instant of collision the direction of motion of $S$ makes an angle of 60° with the wall. The coefficient of restitution between $S$ and the wall is $\frac{1}{4}$. Find:

**a** the speed of $S$ immediately after the collision

**b** the angle of deflection of $S$. *This does not depend on the initial speed. Let us check why!*

Tip: Many questions involve two equations and two unknowns which can be found in one step by:
- $(eqn\ 1)^2 + (eqn\ 2)^2$
- eqn 1 ÷ eqn 2

Ex 5A Q1-4

## Example 2

A small smooth ball is falling vertically. The ball strikes a smooth plane which is inclined at an angle $\alpha$ to the horizontal, where $\tan \alpha = \frac{1}{2}$. Immediately before striking the plane the ball has speed $5\text{ m s}^{-1}$. The coefficient of restitution between the ball and the plane is $\frac{1}{2}$

Find the speed of the ball immediately after the impact.

Ex 5A Q5-8

**Example 3**

A small smooth ball of mass 2 kg is moving in the $xy$-plane and collides with a smooth fixed vertical wall which contains the $y$-axis. The velocity of the ball just before impact is $(-6\mathbf{i} - 4\mathbf{j})\ \mathrm{m\,s^{-1}}$. The coefficient of restitution between the sphere and the wall is $\frac{1}{3}$. Find:

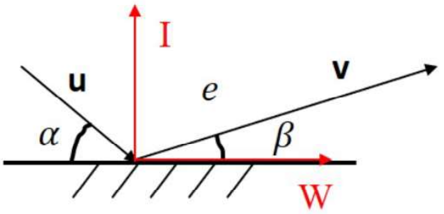
**a** the velocity of the ball immediately after the impact

**b** the kinetic energy lost as a result of the impact

**c** the angle of deflection of the ball. *What knowledge can we use from vectors to help us?*

Ex 5A Q9-10

# Using the Scalar Product - when the wall is not in the 'i' or 'j' directions




Let's explore u.w and u.I

$-e\boldsymbol{u}.\boldsymbol{I} = \boldsymbol{v}.\boldsymbol{I}$
$\boldsymbol{u}.\boldsymbol{w} = \boldsymbol{v}.\boldsymbol{w}$

A smooth sphere S , of mass m, is moving with velocity $(2\boldsymbol{i} + 7\boldsymbol{j})ms^{-1}$ when it collides with a smooth fixed vertical wall.
After the collision the velocity of the sphere, S, is $(\boldsymbol{i} - 3\boldsymbol{j})ms^{-1}$

a) Find the impulse exerted by the wall on the ball.
b) Use the scalar product to find the coefficient of restitution between the sphere and the wall.

Q15, 16
Q11 - you'll need to describe the wall in vector form, and then find out the impulse. This one is harder! Ask if you need me to go through it

**5.** A small ball of mass 0.5 kg is moving on a smooth horizontal plane with velocity $(4\mathbf{i} - \mathbf{j})\,\mathrm{m\,s^{-1}}$ when it strikes a fixed vertical wall. Immediately after the impact the velocity of the ball is $(2\mathbf{i} + 3\mathbf{j})\,\mathrm{m\,s^{-1}}$. The ball is modelled as a particle and the wall is modelled as a smooth plane surface.

(a) Find the magnitude of the impulse of the wall on the ball in the impact.

**(4)**

(b) Find the loss in kinetic energy of the ball due to its impact with the wall.

**(3)**

(c) Find the coefficient of restitution between the ball and the wall.

**(5)**

(d) Verify that the component of the momentum of the ball, parallel to the line of intersection of the wall and the horizontal plane, is unchanged by the impact.

**(2)**

(e) State which modelling assumption ensures that the component of the momentum of the ball, parallel to the line of intersection of the wall and the horizontal plane, is unchanged by the impact.

**(1)**

**11** A small smooth ball of mass 2 kg is moving in the $xy$-plane and collides with a smooth fixed vertical wall which contains the line $y = x$. The velocity of the ball just before impact is $(4\mathbf{i} + 2\mathbf{j})\ \mathrm{m\,s^{-1}}$. The coefficient of restitution between the sphere and the wall is $\frac{1}{3}$. Find:

**a** the velocity of the ball immediately after the impact

**b** the proportion of the original kinetic energy lost as a result of the impact

**c** the angle of deflection of the ball.

# Successive oblique impacts

Sometimes you may be asked to consider two successive impacts.
1)Find the speed and direction of motion after the first impact.
2)Use angle properties to calculate the angle of approach for the second collision using the direction of motion after the first impact.
3)Look at the second impact starting by drawing a new diagram.

There are no new concepts needed to address this type of question.

**Example 4**

Two vertical walls meet at right angles. A smooth sphere slides across a smooth, horizontal floor, bouncing off each wall in turn. Just before the first impact the sphere is moving with speed $4\ \mathrm{m\,s^{-1}}$ at an angle of 30° to the wall. The coefficient of restitution between the sphere and both walls is $\frac{3}{4}$
Find:

**a** the direction of motion and speed of the sphere after the first collision

**b** the direction of motion and speed of the sphere after the second collision.