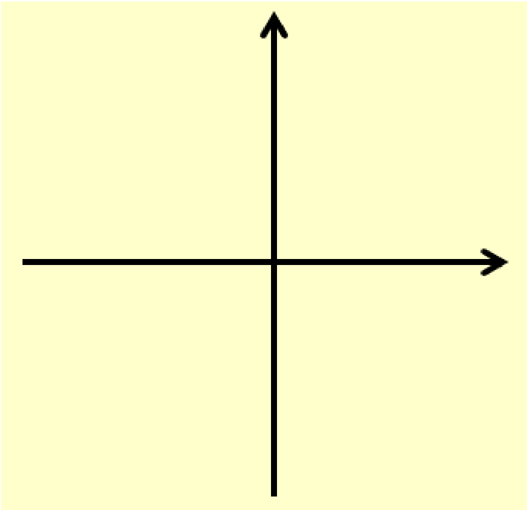
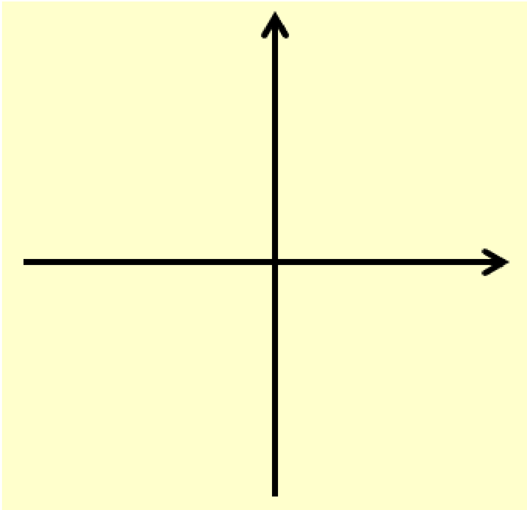
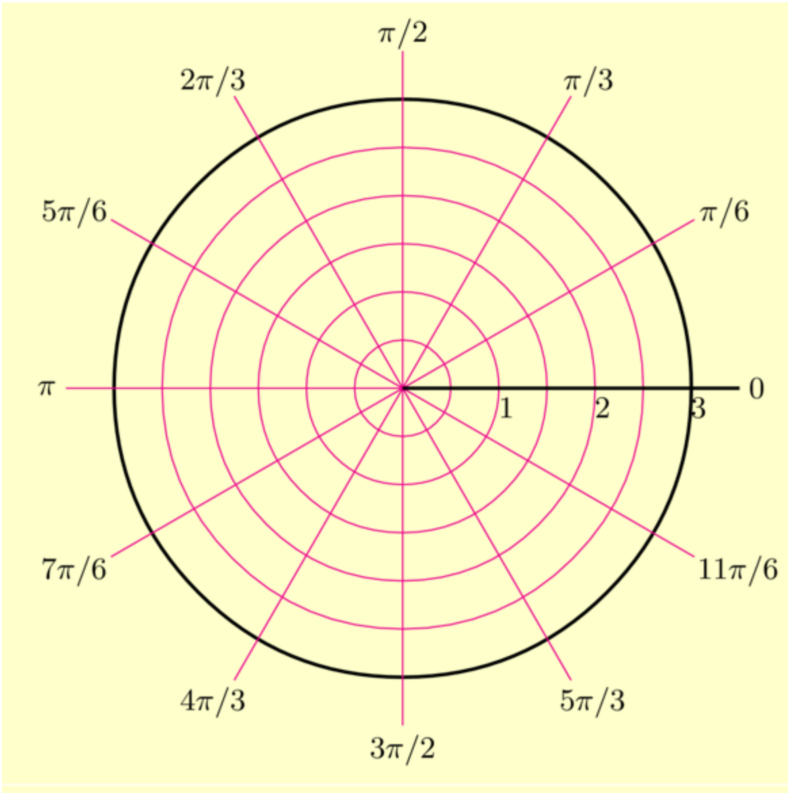
5A Polar Coordinates and Equations



Cartesian



Polar



1. Find the Polar coordinates of the following point:

a) (5,9)

b) (5, -12)

c) $(-\sqrt{3}, -1)$

2. Convert the following Polar coordinate into Cartesian form.

a) $\left(10,\ \frac{4\pi}{3}\right)$

b) $\left(8,\ \frac{2\pi}{3}\right)$

3. Find a Cartesian equation of the following curve:

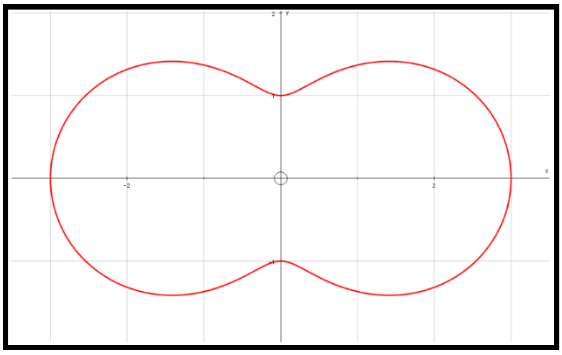
a) $r = 5$

b) $r = 6cosec\theta$

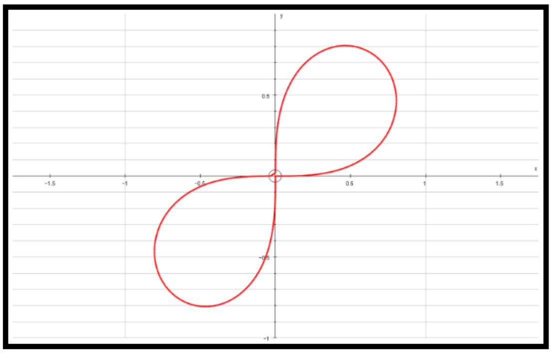
c) $r = 2 + cos2\theta$

d) $r^2 = Sin2\theta,\ 0 < \theta < \frac{\pi}{2}$

$(x^2 + y^2)^{\frac{3}{2}} = 3x^2 + y^2$



$(x^2 + y^2)^2 = 2xy$



4. Find a Polar equivalent for the following Cartesian equation:

a) $y^2 = 4x$

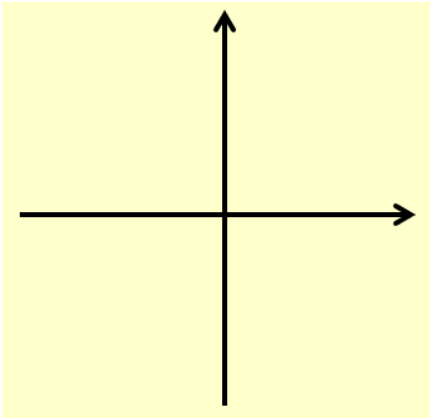
b) $x^2 - y^2 = 5$

c) $y\sqrt{3} = x + 4$

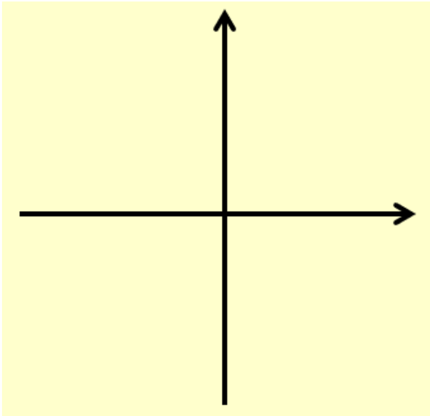
5B Polar Graphs

1. Sketch the Polar equation:

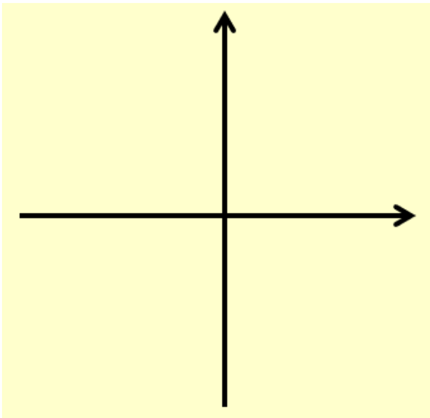
a) $r = a$



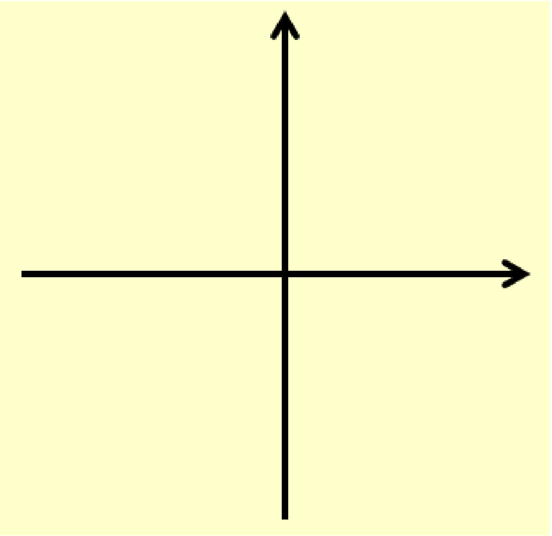
b) $\theta = a$



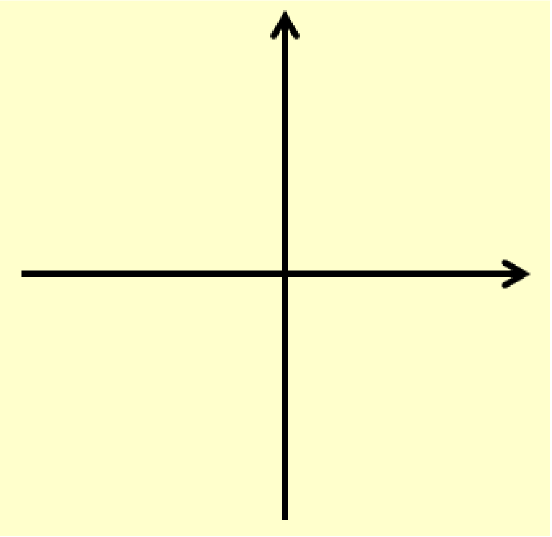
c) $r = a\theta$



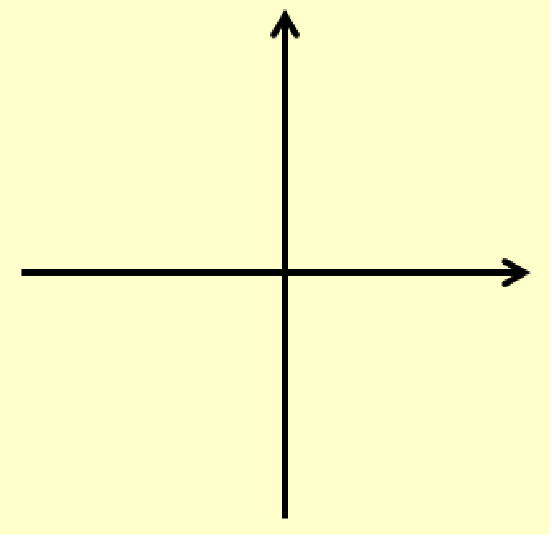
d) $r = a(1 + cos\theta)$



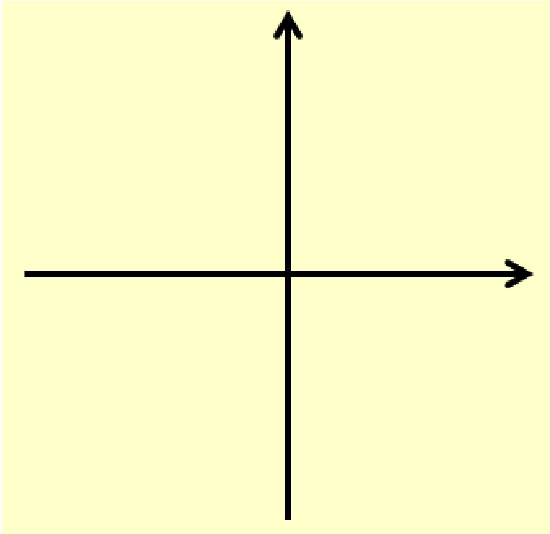
e) $r = asec\theta$



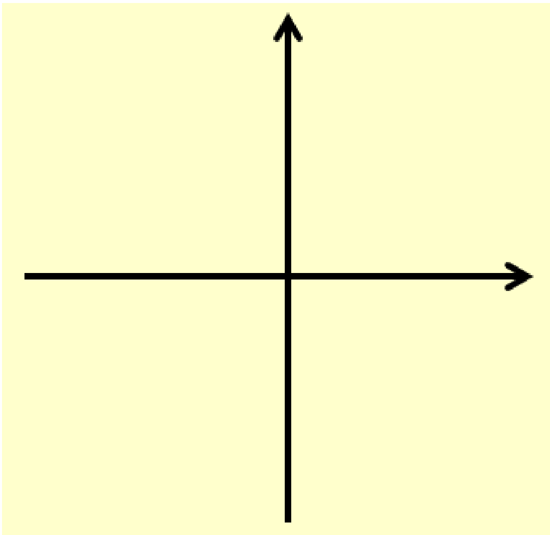
f) $r = sin3\theta$



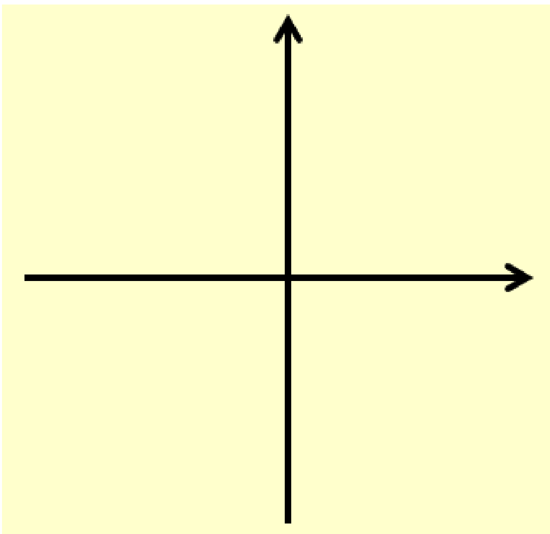
g) $r^2 = a^2 cos2\theta$



h) $r = a(5 + 2cos\theta)$



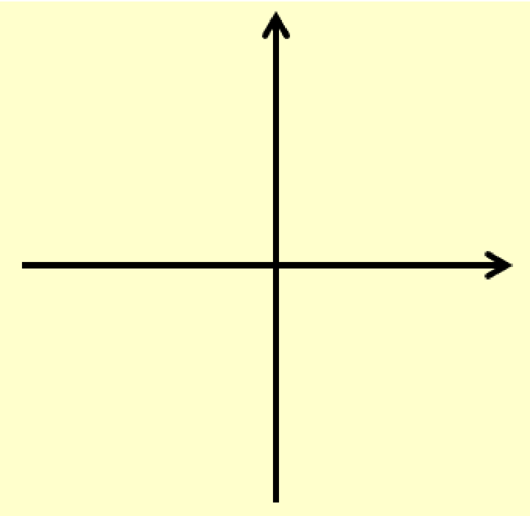
i) $r = a(3 + 2cos\theta)$



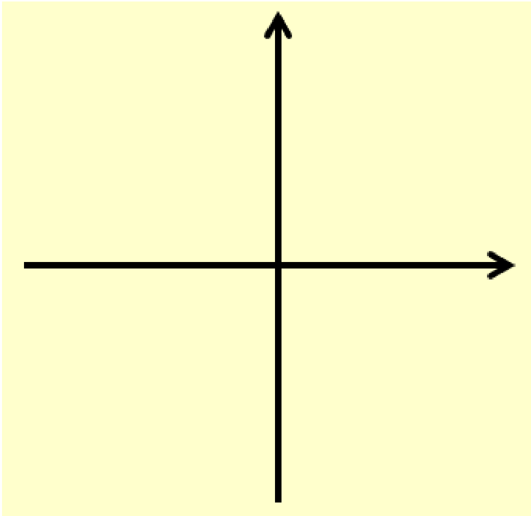
In General

$$r = a(p + q\cos\theta)$$

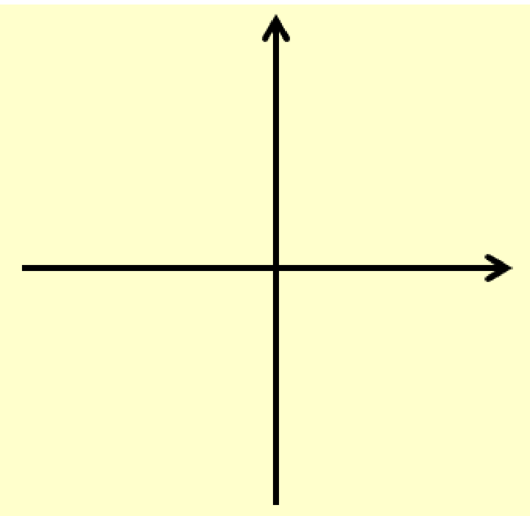
p < q



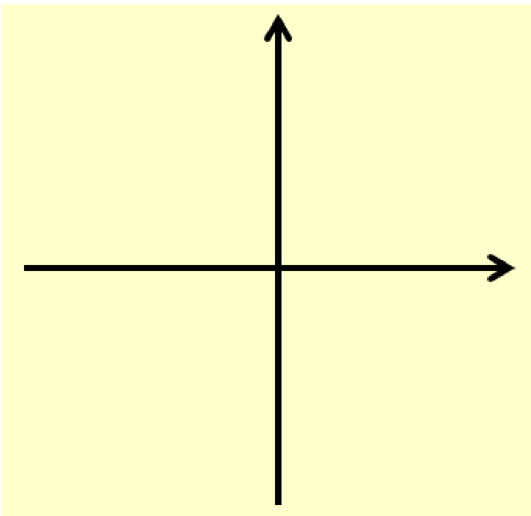
q ≤ p < 2q



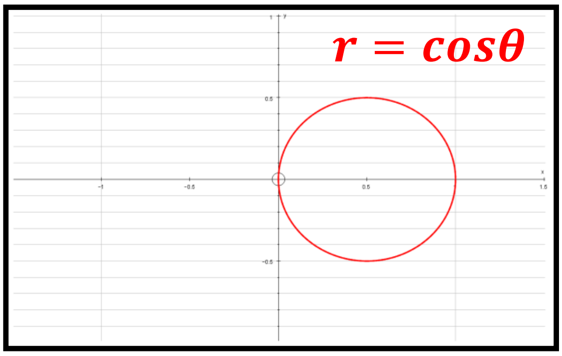
p = q

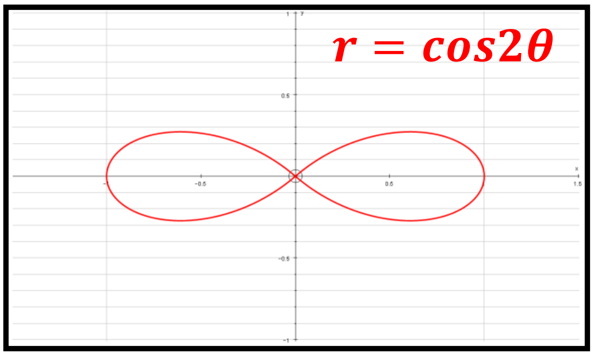


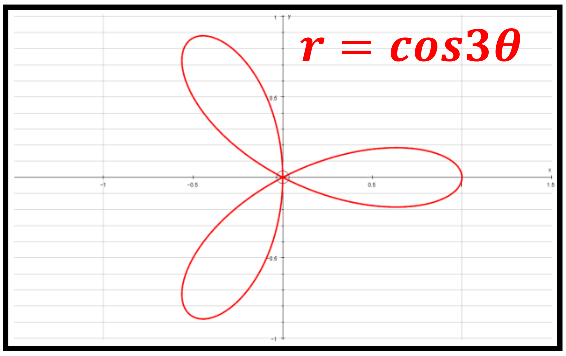
p ≥ 2q

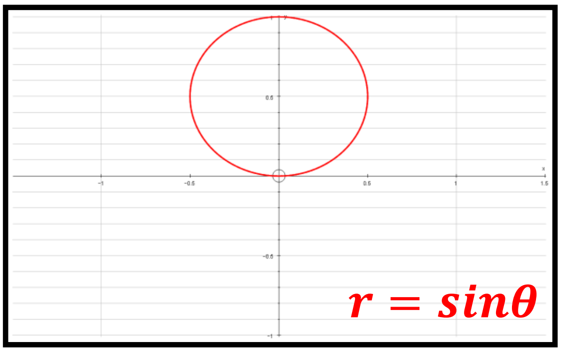


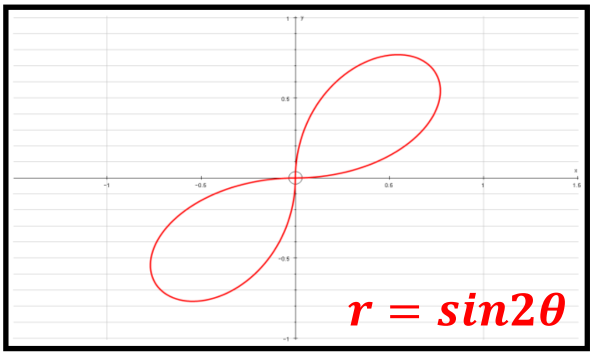
Some graphs to recognise:

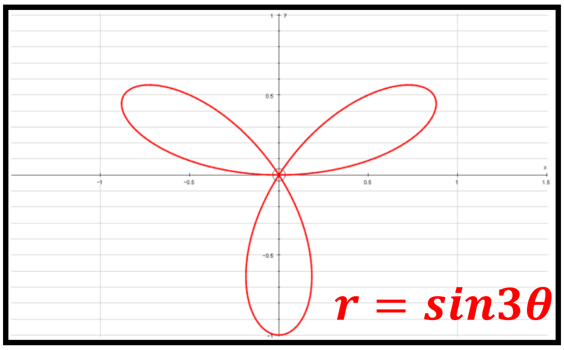


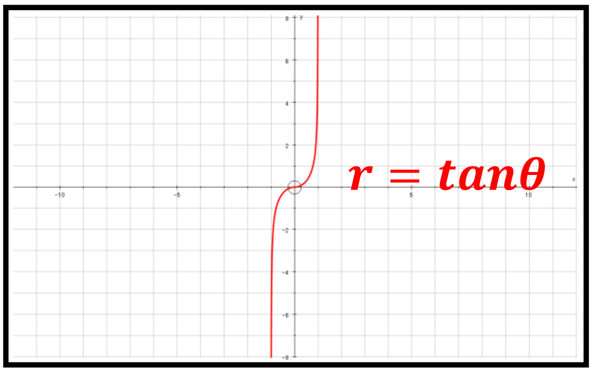


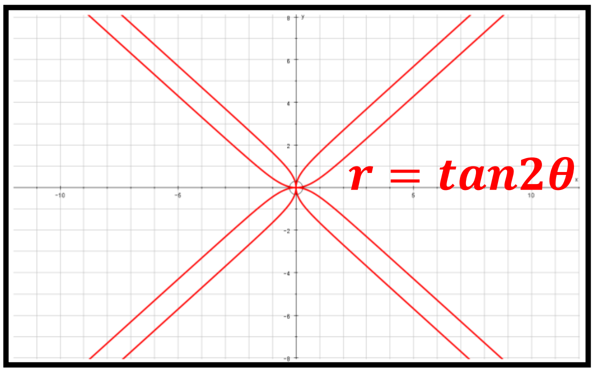


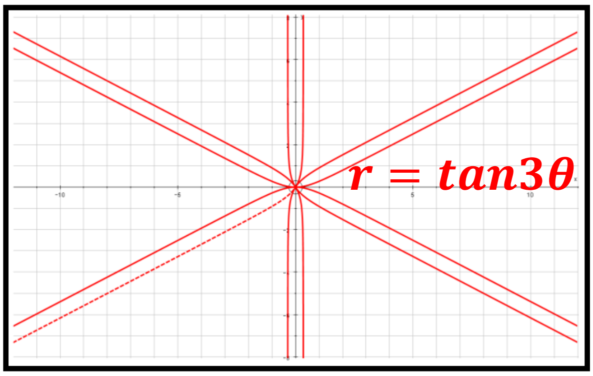


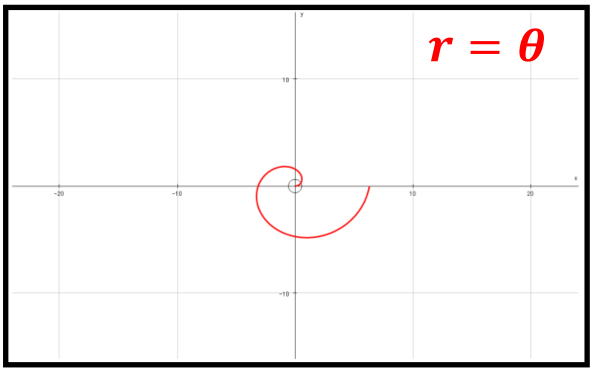


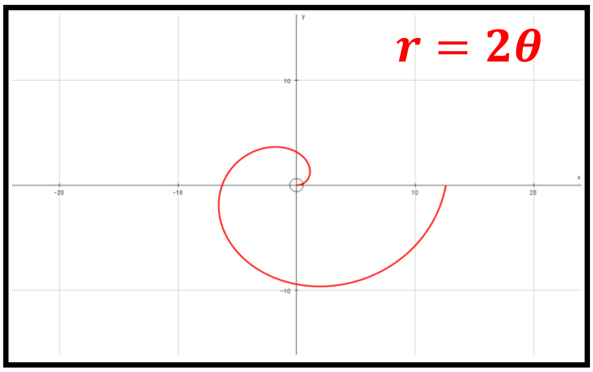


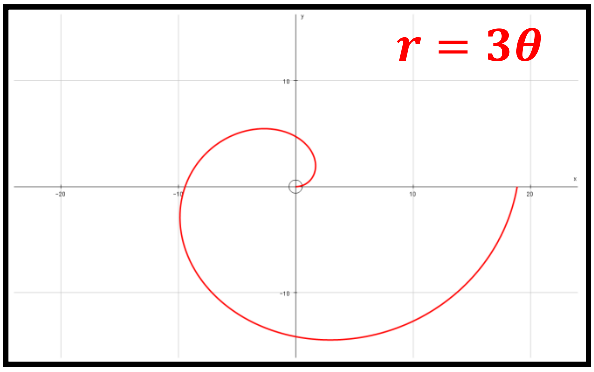


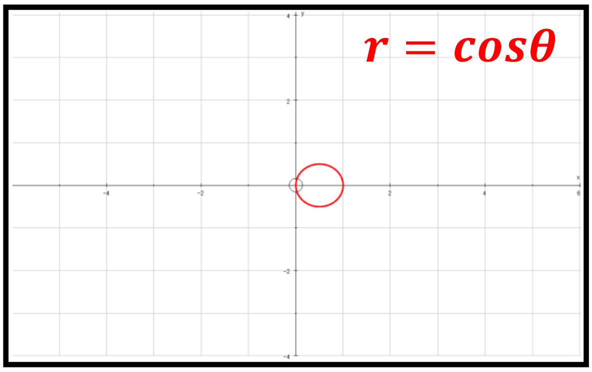


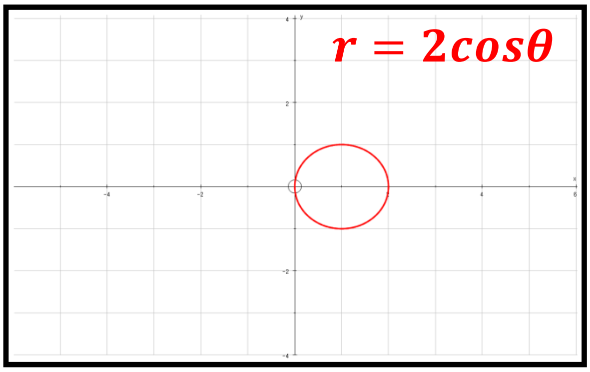

$r = tan3\theta$

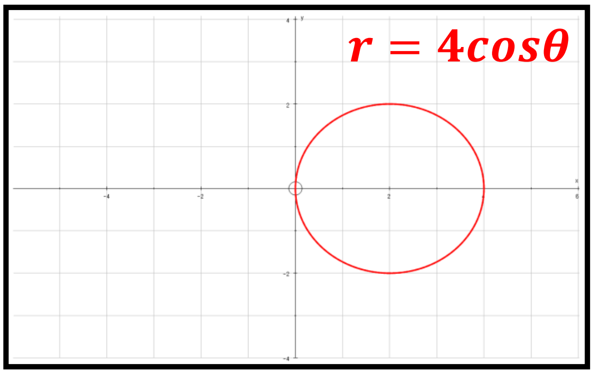

$r = \theta$

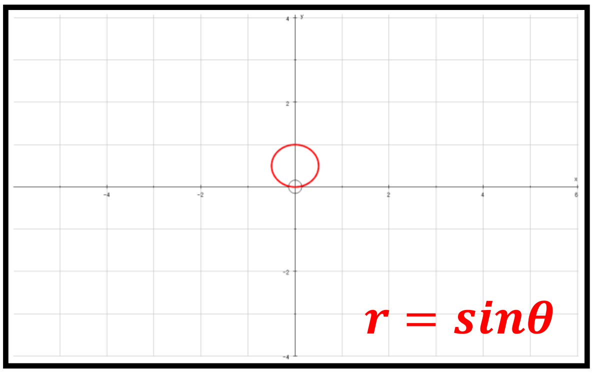

$r = 2\theta$

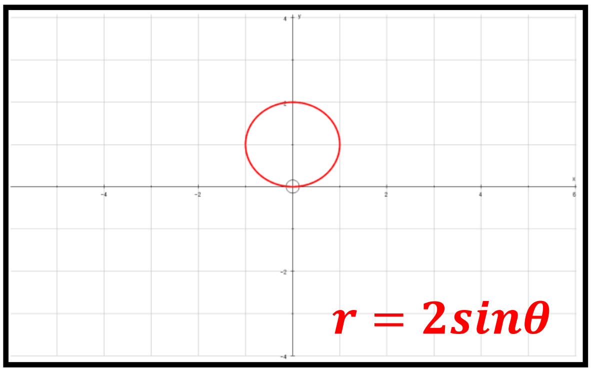

$r = 3\theta$

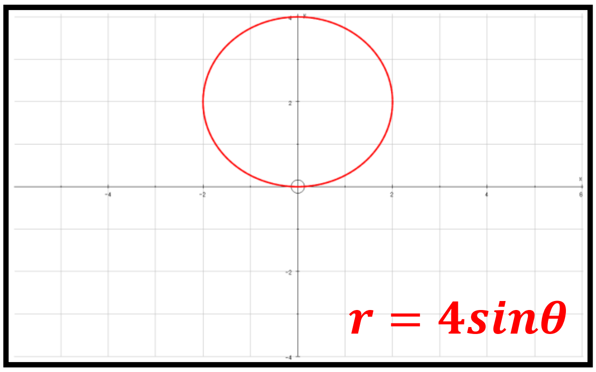

$r = cos\theta$


$r = 2cos\theta$


$r = 4cos\theta$


$r = sin\theta$

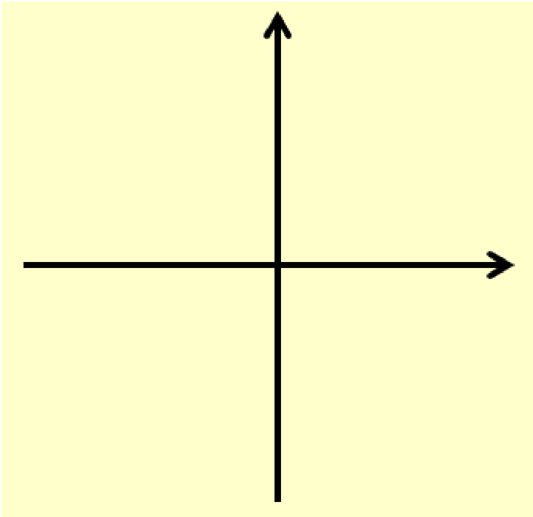

$r = 2sin\theta$


$r = 4sin\theta$

2.

a) Show on an argand diagram the locus of points given by the values of z satisfying:

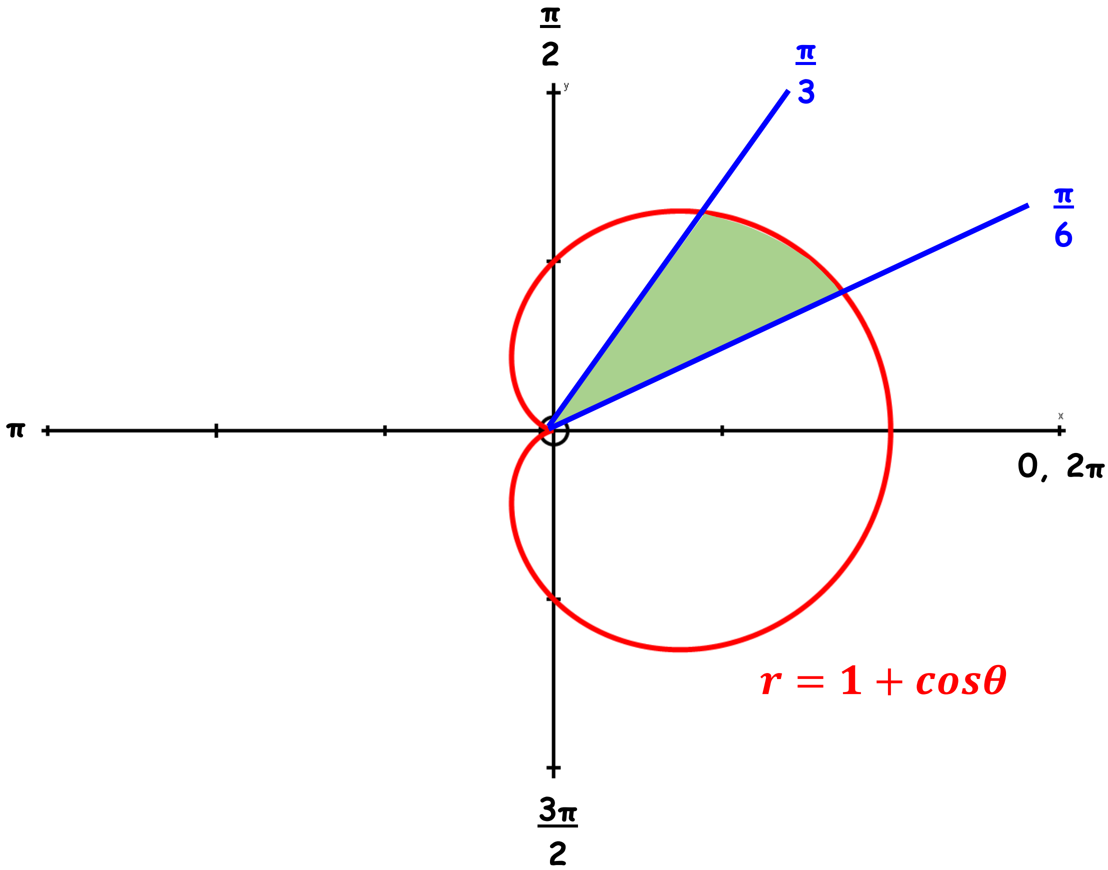
$$|z - 3 - 4i| = 5$$



b) Show that the locus of points can be represented by the polar curve:

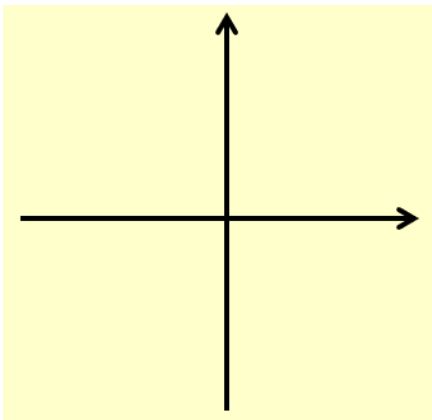
$$r = 6cos\theta + 8sin\theta$$

5C Integrating Polar Curves



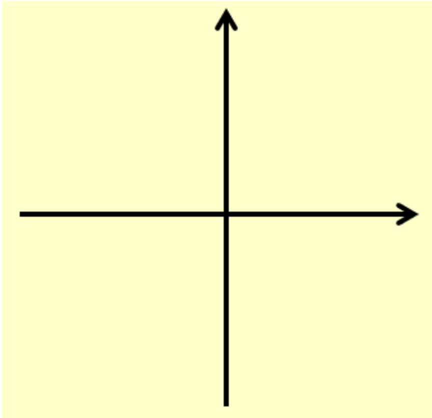
1. Find the area enclosed by the cardioid with equation:

$$r = a(1 + \cos\theta)$$



2. Find the area of <u>one</u> loop of the curve with polar equation:

$$r = a\sin 4\theta$$

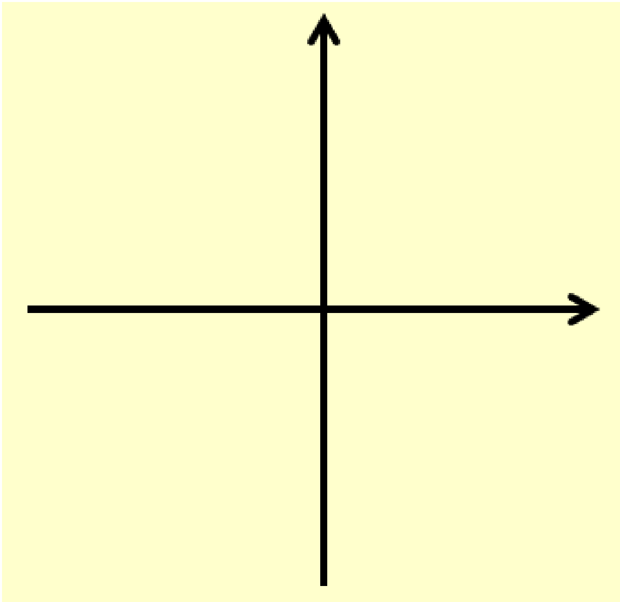


3.

a) On the same diagram, sketch the curves with equations:

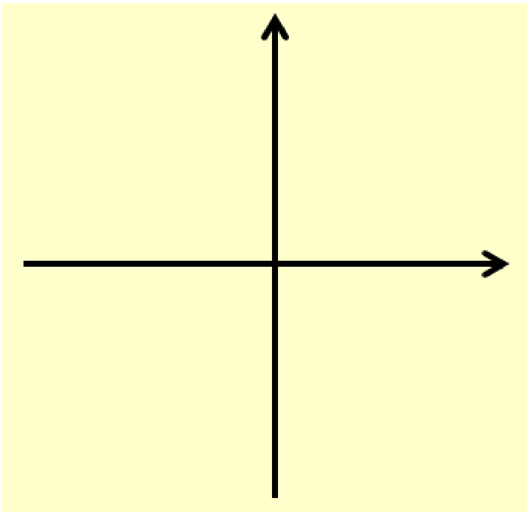
$r = 2 + \cos\theta$

$r = 5\cos\theta$

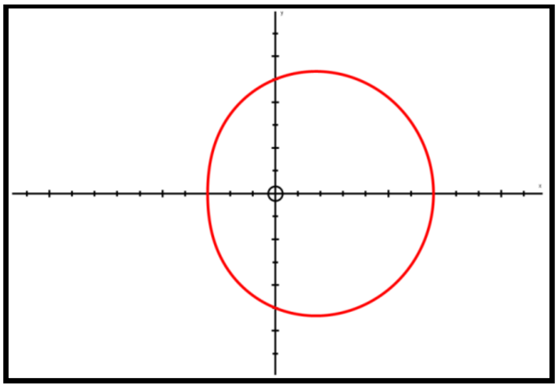


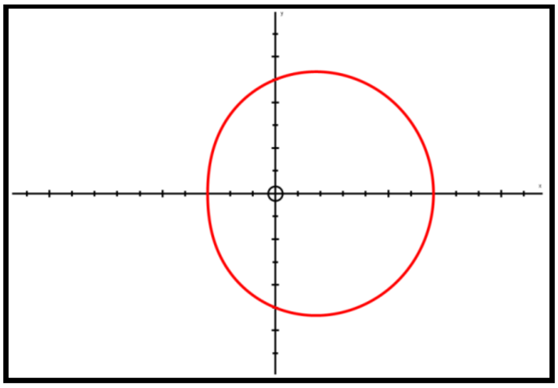
b) Find the polar coordinates of the intersection of these curves

c) Find the exact value of the finite region bounded by the 2 curves



5D Tangents to Polar Curves

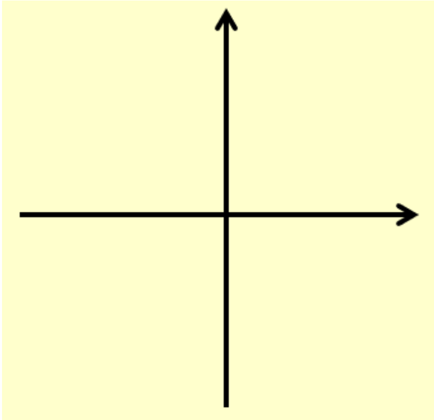




1. Find the coordinates of the points on:

   r = a(1 + cosθ)

   Where the tangents are parallel to the initial line θ = 0.



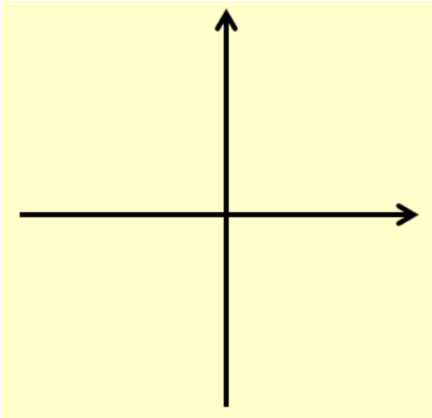
2. Find the coordinates and the equations of the tangents to the curve:

$r = a\sin2\theta, \quad 0 \le \theta \le {}^{\pi}/_{2}$

Where the tangents are:

a) Parallel to the initial line

Give answers to 3 s.f where appropriate:



b) Perpendicular to the initial line

Give answers to 3 s.f where appropriate:

3. Prove that for:

   $r = (p + q\cos\theta)$, p and q both > 0 and $p \geq q$

   to have a 'dimple', $p < 2q$ and also

   $p \geq q$.