

5) Straight line graphs

5.1) $y = mx + c$

5.2) Equations of straight lines

5.3) Parallel and perpendicular lines

5.4) Length and area

5.5) Modelling with straight lines

$$5.1) y = mx + c$$

Worked example

Calculate the gradient between the coordinates:

$(2, 1)$ and $(5, 7)$

$(-2, -1)$ and $(5, 7)$

Your turn

Calculate the gradient between the coordinates:

$(-4, 2)$ and $(6, 8)$

$\frac{3}{5}$

Worked example

Calculate the gradient between the coordinates:

$(2, 1)$ and $(5, -7)$

$(2, -1)$ and $(-5, -7)$

Your turn

Calculate the gradient between the coordinates:

$(-4, 2)$ and $(-6, -8)$

5

Worked example

Write the equation of the line in the form $y = mx + c$ which passes through the points $(2, 3)$ and $(5, 9)$

Write the equation of the line in the form $y = mx + c$ which passes through the points $(2, 3)$ and $(5, -9)$

Your turn

Write the equation of the line in the form $y = mx + c$ which passes through the points $(3, 10)$ and $(-5, 18)$

$$y = -x - 7$$

Worked example

Write the equation of the line in the form $y = mx + c$ which passes through the points $(2, -3)$ and $(7, -5)$

Your turn

Write the equation of the line in the form $y = mx + c$ which passes through the points $(3, -2)$ and $(-7, 5)$

$$y = -\frac{3}{10}x - \frac{11}{10}$$

Worked example

The gradient connecting the two points $(2a, 5)$ and $(7a, 8)$ is 6.
Solve for a

Your turn

The gradient connecting the two points $(3a, 7)$ and $(5a, 12)$ is 6.
Solve for a

$$a = \frac{5}{12}$$

Worked example

The gradient connecting the two points $(2, -5)$ and (a, b) is 4. Find an expression for b in terms of a

Your turn

The gradient connecting the two points $(-3, 4)$ and (a, b) is 2. Find an expression for b in terms of a

$$b = 2a + 10$$

5.2) Equations of straight lines

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Worked example

Determine the gradient and y -intercept of the line with equation $3x + 5y - 4 = 0$

Your turn

Determine the gradient and y -intercept of the line with equation $4x - 3y + 5 = 0$

$$\text{Gradient} = \frac{4}{3}$$

$$y\text{-intercept} = \frac{5}{3}$$

Worked example

Express in the form $ax + by + c = 0$:

$$y = 5x - 2$$

$$y = -2x + 5$$

Your turn

Express $y = 4x + 3$ in the form

$$ax + by + c = 0$$

$$4x - y + 3 = 0$$

Worked example

Express $y = \frac{2}{5}x - \frac{3}{5}$ in the form $ax + by + c = 0$, where a, b, c are integers.

Your turn

Express $y = \frac{1}{3}x - \frac{2}{3}$ in the form $ax + by + c = 0$, where a, b, c are integers.

$$x - 3y - 2 = 0$$

Worked example

Determine the point of intersection of the lines with equations $y = 2x$ and $x + 3y = 5$

Your turn

Determine the point of intersection of the lines with equations $y = 3x$ and $x + 2y = 4$

$$\left(\frac{4}{7}, \frac{12}{7}\right)$$

Worked example

A straight line passes through $(0, 4)$ and has gradient -3 .

It intersects the line with equation $2x - 7y - 6 = 0$ at the point P .

Find the coordinates of P

Your turn

A straight line passes through $(0, 3)$ and has gradient -4 .

It intersects the line with equation $7x - 6y + 2 = 0$ at the point P .

Find the coordinates of P

$$\left(\frac{16}{31}, \frac{29}{31}\right)$$

Worked example

$$y = 2x - 5$$

Gradient:

y -intercept:

x -intercept:

Sketch:

Your turn

$$y = 3x - 4$$

Gradient:

3

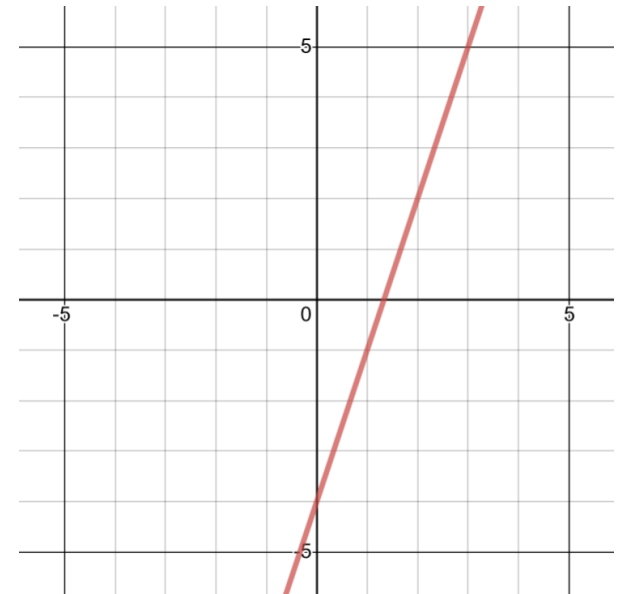
y -intercept:

-4

x -intercept:

$\frac{4}{3}$

Sketch:



Worked example

$$y = -2x + 6$$

Gradient:

y -intercept:

x -intercept:

Sketch:

Your turn

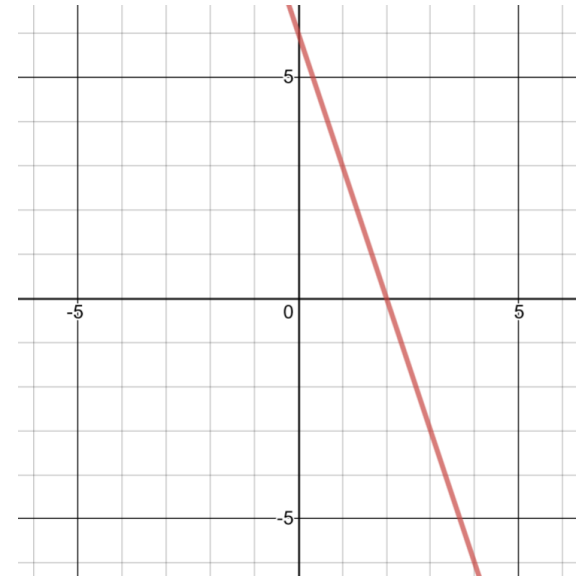
$$y = -3x + 6$$

Gradient: -3

y -intercept: 6

x -intercept: 2

Sketch:



Worked example

$$2x + 3y = 6$$

Gradient:

y -intercept:

x -intercept:

Sketch:

Your turn

$$3x + 2y = 6$$

Gradient:

$$-\frac{3}{2}$$

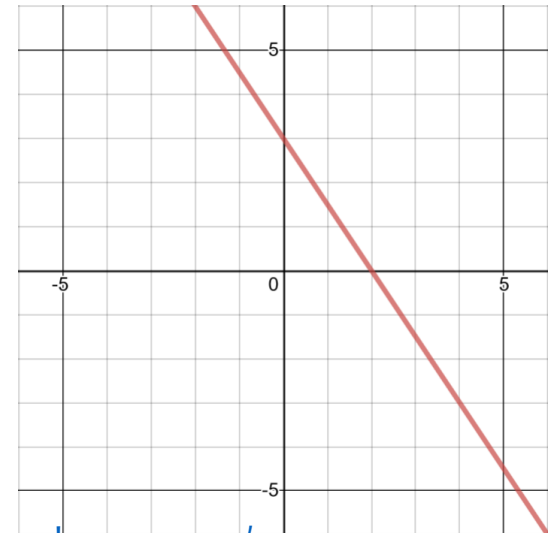
y -intercept:

3

x -intercept:

2

Sketch:

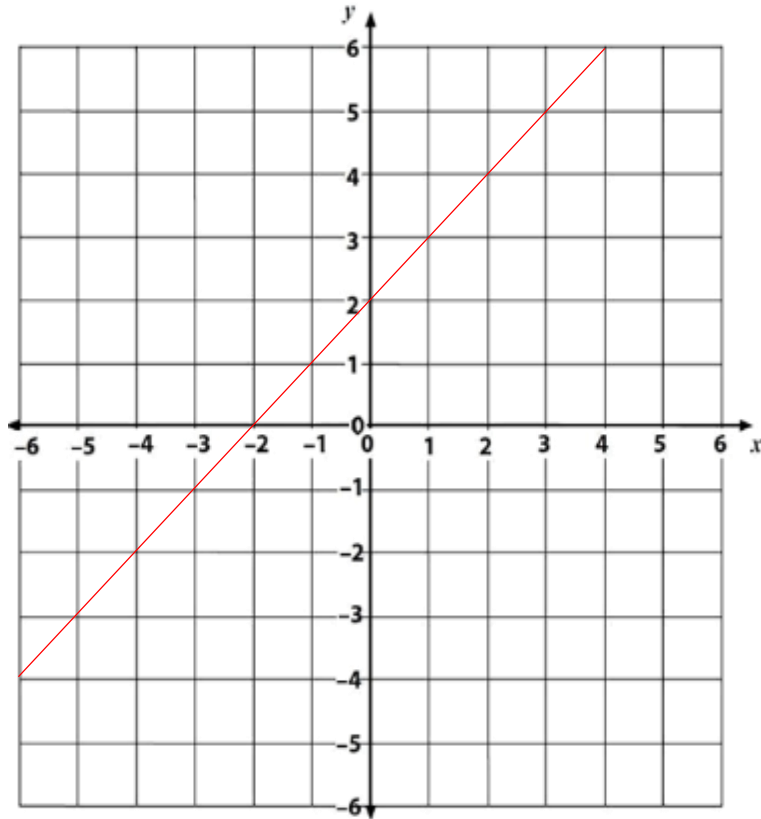


Worked example

Gradient: $m =$

y-intercept: $c =$

$y = mx + c \rightarrow$

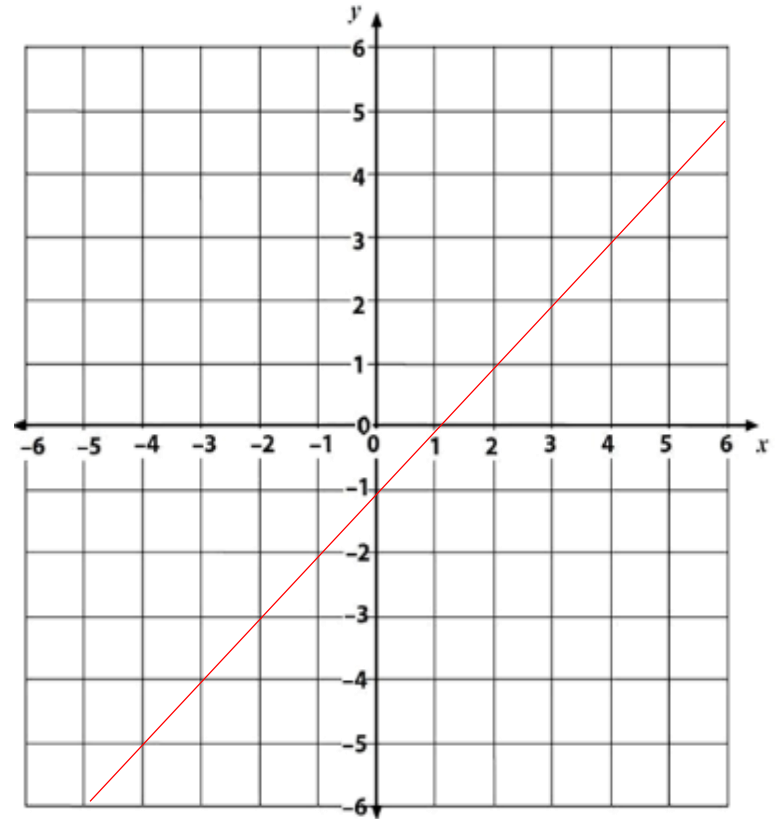


Your turn

Gradient: $m = 1$

y-intercept: $c = -1$

$y = mx + c \rightarrow y = x - 1$

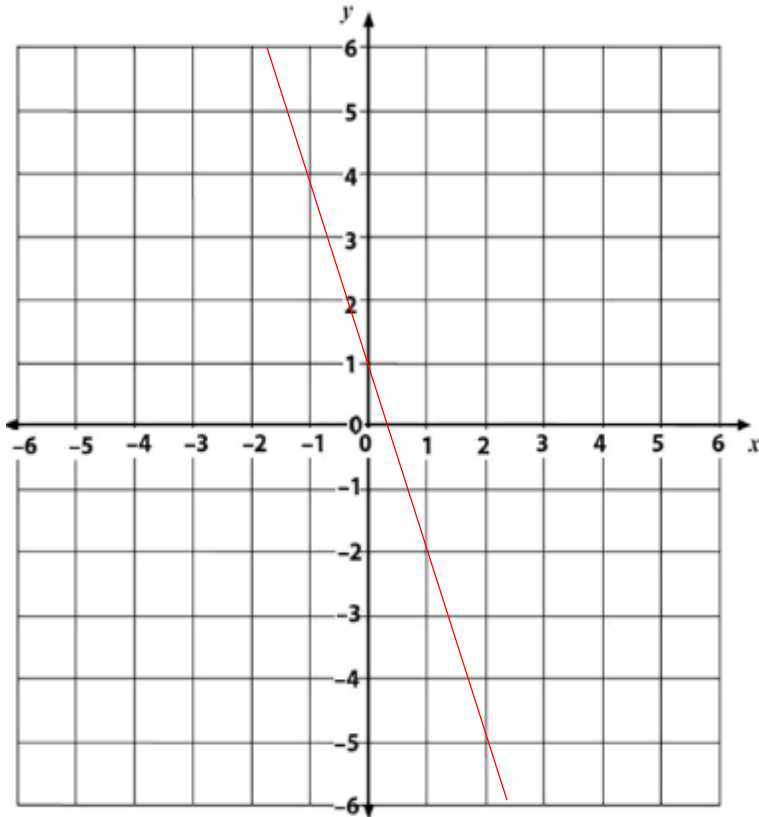


Worked example

Gradient: $m =$

y -intercept: $c =$

$y = mx + c \rightarrow$

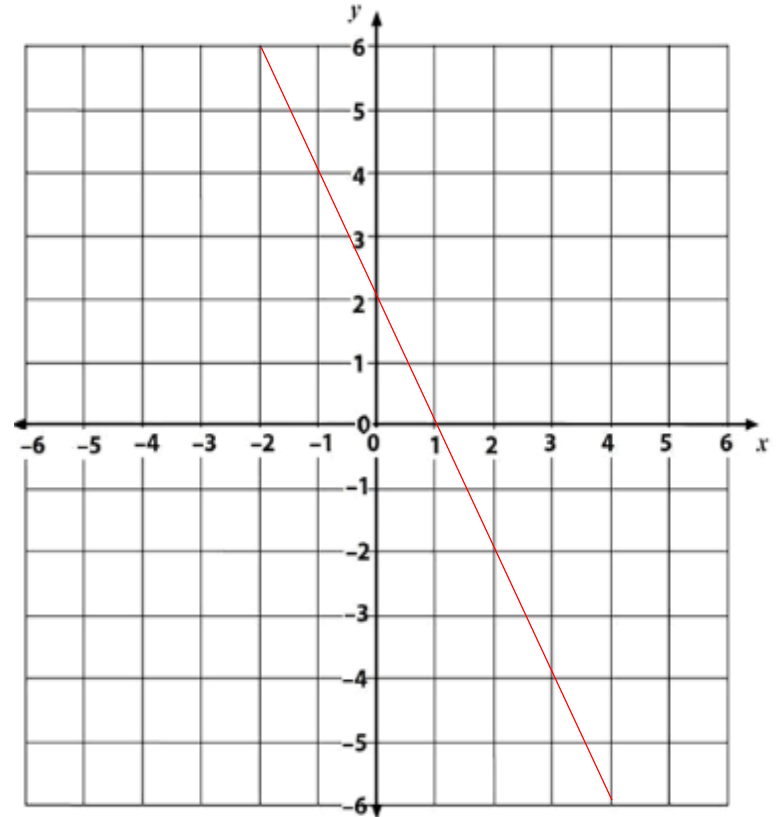


Your turn

Gradient: $m = -2$

y -intercept: $c = 2$

$y = mx + c \rightarrow y = -2x + 1$

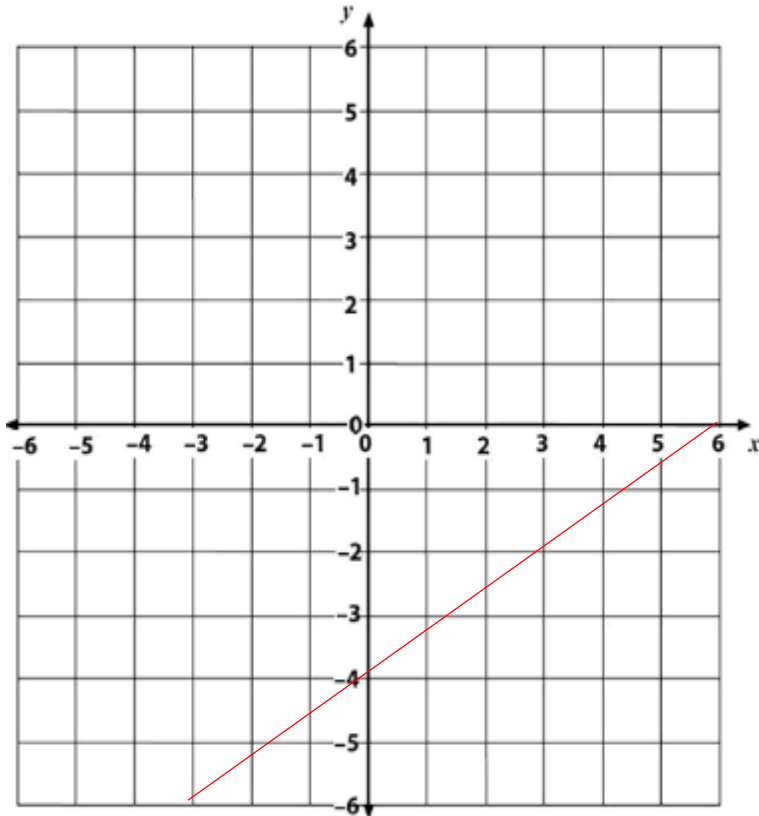


Worked example

Gradient: $m =$

y-intercept: $c =$

$y = mx + c \rightarrow$

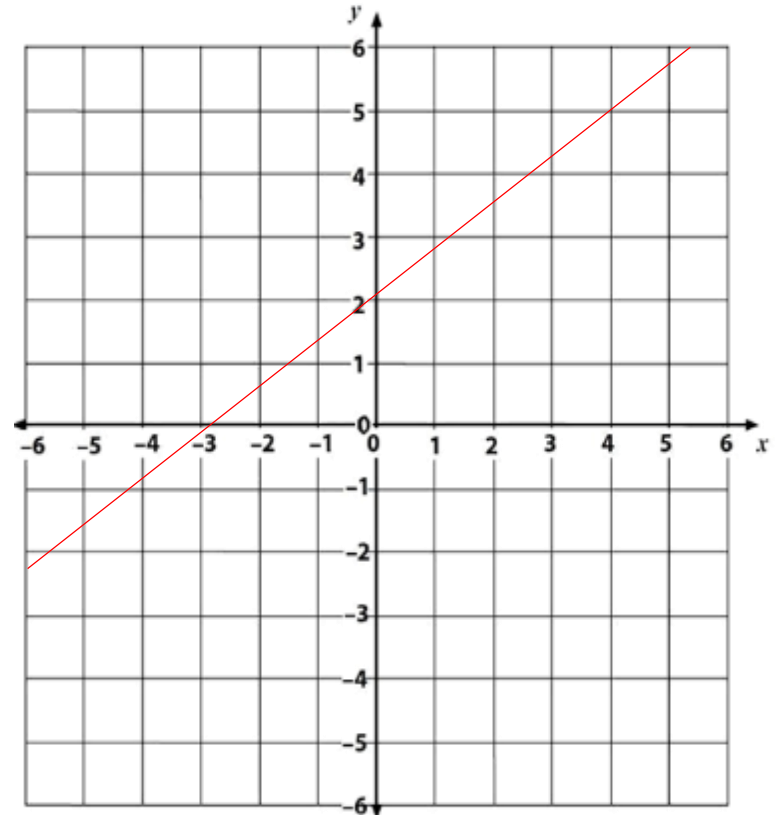


Your turn

Gradient: $m = \frac{3}{4}$

y-intercept: $c = 2$

$y = mx + c \rightarrow y = \frac{3}{4}x + 2$

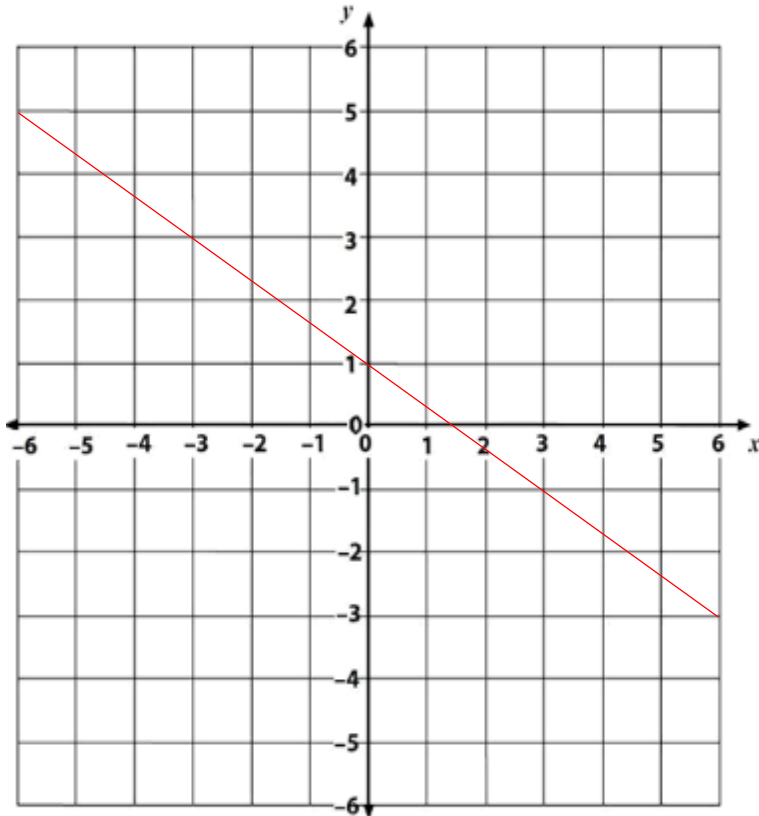


Worked example

Gradient: $m =$

y-intercept: $c =$

$y = mx + c \rightarrow$

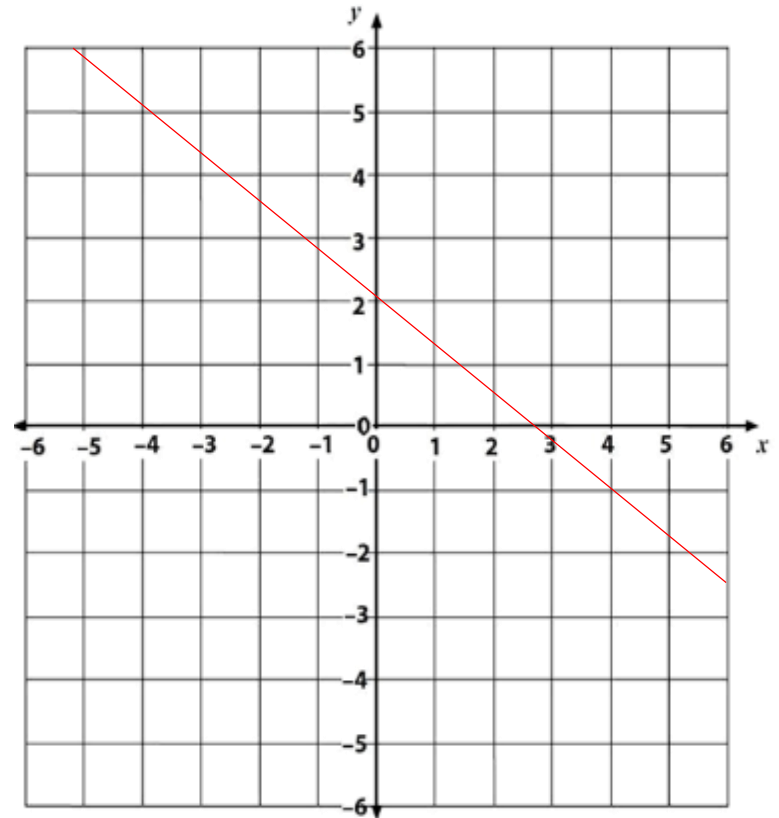


Your turn

Gradient: $m = -\frac{3}{4}$

y-intercept: $c = 2$

$y = mx + c \rightarrow y = -\frac{3}{4}x + 2$



Worked example

Find where the line intercepts the axes:

Line	x-intercept	y-intercept
$y = 2x + 3$		
$y = 3x + 2$		
$y = 3x - 2$		
$y = 2x - 3$		
$y = 3 - 2x$		
$y = 2 - 3x$		
$2x + 3y = 6$		
$3x + 2y = 6$		
$y = ax + b$		

Your turn

Find where the line intercepts the axes:

Line	x-intercept	y-intercept
$y = 4x + 5$	$-\frac{5}{4}$	5
$y = 5x + 4$	$-\frac{4}{5}$	4
$y = 5x - 4$	$\frac{4}{5}$	-4
$y = 4x - 5$	$\frac{5}{4}$	-5
$y = 5 - 4x$	$\frac{5}{4}$	5
$y = 4 - 5x$	$\frac{4}{5}$	4
$4x + 5y = 20$	5	4
$5x + 4y = 20$	4	5
$ax + by = c$	$\frac{c}{a}$	$\frac{c}{b}$

Worked example

Find the equation of the line,
given a point and the gradient:

$(6, 22)$ Gradient 3

$(-6, 22)$ Gradient 3

Your turn

Find the equation of the line,
given a point and the gradient:

$(-2, 5)$ Gradient 4

$$y = 4x + 13$$

Worked example

Write the equation of the line in the form $y = mx + c$ which passes through the points $(2, 3)$ and $(5, -9)$

Your turn

Write the equation of the line in the form $y = mx + c$ which passes through the points $(3, 10)$ and $(-5, 18)$

$$y = -x - 7$$

Worked example

Write the equation of the line in the form $y = mx + c$ which passes through the points $(2, -3)$ and $(7, -5)$

Your turn

Write the equation of the line in the form $y = mx + c$ which passes through the points $(3, -2)$ and $(-7, 5)$

$$y = -\frac{3}{10}x - \frac{11}{10}$$

Worked example

The lines $y = 2x - 7$ and $3x + 2y - 21 = 0$ intersect at the point A .

The point B has coordinates $(2, -8)$.

Find the equation of the line that passes through the points A and B .

Write your answer in the form $ax + by + c = 0$, where a, b and c are integers.

Your turn

The lines $y = 4x - 7$ and $2x + 3y - 21 = 0$ intersect at the point A .

The point B has coordinates $(-2, 8)$.

Find the equation of the line that passes through the points A and B .

Write your answer in the form $ax + by + c = 0$, where a, b and c are integers.

$$3x + 5y - 34 = 0$$

5.3) Parallel and perpendicular lines [Chapter CONTENTS](#)

Worked example

Determine whether the pairs of lines are parallel, perpendicular or neither:

$$5x - 2y - 3 = 0$$

$$y = \frac{5}{2}x$$

$$5x + 3y - 21 = 0$$

$$3x - 5y + 2 = 0$$

Your turn

Determine whether the pairs of lines are parallel, perpendicular or neither:

$$3x - y - 2 = 0$$

$$x + 3y - 6 = 0$$

Perpendicular

$$y = \frac{1}{2}x$$

$$2x - y + 4 = 0$$

Neither parallel nor perpendicular

Worked example

The points A , B and C have coordinates $(0, 12)$, $(-3, 0)$ and $(0, c)$ respectively.

The line through points A and B is perpendicular to the line through points B and C .

Find the value of c

Your turn

The points A , B and C have coordinates $(0, 6)$, $(-2, 0)$ and $(0, c)$ respectively.

The line through points A and B is perpendicular to the line through points B and C .

Find the value of c

$$x = -\frac{2}{3}$$

Worked example

Find the gradient of the perpendicular line to:

$$y = 2x + 3$$

$$y = 2 - 3x$$

$$y = \frac{2}{3}x + 1$$

$$y = 8 - \frac{11}{5}x$$

Your turn

Find the general equation of the perpendicular line to:

$$y = 4x$$

$$y = -\frac{1}{4}x + c$$

$$y = -2x + 4$$

$$y = \frac{1}{2}x + c$$

$$y = \frac{3}{4}x - 5$$

$$y = -\frac{4}{3}x + c$$

$$y = 7 - \frac{11}{3}x$$

$$y = \frac{3}{11}x + c$$

Worked example

Find the equation of the line parallel to $y = 3x + 5$ that passes through $(-2, 5)$

Find the equation of the line parallel to $y = -\frac{1}{3}x - 4$ that passes through $(-2, 5)$

Your turn

Find the equation of the line parallel to $y = -\frac{1}{2}x - 3$ that passes through $(-2, 5)$

$$y = -\frac{1}{2}x + 4$$

Worked example

Find the equation of the line perpendicular to $y = 2x - 4$ that passes through $(-2, 5)$

Find the equation of the line perpendicular to $y = 4x + 5$ that passes through $(-2, 5)$

Your turn

Find the equation of the line perpendicular to $y = 3x + 2$ that passes through $(9, -7)$

$$y = -\frac{1}{3}x - 4$$

Worked example

Find the equation of the line perpendicular to $y = \frac{1}{2}x - 4$ that passes through $(-2, 5)$

Find the equation of the line perpendicular to $y = -\frac{2}{3}x + 5$ that passes through $(-2, 5)$

Your turn

Find the equation of the line perpendicular to $y = -\frac{4}{3}x + 3$ that passes through $(-12, -5)$

$$y = \frac{3}{4}x + 4$$

Worked example

Find the equation of the line perpendicular to $x + 2y = 5$ which passes through the point $(3, 7)$

Find the equation of the line perpendicular to $3x + 2y = 5$ which passes through the point $(3, 7)$

Your turn

Find the equation of the line perpendicular to $2x + 3y = 5$ which passes through the point $(4, 7)$

$$y = \frac{3}{2}x + 1$$

Worked example

Write down an equation of a line parallel to $y = 3x - 4$ which passes through the origin.

Write down an equation of a line parallel to $y = 3 - 4x$ which passes through the origin.

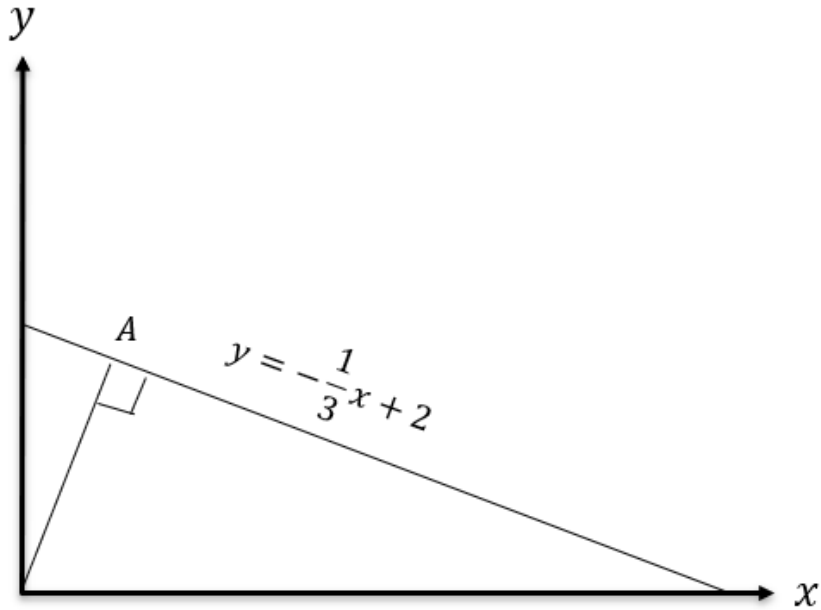
Your turn

Write down an equation of a line parallel to $y = -2x + 5$ which passes through the origin.

$$y = -2x$$

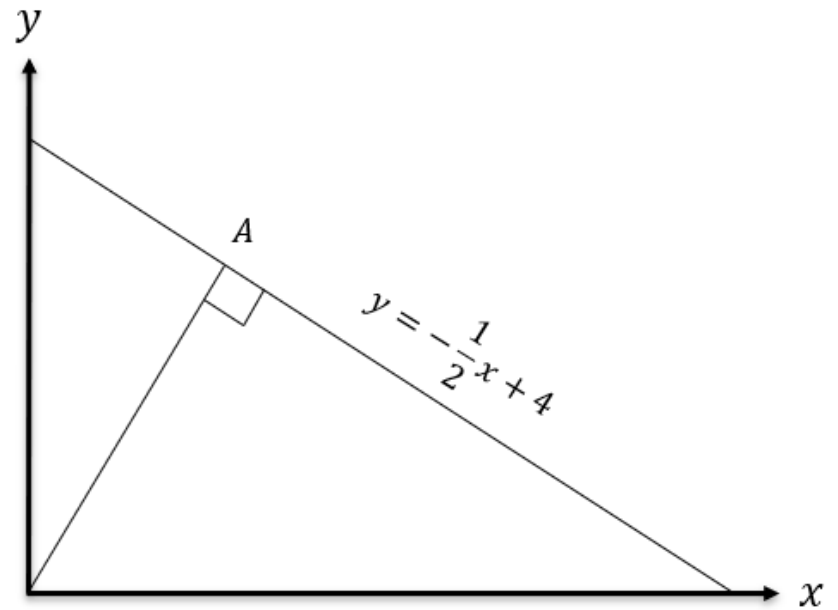
Worked example

Determine the coordinates of A



Your turn

Determine the coordinates of A



$$\left(\frac{8}{5}, \frac{16}{5}\right)$$

5.4) Length and area

[Chapter CONTENTS](#)

Worked example

Find the distance between:
 $(2, 4)$ and $(8, 8)$

$(-2, 4)$ and $(-9, 9)$

Your turn

Find the distance between:
 $(2, -4)$ and $(11, 8)$

15

Worked example

In your head, find the distance between:
 $(8,2)$ and $(5,6)$

$(-1,-7)$ and $(11,2)$

$(-23,0)$ and $(1,7)$

Your turn

In your head, find the distance between:
 $(1,10)$ and $(4,14)$

5

$(-4,-2)$ and $(-12,4)$

10

$(0,-9)$ and $(5,3)$

13

Worked example

The straight line l_1 with equation $2x - y = 0$ and the straight line l_2 with equation $3x + 2y - \frac{7}{2} = 0$ intersect at point A .

O is the origin. B is the point where l_2 meets the x -axis.

Work out the area of triangle AOB

Your turn

The straight line l_1 with equation $4x - y = 0$ and the straight line l_2 with equation $2x + 3y - 21 = 0$ intersect at point A .

O is the origin. B is the point where l_2 meets the x -axis.

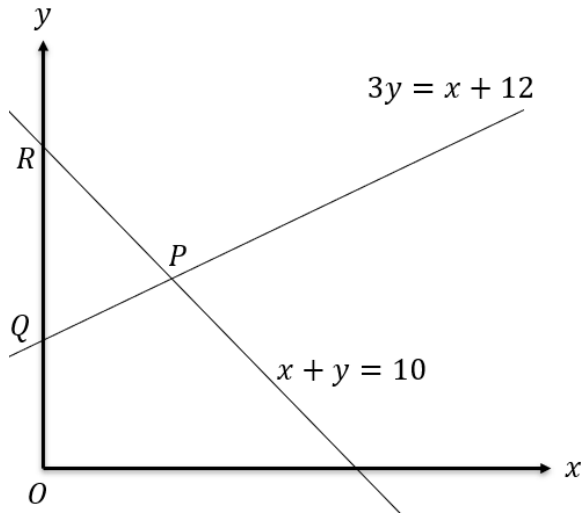
Work out the area of triangle AOB

$$\frac{63}{2}$$

Worked example

Determine:

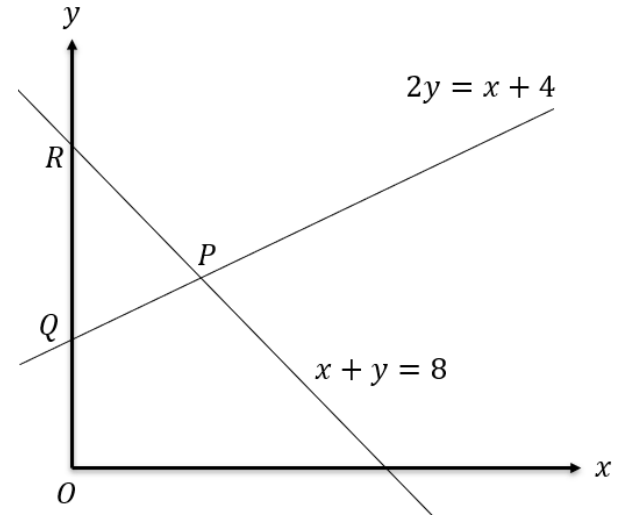
- The length of PQ
- The area of triangle PQR



Your turn

Determine:

- The length of PQ
- The area of triangle PQR

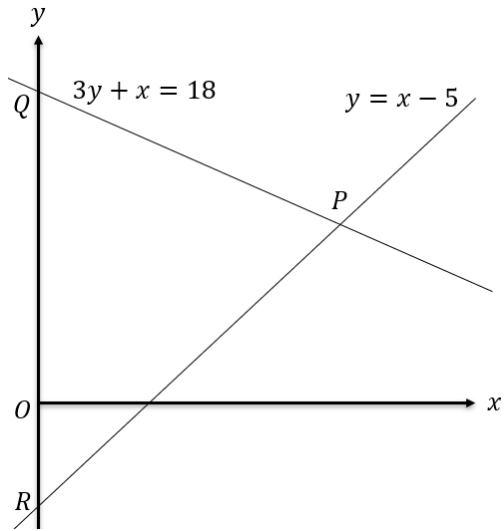


- $2\sqrt{5}$
- 12

Worked example

Determine:

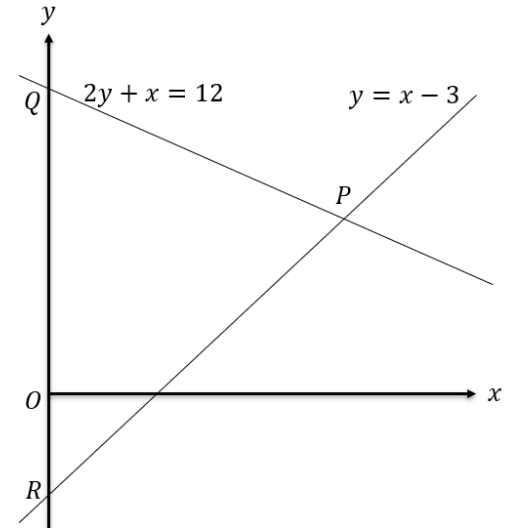
- The length of PQ
- The area of triangle PQR



Your turn

Determine:

- The length of PQ
- The area of triangle PQR



- $3\sqrt{5}$
- 27

5.5) Modelling with straight lines

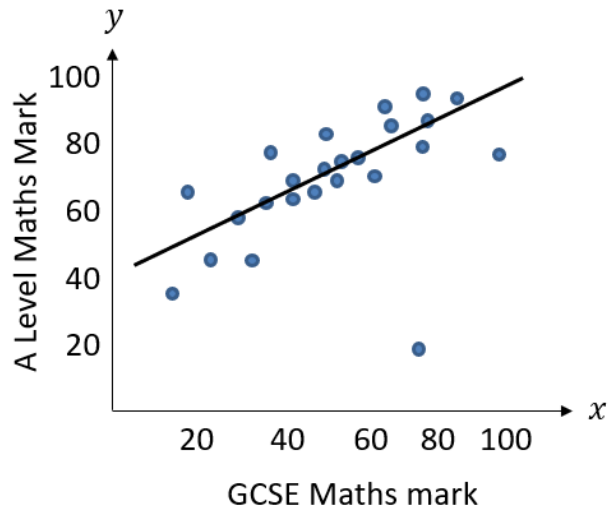
[Chapter CONTENTS](#)

Worked example

The A Level Maths mark, y %, and GCSE Maths mark, x %, is recorded for several students.

Assume the line goes through $(0, 40)$ and $(60, 80)$.

- Write a linear model
- Interpret the gradient and y -intercept in this context
- Predict the A Level Maths mark of a student who got 100% for their GCSE Maths mark

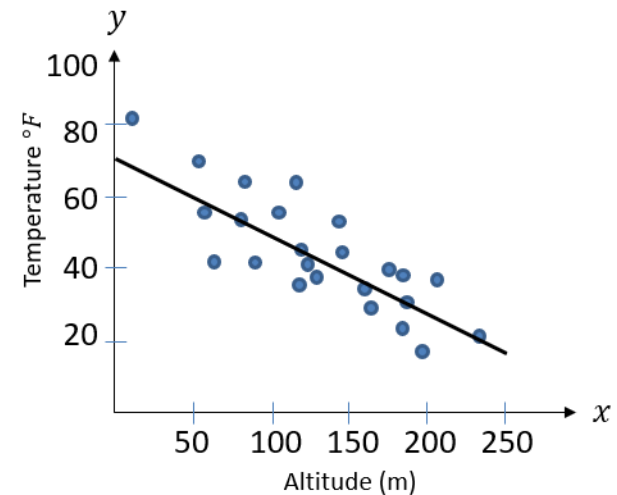


Your turn

The temperature y at different points on a mountain is recorded at different altitudes x .

Assume the line goes through $(0, 70)$ and $(250, 20)$.

- Write a linear model
- Interpret the gradient and y -intercept in this context
- Predict at what altitude the temperature reaches $0^\circ F$



- $y = 0.2x + 70$
- The temperature at sea level is $70^\circ F$. The temperature decreases by $0.2^\circ F$ for each extra metre in altitude.
- 350 m

Worked example

In 2010 the population of rabbits in an area was 200. Locals projected that the number of rabbits would increase by 4 per year.

- a) Write a linear model for the population, p , of rabbits t years after 2010
- b) Write down a reason why this might not be a realistic model.

Your turn

In 2000 there were 18500 people in a village. Planners projected that the number of people living in the village would increase by 350 per year.

- a) Write a linear model for the population, p , of the village t years after 2000
- b) Write down a reason why this might not be a realistic model.

a) $p = 350t + 18500$

b) The population is unlikely to increase by the same amount each year. An exponential model would be more suitable