5) Probability

5.1) Calculating probabilities

Worked example	Your turn
Two fair spinners each have five sectors numbered 1 to 5. The two spinners are spun together and the sum of the numbers indicated on each spinner is recorded. Find the probability of the spinners indicating a sum of: a) exactly 6 b) more than 6	Two fair spinners each have four sectors numbered 1 to 4. The two spinners are spun together and the sum of the numbers indicated on each spinner is recorded. Find the probability of the spinners indicating a sum of: a) exactly 5 b) more than 5 a) $\dfrac{1}{4}$ b) $\dfrac{3}{8}$

Worked example

The table shows the times taken, in minutes, for a group of students to complete a number puzzle.

Time, t (min)	$5 \leq t < 8$	$8 \leq t < 11$	$11 \leq t < 12$	$12 \leq t < 14$	$14 \leq t < 15$
Frequency	4	16	7	9	5

A student is chosen at random. Find the probability that they completed the number puzzle in:
a)   under 12 minutes
b)   over 9.5 minutes.

Your turn

The table shows the times taken, in minutes, for a group of students to complete a number puzzle.

Time, t (min)	$5 \leq t < 7$	$7 \leq t < 9$	$9 \leq t < 11$	$11 \leq t < 13$	$13 \leq t < 15$
Frequency	6	13	12	5	4

A student is chosen at random. Find the probability that they completed the number puzzle in:
a)   under 9 minutes
b)   over 10.5 minutes.

a) $\dfrac{19}{40}$

b) $\dfrac{3}{10}$

Worked example	Your turn
A participant is chosen at random. What is the probability they took longer than 60 seconds?	A participant is chosen at random. What is the probability they weigh more than $14\ kg$?

Frequency density

Time (seconds)

Frequency density

Weight, kilograms

$$\frac{14}{25} = 0.56$$

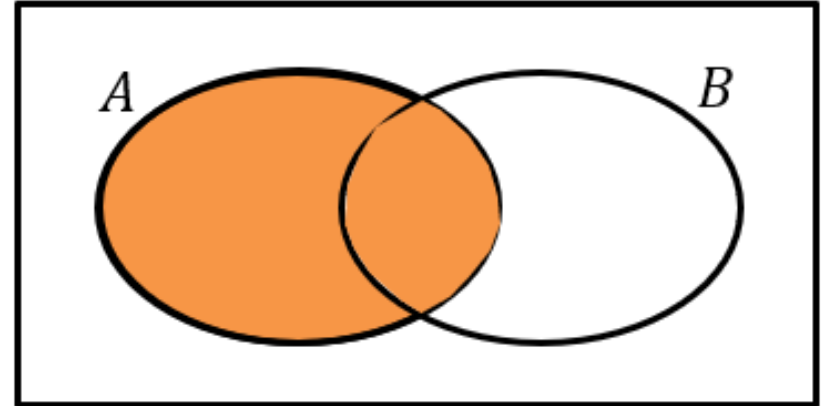# 5.2) Venn diagrams

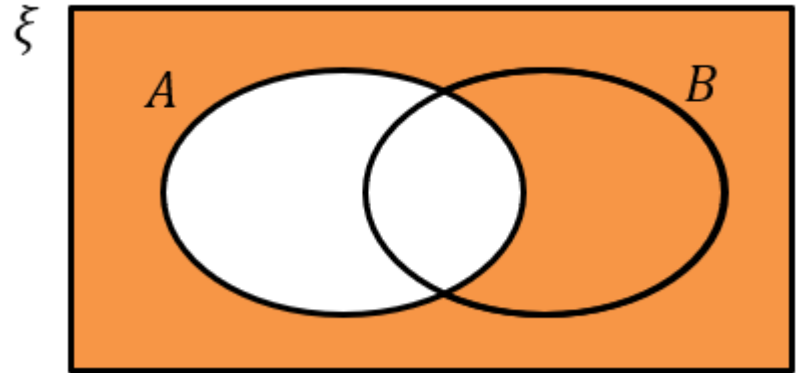| Worked example	Your turn
Draw a Venn diagram for two events C and D. Shade the region represented by: $$D$$	Draw a Venn diagram for two events A and B. Shade the Venn diagram the region represented by:

Worked example

Draw a Venn diagram for two events C and D.
Shade the region represented by:

Not D

Your turn

Draw a Venn diagram for two events A and B.
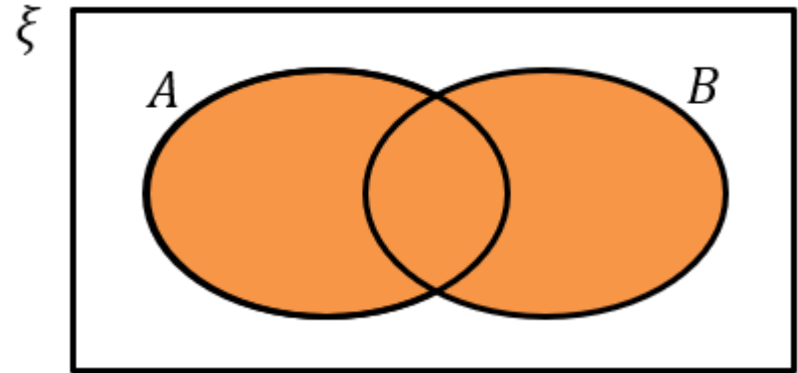Shade the Venn diagram the region represented by:
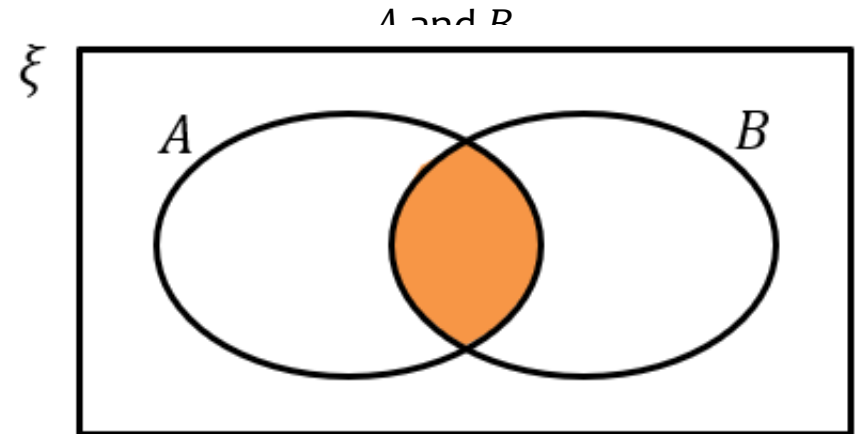
Worked example

Draw a Venn diagram for two events C and D.
Shade the region represented by:
$$C \text{ or } D$$

Your turn

Draw a Venn diagram for two events A and B.
Shade the Venn diagram the region represented by:

| Worked example	Your turn
Draw a Venn diagram for two events C and D. Shade the region represented by: $\qquad C$ and D	Draw a Venn diagram for two events A and B. Shade the Venn diagram the region represented by: $\qquad A$ and B

Worked example	Your turn
Draw a Venn diagram for two events C and D. Shade the region represented by: $$D \text{ and not } C$$	Draw a Venn diagram for two events A and B. Shade the Venn diagram the region represented by:
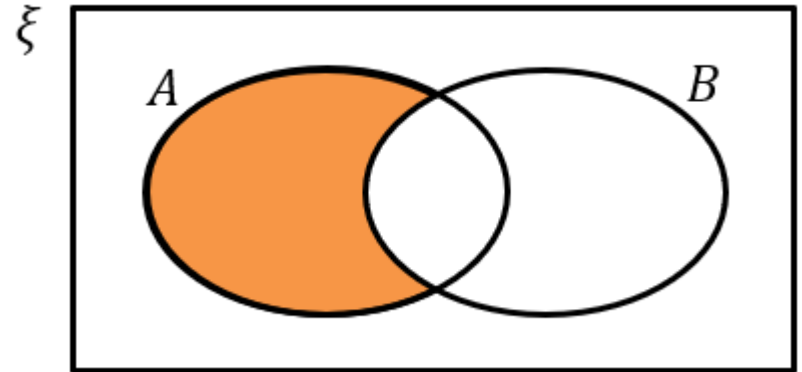
Worked example	Your turn
In a class of 30 students, 6 play the piano, 11 play the guitar and 3 play both instruments. A student is chosen at random from the class. Find the probability that the student: a) Does not play the piano b) Does not play the piano or the guitar	In a class of 30 students, 7 are in the choir, 5 are in the school band and 2 are in the choir and the band. A student is chosen at random from the class. Find the probability that the student: a) Is not in the band b) Is not in the choir or the band a) $\dfrac{5}{6}$ b) $\dfrac{2}{3}$

Worked example	Your turn
In a class of 30 students, 6 play the piano only, 11 play the guitar only and 3 play neither instrument. A student is chosen at random from the class. Find the probability that the student plays both instruments.	In a class of 30 students, 7 are in the choir only, 5 are in the school band only and 2 are in neither group. A student is chosen at random from the class. Find the probability that the student is in both groups.
	$$\frac{16}{30} = \frac{8}{15}$$

Worked example	Your turn
In a class of 30 students, 6 play the piano, 11 play the guitar and 16 play neither instrument. A student is chosen at random from the class. Find the probability that the student plays both instruments.	In a class of 30 students, 7 are in the choir, 5 are in the school band and 21 are in neither group. A student is chosen at random from the class. Find the probability that the student is in both groups. $$\frac{3}{30} = \frac{1}{10}$$

Worked example	Your turn
Given that $P(D) = 0.7$ and $P(C \text{ or } D) = 0.95$, find the probability of: a) $P(C \text{ and not } D)$ b) $P(\text{neither } C \text{ nor } D)$	Given that $P(A) = 0.6$ and $P(A \text{ or } B) = 0.85$, find the probability of: a) $P(A \text{ and not } B)$ b) $P(\text{neither } A \text{ nor } B)$ a) 0.25 b) 0.15

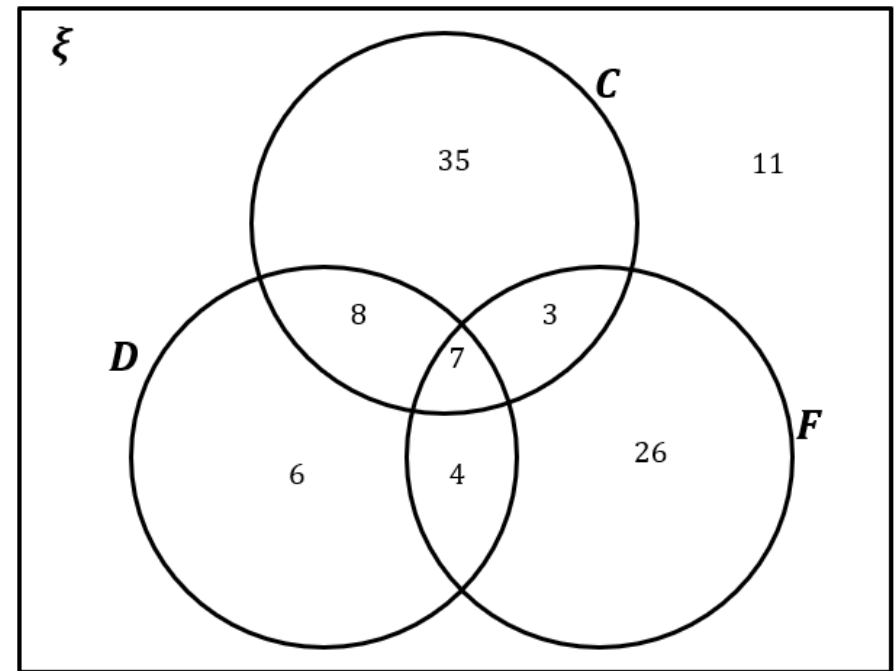| Worked example	Your turn
The probability of a person having read book A is 0.46. The probability that they have read book B is 0.18. The probability that they have read book A or B or both is 0.51. A person is chosen at random. Find the probability that the person has a) Read both book A and book B b) Read book B but not book A c) Read neither book	The probability of a person having read book A is 0.37. The probability that they have read book B is 0.25. The probability that they have read book A or B or both is 0.54. A person is chosen at random. Find the probability that the person has a) Read both book A and book B b) Read book A but not book B c) Read neither book a) 0.16 b) 0.21 c) 0.46

Worked example

A gym owner surveys 100 of their clients. They find that 65 run, 40 run and swim, 35 run and cycle, 48 swim, 30 swim and cycle, 25 do all three types of exercise and 60 cycle.
Draw a Venn Diagram to represent this data.

Your turn

A vet surveys 100 of their clients. They find that 25 own dogs, 15 own dogs and cats, 11 own dogs and tropical fish, 53 own cats, 10 own cats and tropical fish, 7 own dogs, cats and tropical fish, 40 own tropical fish.
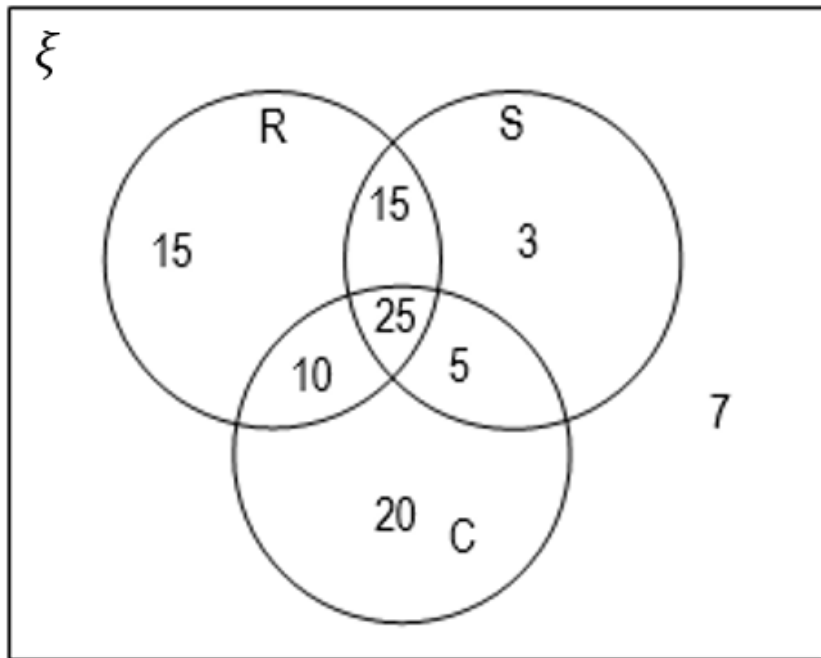Draw a Venn Diagram to represent this data.

Worked example	Your turn

Worked example

A gym owner surveys 100 of their clients.
A client is chosen at random.
Find the probability that the client:
a) Cycles only
b) Does not swim
c) Does not do any of these three exercises
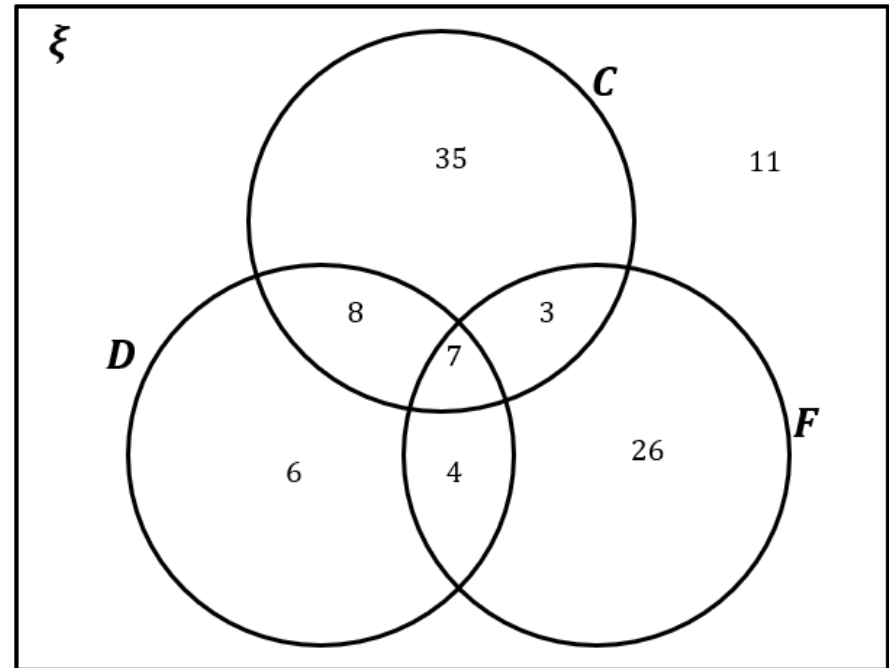d) Runs and swims but does not cycle.

Your turn

A gym owner surveys 100 of their clients.
A client is chosen at random.
Find the probability that the client:
a) Owns dogs only $\frac{6}{100} = \frac{3}{50} = 0.06$
b) Does not own fish $\frac{60}{100} = \frac{3}{5} = 0.6$
c) Does not own dogs, cats or fish $\frac{11}{100} = 0.11$
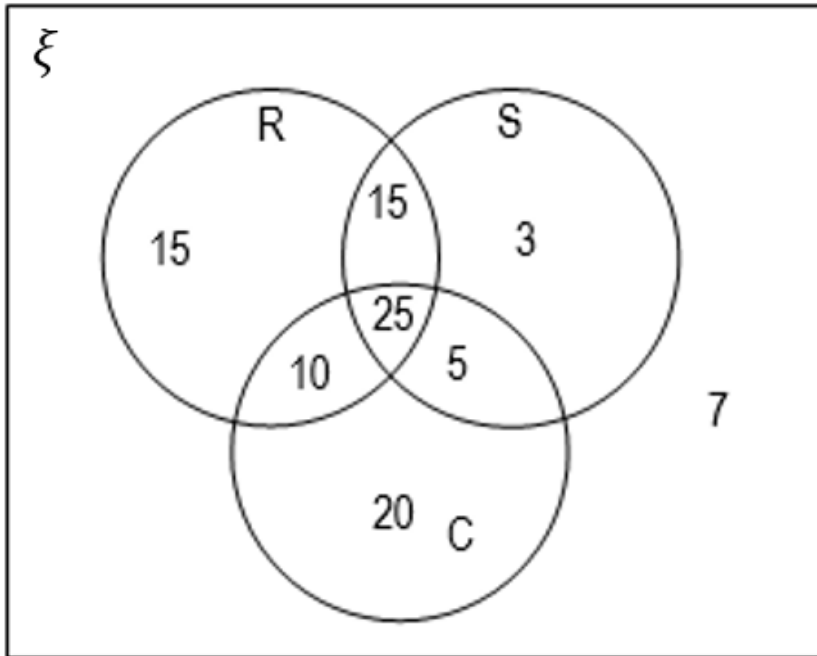d) Owns fish and cats but not dogs $\frac{10}{100} = \frac{1}{10} = 0.1$

Worked example

A gym owner surveys 100 of their clients.
A client is chosen at random.
Find the probability that the client:
a) Does exactly one of the exercises
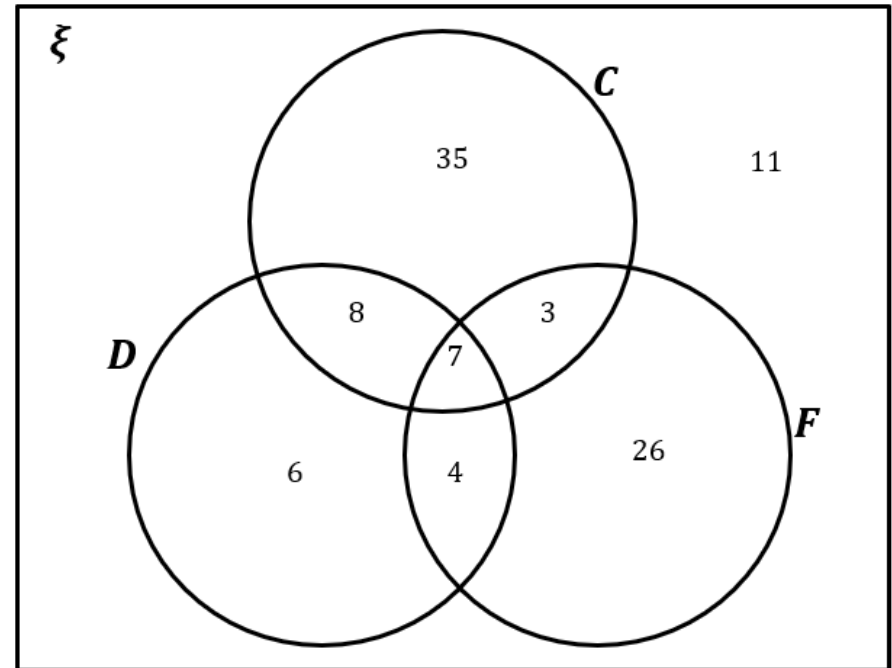b) Does at least two of the exercises

Your turn

A gym owner surveys 100 of their clients.
A client is chosen at random.
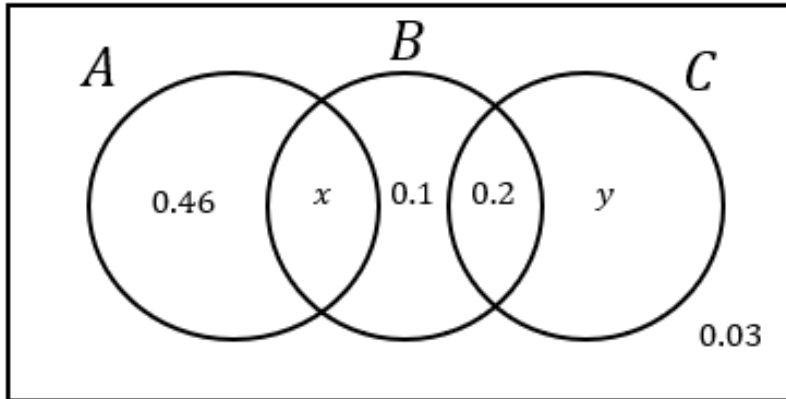Find the probability that the client:
a) Owns exactly one type of pet. $\frac{67}{100} = 0.67$
b) Owns at least two of the types of pet. $\frac{22}{100} = \frac{11}{50} = 0.22$

Diagrams/Graphs used with permission from DrFrostMaths: https://www.drfrostmaths.com/
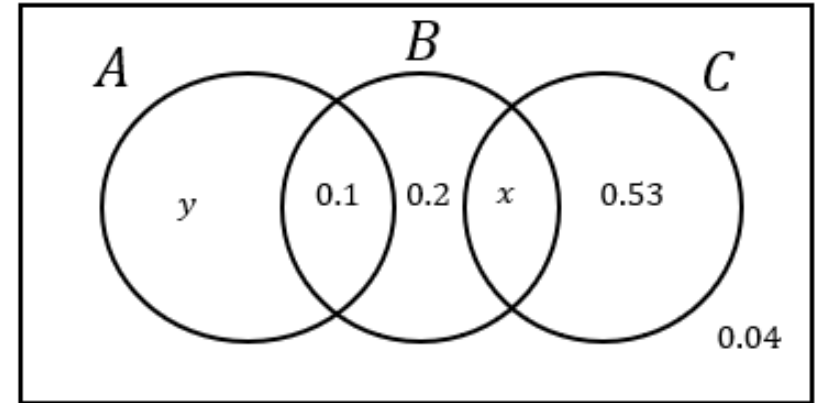
Worked example

The Venn diagram shows the probabilities of group members taking part in activities A, B and C.
Given that $P(B) = 0.39$, find $P(C)$

A   B   C

0.46 x 0.1   0.2 y

0.03

Your turn

The Venn diagram shows the probabilities of group members taking part in activities A, B and C.
Given that $P(B) = 0.35$, find $P(A)$

A   B   C

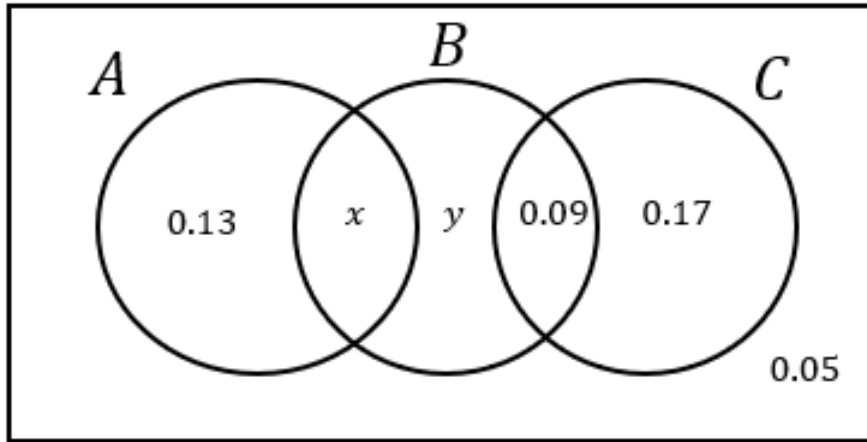y 0.1   0.2 x 0.53

0.04

0.18

Worked example

The Venn diagram shows the probabilities of group members taking part in activities A, B and C.
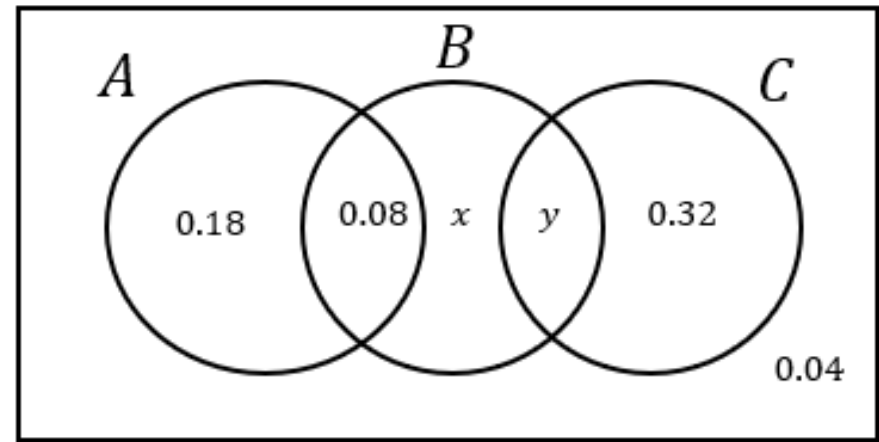Given that $P(A) = P(B)$, find the values of x and y

Your turn

The Venn diagram shows the probabilities of group members taking part in activities A, B and C.
Given that $P(B) = P(C)$, find the values of x and y

$$x = 0.24 \, , y = 0.14$$

5.3) Mutually exclusive and independent events

Worked example	Your turn
Events C and D are mutually exclusive and $P(C) = 0.6$ and $P(D) = 0.3$. a) Find $P(C \text{ or } D)$ b) Find $P(D \text{ but not } C)$ c) Find $P(\text{neither } C \text{ nor } D)$	Events A and B are mutually exclusive and $P(A) = 0.2$ and $P(B) = 0.4$. a) Find $P(A \text{ or } B)$ b) Find $P(A \text{ but not } B)$ c) Find $P(\text{neither } A \text{ nor } B)$ a) 0.6 b) 0.2 c) 0.4

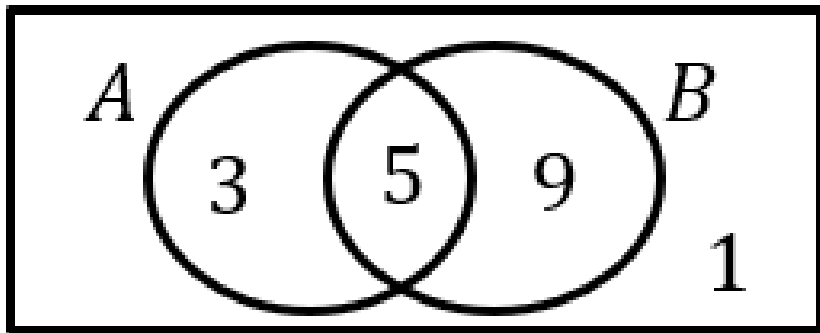| Worked example	Your turn
Events C and D are independent. $P(C) = \frac{5}{7}$ and $P(D) = \frac{3}{8}$. Find $P(C \text{ and } D)$.	Events A and B are independent. $P(A) = \frac{1}{3}$ and $P(B) = \frac{1}{5}$. Find $P(A \text{ and } B)$. $$\frac{1}{15}$$

Worked example	Your turn
The Venn diagram shows the number of people who like each of two different books. Determine if A and B are independent.	The Venn diagram shows the number of people who like each of two different colours. Determine if A and B are independent.

Worked example diagram:

A 3 5 9 B 1

Your turn diagram:

A 6 4 5 B 0
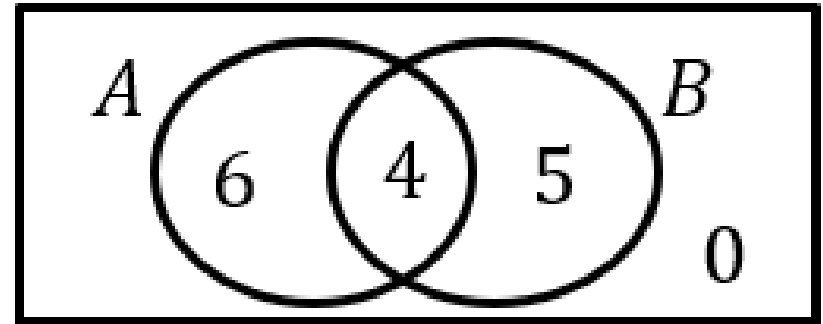
$$P(A) = \frac{10}{15} = \frac{2}{3}$$

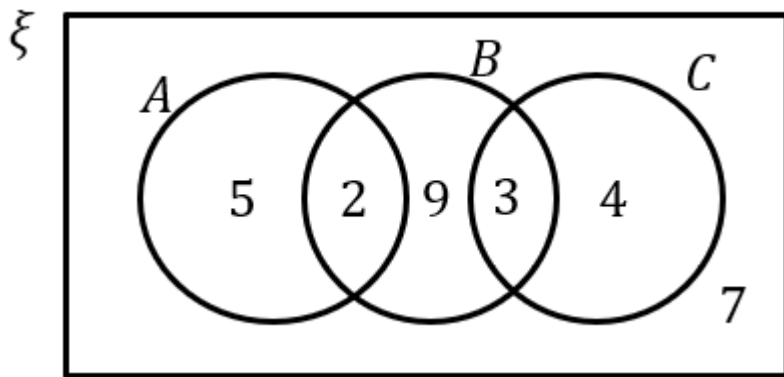$$P(B) = \frac{9}{15} = \frac{3}{5}$$

$$P(A \text{ and } B) = \frac{4}{15}$$

$$P(A) \times P(B) = \frac{2}{3} \times \frac{3}{5} = \frac{6}{15} \neq \frac{4}{15}$$
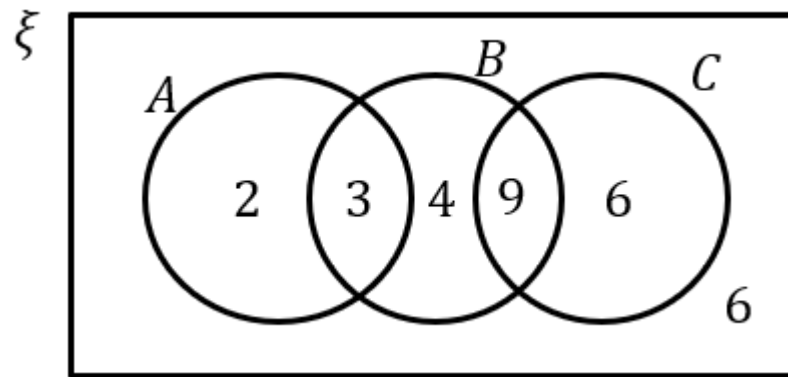
$\therefore$ not independent.

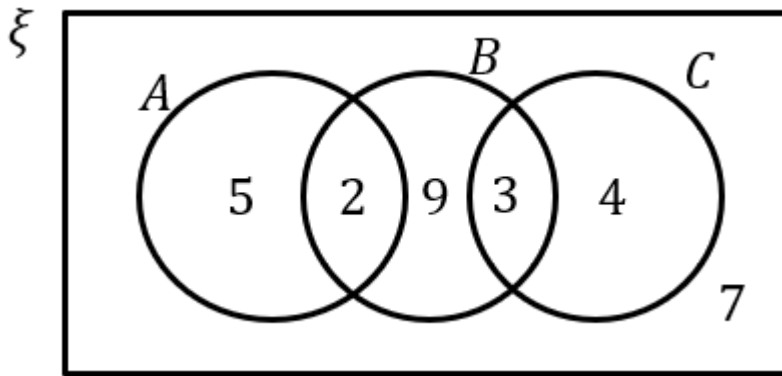| Worked example	Your turn
The Venn diagram shows the number of students in a particular class who watch any of three popular TV programmes. Find the probability that a student chosen at random watches A or B or both.	The Venn diagram shows the number of students in a particular class who watch any of three popular TV programmes. Find the probability that a student chosen at random watches B or C or both.

Worked example diagram:
ξ

A B C

5   2   9   3   4

7

Your turn diagram:
ξ

A B C

2   3   4   9   6

6

$$\frac{26}{30} = \frac{13}{15}$$

Worked example | Your turn

| Worked example	Your turn
The Venn diagram shows the number of students in a particular class who watch any of three popular TV programmes. Determine whether watching A and watching B are statistically independent.	The Venn diagram shows the number of students in a particular class who watch any of three popular TV programmes. Determine whether watching A and watching B are statistically independent.

ξ

A B C

5   (2)   9   (3)   4

7

ξ

A B C

2   (3)   4   (9)   6

6
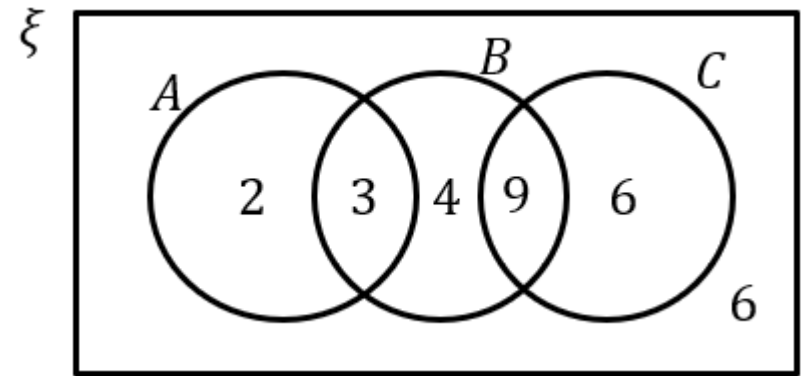
$$P(A) = \frac{7}{30}$$

$$P(B) = \frac{19}{30}$$

$$P(A \text{ and } B) = \frac{4}{30}$$

$$P(A) \times P(B) = \frac{7}{30} \times \frac{19}{30} = \frac{133}{900} \neq \frac{4}{30}$$

$\therefore$ not independent.

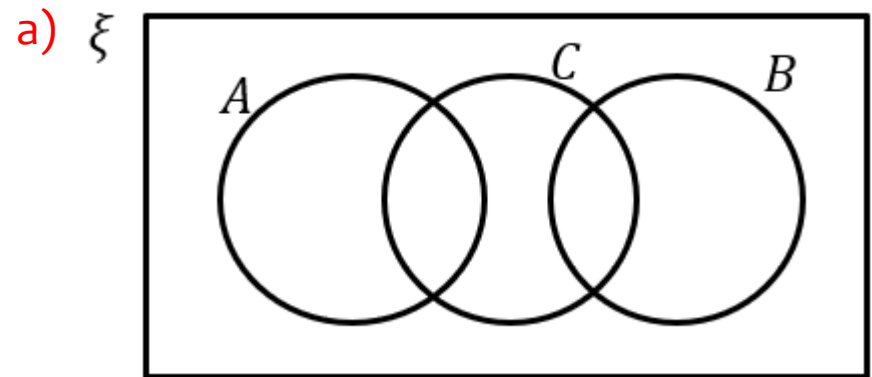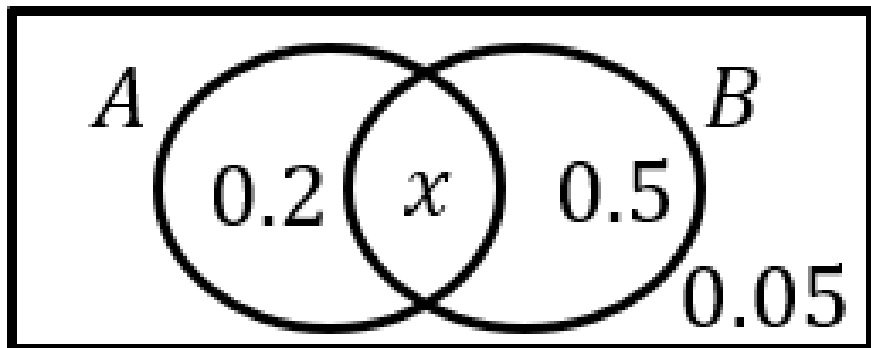| Worked example	Your turn
There are three events D, E, F. The events D and E are mutually exclusive. a) Draw a Venn diagram which represents this information. b) If $P(D) = 0.2$ and $P(E) = 0.7$, determine $P(neither\ D\ nor\ E)$	There are three events A, B, C. The events A and B are mutually exclusive. a) Draw a Venn diagram which represents this information. b) If $P(A) = 0.1$ and $P(B) = 0.6$, determine $P(neither\ A\ nor\ B)$

a) ξ

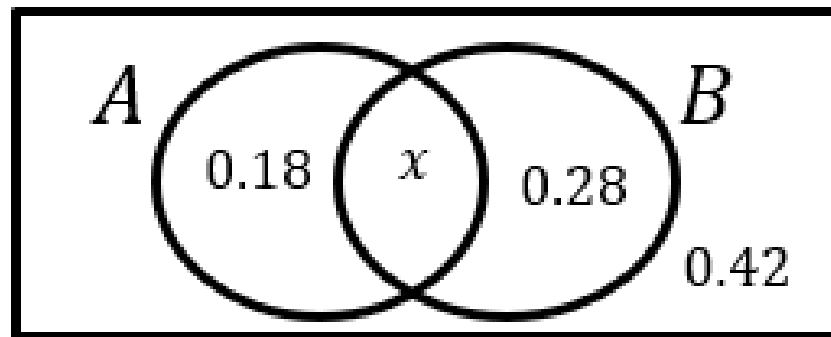

b) 0.3

Worked example	Your turn
Determine if events A and B are independent.	Determine if events A and B are independent.

Determine if events A and B are independent.

A   0.2 x 0.5   B
                    0.05

Determine if events A and B are independent.

A   0.18 x 0.28   B
                    0.42

$x = 1 - 0.18 - 0.28 - 0.42 = 0.12$
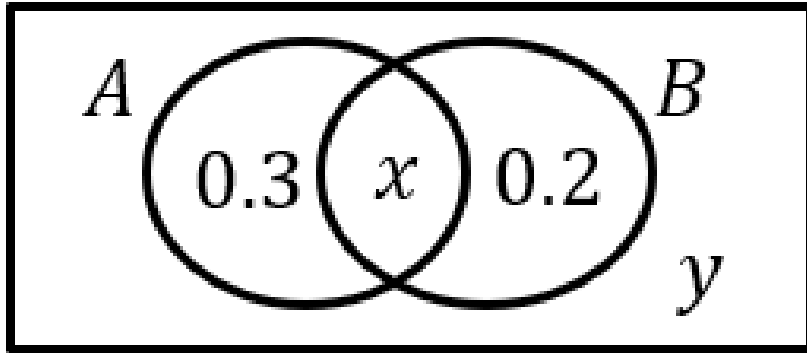$P(A) = 0.3$
$P(B) = 0.4$
$P(A \text{ and } B) = 0.12$
$P(A) \times P(B) = 0.3 \times 0.4 = 0.12$
$\therefore$ independent.

Worked example

Given that A and B are independent, determine the possible values for x and y

A 0.3 x 0.2 B y

Your turn

Given that A and B are independent, determine the possible values of x

A 0.1 x 0.4 B y
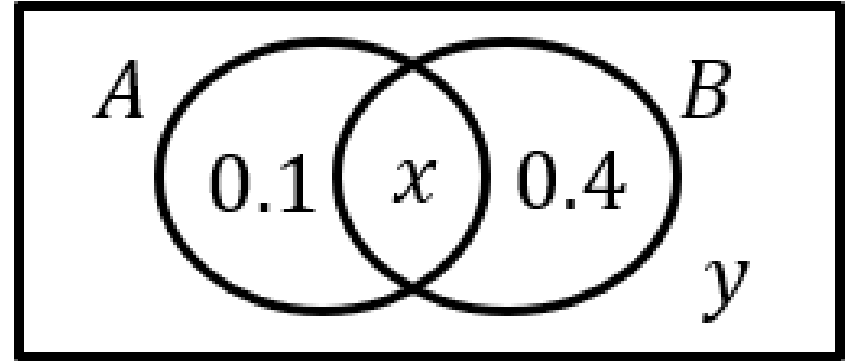
$x = 0.3, y = 0.2$
$x = 0.2, y = 0.3$

5.4) Tree diagrams

Worked example	Your turn
There are three red and nine yellow counters in a bag. A counter is taken from the bag at random and not replaced. A second counter is then taken from the bag. Determine the probability that: a) Both counters are green. b) The counters are different colours.	There are seven green and five blue beads in a bag. A bead is taken from the bag at random and not replaced. A second bead is then taken from the bag. Determine the probability that: a) Both beads are green. b) The beads are different colours. a) $\dfrac{7}{22}$ b) $\dfrac{35}{66}$

Worked example	Your turn
There are 5 blue and 4 red beads in a bag. I take two beads at random. Determine the probability that: a) They are of the same colour. b) They are of different colours.	There are 3 yellow and 2 green counters in a bag. I take two counters at random. Determine the probability that: a) They are of the same colour. b) They are of different colours. a) $\dfrac{2}{5}$ b) $\dfrac{3}{5}$

Worked example	Your turn
A bag contains 15 tokens, 3 coloured blue, 5 coloured red and 7 coloured yellow. Three tokens are drawn from the bag without replacement. Find the probability that the third token is yellow, given that the first two are yellow.	A bag contains 14 tokens, 4 coloured purple, 7 coloured orange and 3 coloured green. Three tokens are drawn from the bag without replacement. Find the probability that the third token is purple, given that the first two are purple. $$\frac{2}{12} = \frac{1}{6}$$

Worked example	Your turn
A bag contains 15 tokens, 3 coloured blue, 5 coloured red and 7 coloured yellow. Three tokens are drawn from the bag without replacement. Find the probability that all three tokens are different colours.	A bag contains 14 tokens, 4 coloured purple, 7 coloured orange and 3 coloured green. Three tokens are drawn from the bag without replacement. Find the probability that all three tokens are different colours. $$\frac{3}{13}$$

Worked example	Your turn
The probability I hit a target on each shot is 0.4. I keep firing until I hit the target. Determine the probability I hit the target on the 6th shot.	The probability I hit a target on each shot is 0.3. I keep firing until I hit the target. Determine the probability I hit the target on the 5th shot.

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