5) Radians

5.1) Radian measure

Chapter CONTENTS

Worked example	Your turn
Convert to radians:	Convert to radians:
$180°$	$360°$ 2π
$90°$	$45°$ $\frac{\pi}{4}$
$60°$	$120°$ $\frac{\pi}{3}$
$135°$	$315°$ $\frac{7\pi}{4}$
$720°$	$72°$ $\frac{2\pi}{5}$

Worked example

Convert to degrees:

3π

$\frac{\pi}{2}$

$\frac{\pi}{6}$

$\frac{5\pi}{4}$

$\frac{3\pi}{5}$

Your turn

Convert to degrees:

5π

$900°$

$\frac{\pi}{3}$

$60°$

$\frac{5\pi}{6}$

$150°$

$\frac{9\pi}{4}$

$405°$

$\frac{4\pi}{5}$

$144°$

Worked example

Sketch the graph for $0 \leq x \leq 2\pi$ of:

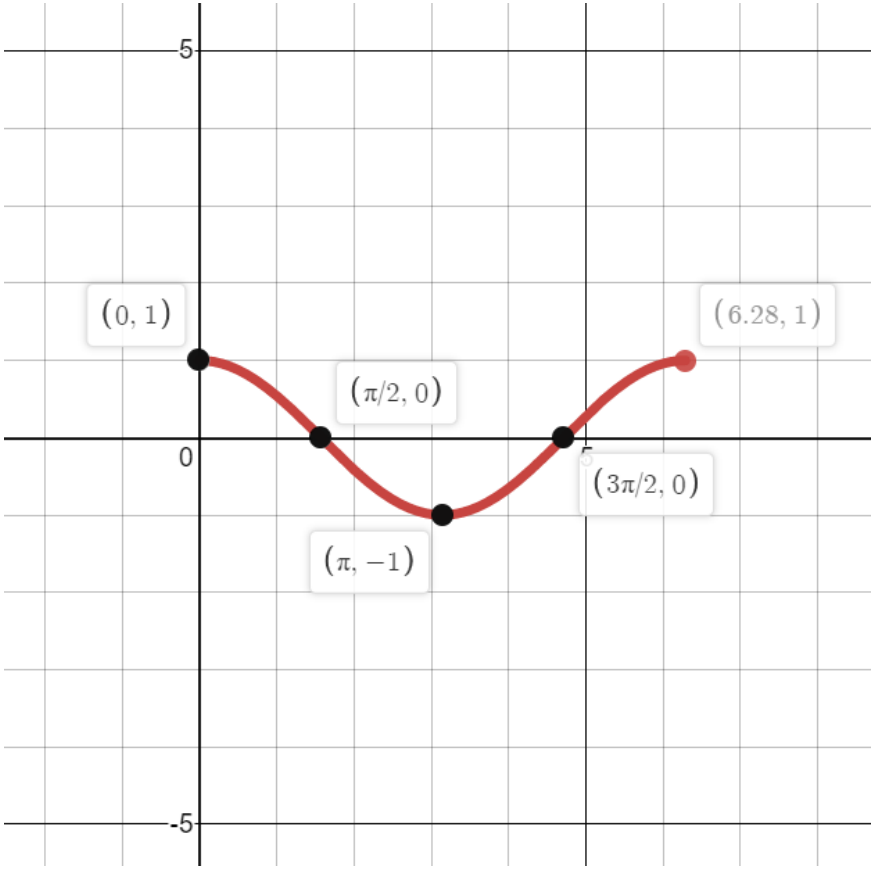
$$y = \sin x$$

$$y = \tan x$$

Your turn

Sketch the graph for $0 \leq x \leq 2\pi$ of:

$$y = \cos x$$



Graphs used with permission from DESMOS: https://www.desmos.com/

Worked example

Find the exact values, without a calculator:

$$\cos\left(\frac{7\pi}{6}\right)$$

$$\sin\left(-\frac{4\pi}{3}\right)$$

Your turn

Find the exact values, without a calculator:

$$\cos\left(\frac{4\pi}{3}\right)$$

$$-\frac{1}{2}$$

$$\sin\left(-\frac{7\pi}{6}\right)$$

$$\frac{1}{2}$$

Worked example

Sketch the graph for $0 \leq x \leq 2\pi$ of:

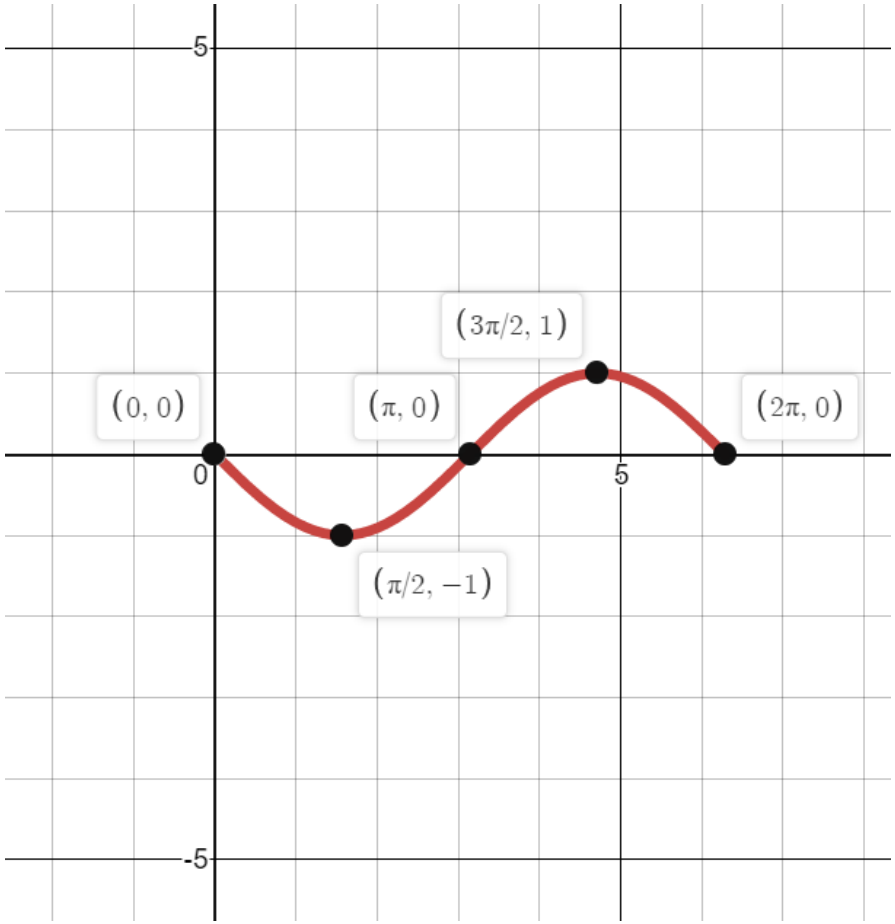
$$y = \sin(x + \frac{\pi}{4})$$

$$y = \tan\left(x - \frac{\pi}{3}\right)$$

Your turn

Sketch the graph for $0 \leq x \leq 2\pi$ of:

$$y = \cos\left(x + \frac{\pi}{2}\right)$$



Graphs used with permission from DESMOS: https://www.desmos.com/

Worked example

Sketch the graph for $0 \leq x \leq 2\pi$ of:

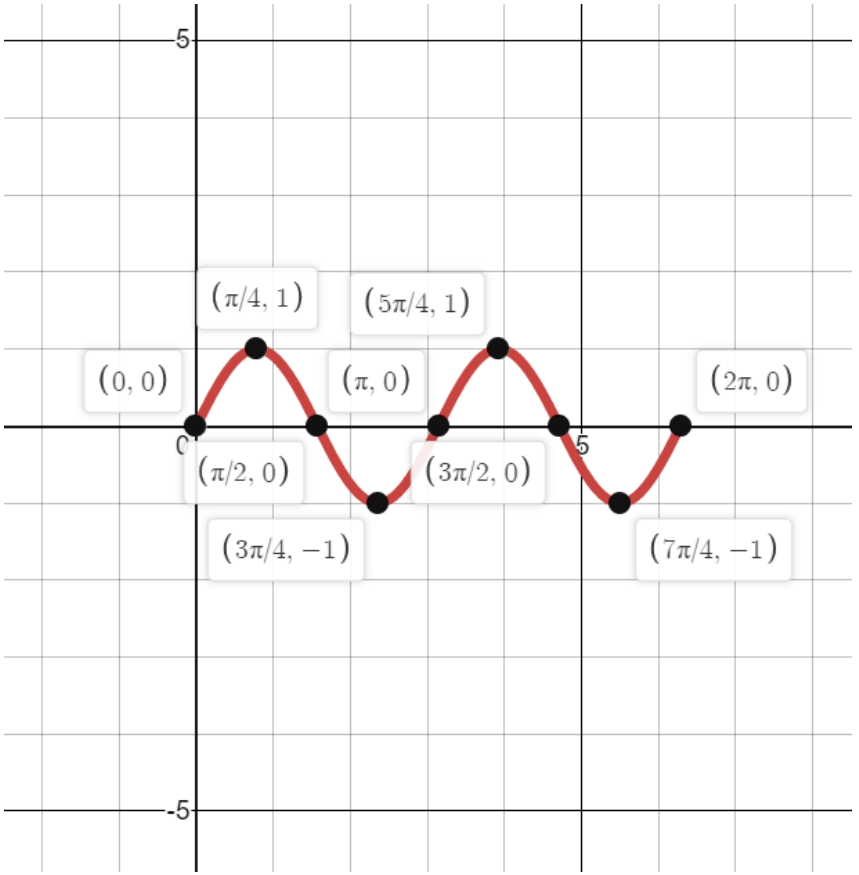
$$y = \cos(4x)$$

$$y = \tan(3x)$$

Your turn

Sketch the graph for $0 \leq x \leq 2\pi$ of:

$$y = \sin(2x)$$



Graphs used with permission from DESMOS: https://www.desmos.com/

5.2) Arc length

Chapter CONTENTS

Worked example

Find the length of the arc of a circle of radius 5.2 cm, given that the arc subtends an angle of 0.4 radians at the centre of the circle.

Find the length of the arc of a circle of radius 10.4 cm, given that the arc subtends an angle of 0.2 radians at the centre of the circle.

Your turn

Find the length of the arc of a circle of radius 5.2 cm, given that the arc subtends an angle of 0.8 radians at the centre of the circle.

$4.16\ cm$

Worked example

An arc AB of a circle with radius 0.35 cm and centre O has a length of 2.45 cm. Find the angle $\angle AOB$ subtended by the arc at the centre of the circle

An arc AB of a circle with radius 0.7 cm and centre O has a length of 4.9 cm. Find the angle $\angle AOB$ subtended by the arc at the centre of the circle

Your turn

An arc AB of a circle with radius 7 cm and centre O has a length of 2.45 cm. Find the angle $\angle AOB$ subtended by the arc at the centre of the circle

$0.35\ rad$

Worked example

The border of a garden pond consists of a straight edge AB of length 4.8 m, and a curved part C, also connecting A and B. The curve part is an arc of a circle, centre O, radius 4 m.
Find the length of C.

Your turn

The border of a garden pond consists of a straight edge AB of length 2.4 m, and a curved part C, also connecting A and B. The curve part is an arc of a circle, centre O, radius 2 m.
Find the length of C.

9.99 m

Worked example

A triangle ABC is such that $AB = 4\ cm$, $AC = 5.5\ cm$ and $\angle BAC = 0.35$ radians.
The arc BD, where D lies on AC, is an arc of a circle with centre A and radius 4 cm.
A region R, is bounded by the straight lines BC and CD and the arc BD.
Find the perimeter of R

Your turn

A triangle ABC is such that $AB = 8\ cm$, $AC = 11\ cm$ and $\angle BAC = 0.7$ radians.
The arc BD, where D lies on AC, is an arc of a circle with centre A and radius 8 cm.
A region R, is bounded by the straight lines BC and CD and the arc BD.
Find the perimeter of R

$15.7\ cm$ (3 sf)

Worked example

A sector of a circle of radius 30 cm contains an angle of θ radians. Given that the perimeter of the sector is 84 cm, find the value of θ

Your turn

A sector of a circle of radius 15 cm contains an angle of θ radians. Given that the perimeter of the sector is 42 cm, find the value of θ

$\theta = 0.8$ rad

Worked example

The perimeter of a sector OAB is four times the length of the arc AB. Find the size of angle AOB

Your turn

The perimeter of a sector OAB is four times the length of the arc AB. Find the size of angle AOB

$\theta = 2$ rad

5.3) Areas of sectors and segments

Chapter CONTENTS

Worked example

A circle, centre O, radius 5.2 cm has a minor sector OAB where the arc AB subtends an angle of 0.4 radians at the centre of the circle.
Find the area of the sector.

A circle, centre O, radius 5.2 cm has a minor sector OAB where the arc AB subtends an angle of 0.2 radians at the centre of the circle.
Find the area of the sector.

Your turn

A circle, centre O, radius 5.2 cm has a minor sector OAB where the arc AB subtends an angle of 0.8 radians at the centre of the circle.
Find the area of the sector.

$10.816\ cm^2$

Worked example

A circle, centre O, radius 5.2 cm has a minor sector OAB where the arc AB subtends an angle of 0.4 radians at the centre of the circle.
A segment is enclosed by a chord AB and the arc AB.
Find the area of the segment.

A circle, centre O, radius 5.2 cm has a minor sector OAB where the arc AB subtends an angle of 0.2 radians at the centre of the circle.
A segment is enclosed by a chord AB and the arc AB.
Find the area of the segment.

Your turn

A circle, centre O, radius 5.2 cm has a minor sector OAB where the arc AB subtends an angle of 0.8 radians at the centre of the circle.
A segment is enclosed by a chord AB and the arc AB.
Find the area of the segment.

$1.12\ cm^2$ (3 sf)

Worked example	Your turn
The area of the minor sector AOB is 14.45 cm². Given that $\angle AOB = 0.4$ radians and O is the centre of the circle, calculate the length of the radius	The area of the minor sector AOB is 28.9 cm². Given that $\angle AOB = 0.8$ radians and O is the centre of the circle, calculate the length of the radius $8.5\ cm$

Worked example	Your turn
A sector of a circle of radius 110 m and perimeter 352 m. Calculate the area of the sector	A sector of a circle of radius 110 m and perimeter 176 m. Calculate the area of the sector $1815\ m^2$

Worked example

OAB is a sector of a circle, centre O, radius $8m$.
The chord AB is 10m long.
Find the area of the segment.

Your turn

OAB is a sector of a circle, centre O, radius $4m$.
The chord AB is 5m long.
Find the area of the segment.

$3.00\ m^2$ (3 sf)

Worked example

AB is the diameter of a semicircle, centre O, radius r cm.
C is a point on the semicircle.
<BOC $= \theta$ radians.
Given that the area of ΔAOC is six times the segment enclosed by CB, show that $6\theta - 7\sin\theta = 0$

Your turn

AB is the diameter of a semicircle, centre O, radius r cm.
C is a point on the semicircle.
<BOC $= \theta$ radians.
Given that the area of ΔAOC is three times the segment enclosed by CB, show that $3\theta - 4\sin\theta = 0$

Shown

Worked example

OAB is a sector of a circle, centre O, radius 18 cm and angle 0.35 radians.
C lies outside the sector.
AC is a straight line, perpendicular to OA.
OBC is a straight line.
Find the area of the region bounded by the arc AB and the lines AC and BC

Your turn

OAB is a sector of a circle, centre O, radius 9 cm and angle 0.7 radians.
C lies outside the sector.
AC is a straight line, perpendicular to OA.
OBC is a straight line.
Find the area of the region bounded by the arc AB and the lines AC and BC

$5.76\ cm^2$ (3 sf)

Worked example

OPQ is a sector of a circle, centre O, radius 20 cm where <POQ $= 0.6$ radians.
The point R is on OQ such that the ratio OR:RQ is $1:3$
A region is bounded by the arc PQ, QR and a line RP.

a) Find the perimeter of the region
b) Find the area of the region

Your turn

OPQ is a sector of a circle, centre O, radius 10 cm where <POQ $= 0.3$ radians.
The point R is on OQ such that the ratio OR:RQ is $1:3$
A region is bounded by the arc PQ, QR and a line RP.

a) Find the perimeter of the region
b) Find the area of the region

a) $18.1\ cm$ (3 sf)
b) $11.3\ cm^2$ (3 sf)

5.4) Solving trigonometric equations

Chapter CONTENTS

Worked example

Solve in the interval $0 \leq \theta \leq 2\pi$:

$$\cos\theta = \frac{1}{2}$$

$$\tan\theta = 1$$

Your turn

Solve in the interval $0 \leq \theta \leq 2\pi$:

$$\sin\theta = \frac{1}{2}$$

$$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$$

Worked example

Solve in the interval $0 \leq \theta \leq 2\pi$:

$$\cos\theta + 1 = \frac{1}{2}$$

$$\tan\theta - 2 = 1$$

Your turn

Solve in the interval $0 \leq \theta \leq 2\pi$:

$$\sin\theta + 1 = \frac{1}{2}$$

$$\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$$

Worked example

Solve in the interval $0 \leq \theta \leq 2\pi$:

$$5\cos\theta + 2 = 2.3$$

$$4\tan\theta - 5 = 1$$

Your turn

Solve in the interval $0 \leq \theta \leq 2\pi$:

$$3\sin\theta + 1 = 0.4$$

$\theta = 3.34, 6.08$ (3 sf)

Worked example

Solve in the interval $0 \leq \theta \leq 2\pi$:

$$\cos(\theta - \frac{\pi}{2}) = \frac{1}{2}$$

$$\tan(\theta + \frac{\pi}{3}) = 1$$

Your turn

Solve in the interval $0 \leq \theta \leq 2\pi$:

$$\sin(\theta - \frac{\pi}{4}) = \frac{1}{2}$$

$$\theta = \frac{5\pi}{12}, \frac{13\pi}{12}$$

Worked example

Solve in the interval $0 \leq \theta \leq 2\pi$:

$$\cos 5\theta = \frac{\sqrt{3}}{2}$$

$$\tan 4\theta = \sqrt{3}$$

Your turn

Solve in the interval $0 \leq \theta \leq 2\pi$:

$$\sin 3\theta = \frac{\sqrt{3}}{2}$$

$$\theta = \frac{\pi}{9}, \frac{2\pi}{9}, \frac{7\pi}{9}, \frac{8\pi}{9}, \frac{13\pi}{9}, \frac{14\pi}{9}$$

Worked example

Solve in the interval $0 \leq \theta \leq 2\pi$:

$$\cos^2\theta = \frac{3}{4}$$

$$\tan^2\theta = 3$$

Your turn

Solve in the interval $0 \leq \theta \leq 2\pi$:

$$\sin^2\theta = \frac{1}{4}$$

$$\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

Worked example

Solve in the interval $0 \leq \theta \leq 2\pi$:

$$2\cos^2\theta + 5\cos\theta - 3 = 0$$

$$2\tan^2\theta - 5\tan\theta - 3 = 0$$

Your turn

Solve in the interval $0 \leq \theta \leq 2\pi$:

$$2\sin^2\theta - 5\sin\theta - 3 = 0$$

$$\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$$

Worked example

Solve in the interval $0 \leq \theta \leq 2\pi$:

$$5\cos^2\theta + 2\cos\theta = 0$$

$$4\tan^2\theta - 3\tan\theta = 0$$

Your turn

Solve in the interval $0 \leq \theta \leq 2\pi$:

$$5\sin^2\theta - 2\sin\theta = 0$$

$\theta = 0, 0.412, 2.73, \pi, 2\pi$

Worked example

Solve in the interval $0 \leq \theta \leq 2\pi$:

$$5\cos\theta \, sin\theta + 2\cos\theta = 0$$

Your turn

Solve in the interval $0 \leq \theta \leq 2\pi$:

$$5\cos\theta \, sin\theta + 2\sin\theta = 0$$

$$\theta = 0, 1.98, \pi, 4.30, 2\pi$$

Worked example

Solve in the interval $0 \leq \theta < 2\pi$:

$$4\tan x = 5\cos x$$

Your turn

Solve in the interval $0 \leq \theta < 2\pi$:

$$2\tan x = 3\sin x$$

$$\theta = 0, 0.841, \pi, 5.44$$

Worked example

Find all the solutions, in the interval $0 \leq x < 2\pi$, of the equation

$$2\sin^2 x + 1 = -5\cos x,$$

giving each solution in terms of π.

Your turn

Find all the solutions, in the interval $0 \leq x < 2\pi$, of the equation

$$2\cos^2 x + 1 = 5\sin x,$$

giving each solution in terms of π.

$$x = \frac{\pi}{6}, \frac{5\pi}{6}$$

5.5) Small angle approximations

Chapter CONTENTS

Worked example

When θ is small, find the approximate value of:

a) $\frac{\sin\theta+\tan 4\theta}{3\theta}$

b) $\frac{\cos 6\theta-1}{\theta\tan 3\theta}$

c) $\sin 3\theta+\tan 4\theta-\cos 5\theta$

Your turn

When θ is small, find the approximate value of:

a) $\frac{\sin 2\theta+\tan\theta}{2\theta}$

b) $\frac{\cos 4\theta-1}{\theta\sin 2\theta}$

c) $\sin 5\theta+\tan 2\theta-\cos 2\theta$

a) $\frac{3}{2}$

b) -4

c) -1

Worked example	Your turn
Find the percentage error when calculating the value of $\cos(0.123\ rad)$ using the small-angle approximations	Find the percentage error when calculating the value of $\cos(0.246\ rad)$ using the small-angle approximations 0.015701% (6 dp)

Worked example

When θ is small, find the approximate value of:

$$\frac{\sin\theta - \cos 6\theta - 4}{\sin 2\theta - 1}$$

Your turn

When θ is small, find the approximate value of:

$$\frac{1 - 2\tan\theta - 4\cos 2\theta}{\tan 2\theta + 1}$$

-3