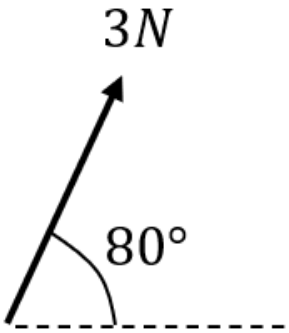
5) Forces and friction

5.1) Resolving forces

Chapter CONTENTS

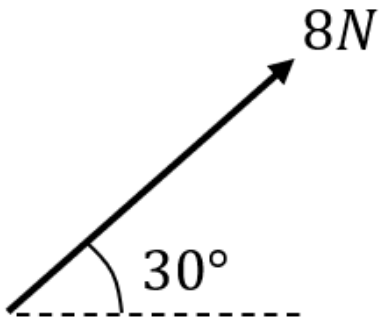
Worked example

Convert each force to the form $a\boldsymbol{i} + b\boldsymbol{j}$, where $\boldsymbol{i}$ and $\boldsymbol{j}$ are the positive x and y directions respectively.



Your turn

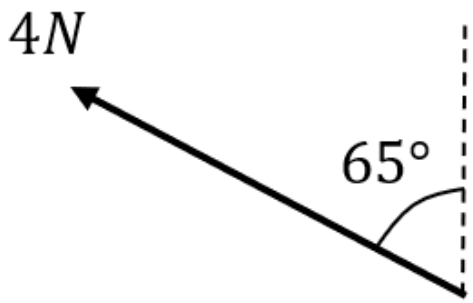
Convert each force to the form $a\boldsymbol{i} + b\boldsymbol{j}$, where $\boldsymbol{i}$ and $\boldsymbol{j}$ are the positive x and y directions respectively.



$4\sqrt{3}\,\boldsymbol{i} + 4\boldsymbol{j}$

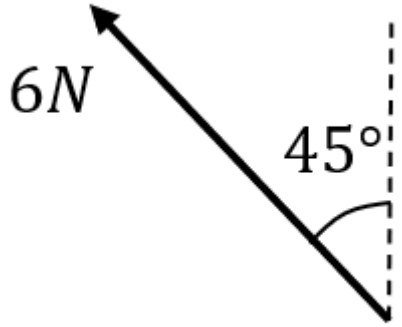
Worked example

Convert each force to the form $a\boldsymbol{i} + b\boldsymbol{j}$, where $\boldsymbol{i}$ and $\boldsymbol{j}$ are the positive x and y directions respectively.



Your turn

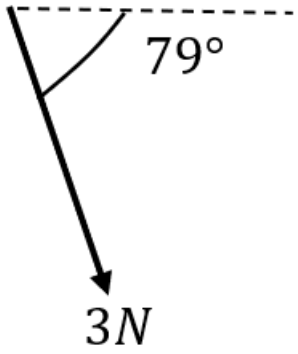
Convert each force to the form $a\boldsymbol{i} + b\boldsymbol{j}$, where $\boldsymbol{i}$ and $\boldsymbol{j}$ are the positive x and y directions respectively.



$-3\sqrt{2}\,\boldsymbol{i} + 3\sqrt{2}\boldsymbol{j}$

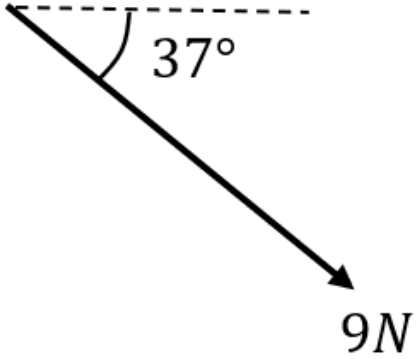
Worked example

Convert each force to the form $a\boldsymbol{i} + b\boldsymbol{j}$, where $\boldsymbol{i}$ and $\boldsymbol{j}$ are the positive x and y directions respectively.



Your turn

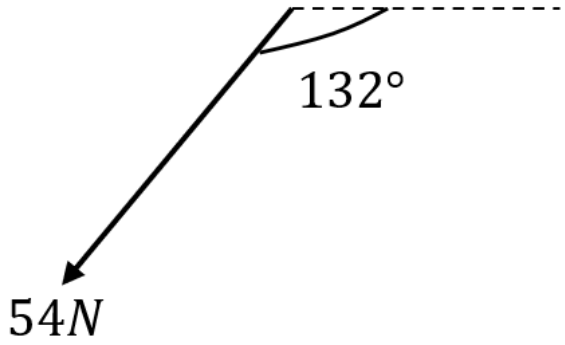
Convert each force to the form $a\boldsymbol{i} + b\boldsymbol{j}$, where $\boldsymbol{i}$ and $\boldsymbol{j}$ are the positive x and y directions respectively.



$7.19\boldsymbol{i} - 5.42\boldsymbol{j}$ (3 sf)

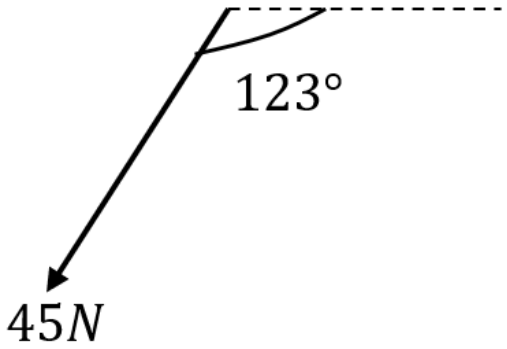
Worked example

Convert each force to the form $a\boldsymbol{i} + b\boldsymbol{j}$, where $\boldsymbol{i}$ and $\boldsymbol{j}$ are the positive x and y directions respectively.



Your turn

Convert each force to the form $a\boldsymbol{i} + b\boldsymbol{j}$, where $\boldsymbol{i}$ and $\boldsymbol{j}$ are the positive x and y directions respectively.



$-24.5\boldsymbol{i} - 37.7\boldsymbol{j}$

Worked example

A box of mass 10kg lies on a smooth horizontal floor.
A force of 8N is applied at an angle of $50°$ causing the box to accelerate horizontally along the floor.

(a) Work out the acceleration of the box.

(b) Calculate the normal reaction between the box and the floor.

Your turn

A box of mass 8kg lies on a smooth horizontal floor.
A force of 10N is applied at an angle of $30°$ causing the box to accelerate horizontally along the floor.

(a) Work out the acceleration of the box.

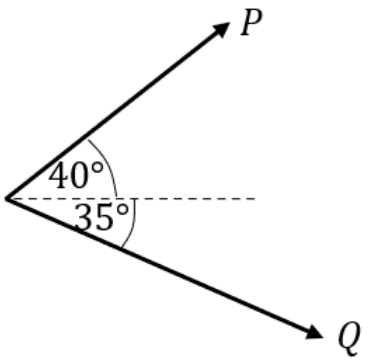
(b) Calculate the normal reaction between the box and the floor.

a) $\frac{5\sqrt{3}}{8}\ ms^{-2} = 1.1\ ms^{-2}$ (2 sf)

b) $73\ N$ (2 sf)

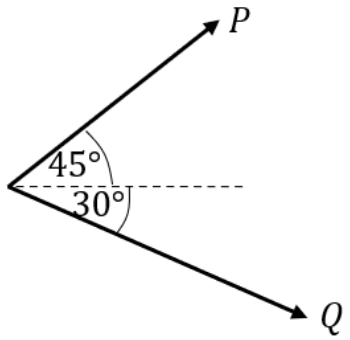
Worked example

Two forces P and Q act on a particle as shown.
P has a magnitude of 5N and Q has a magnitude of 4N.
Work out the magnitude and direction of the resultant force.



Your turn

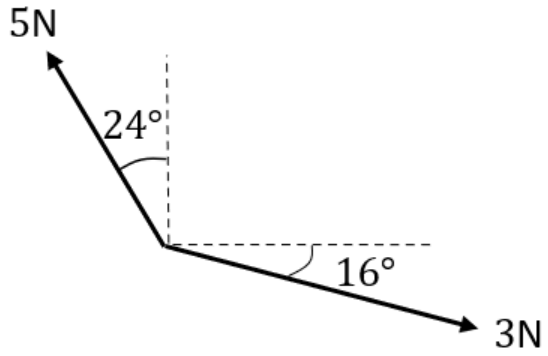
Two forces P and Q act on a particle as shown. P has a magnitude of 10N and Q has a magnitude of 8N.
Work out the magnitude and direction of the resultant force.



14.3 N (3 sf) acting at an angle of 12.4° (3 sf) above the horizontal.

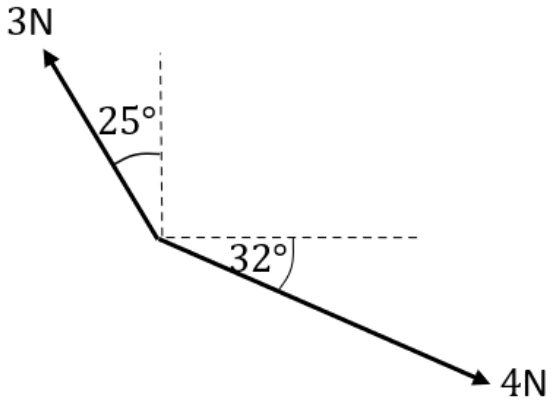
Worked example

Two forces act on a particle as shown.
Determine the magnitude and direction (anticlockwise from the positive x direction) of the resultant force.



Your turn

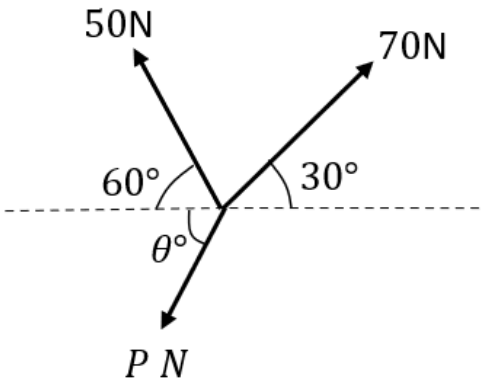
Two forces act on a particle as shown.
Determine the magnitude and direction (anticlockwise from the positive x direction) of the resultant force.



$2.21\ N$ (3 sf) acting at an angle of $15.8°$ (3 sf)

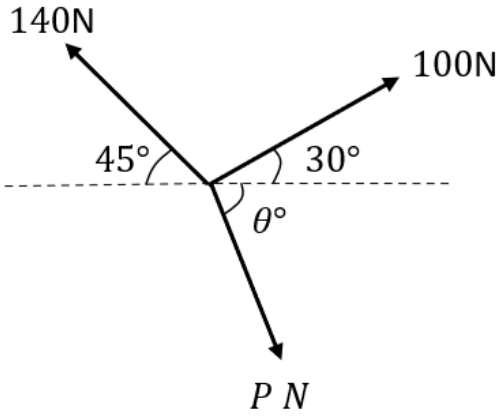
Worked example

Three forces act on a particle as shown.
Given that the particle is in equilibrium, calculate the magnitude of P



Your turn

Three forces act on a particle as shown.
Given that the particle is in equilibrium, calculate the magnitude of P



150 N (3 sf)

5.2) Inclined planes

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Worked example

A block of mass 15kg slides down a smooth slope angled at $10°$ to the horizontal.

(a) Calculate the magnitude of the normal reaction of the slope on the block.

(b) Find the acceleration of the block.

Your turn

A block of mass 10kg slides down a smooth slope angled at $15°$ to the horizontal.

(a) Calculate the magnitude of the normal reaction of the slope on the block.

(b) Find the acceleration of the block.

a) $95\ N$ (2 sf)

b) $2.5\ ms^{-2}$ (2 sf)

Worked example

A particle of mass m is pushed up a smooth slope, inclined at $60°$ by a force of magnitude $10g$ N acting at angle of $30°$ to the slope, causing the particle to accelerate up the slope at 0.25 ms^{-2}.

Show that the mass of the particle is $\left(\frac{20\sqrt{3}g}{1+2\sqrt{3}g}\right)$ kg

Your turn

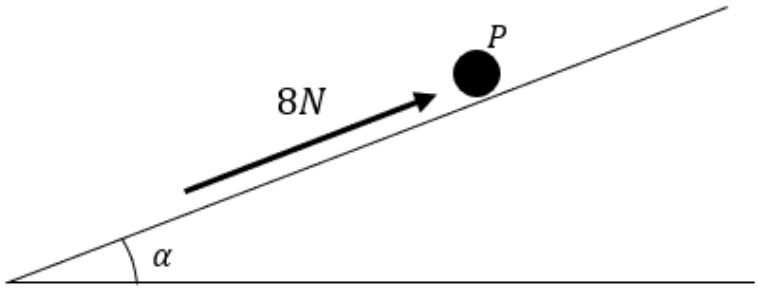
A particle of mass m is pushed up a smooth slope, inclined at $30°$ by a force of magnitude $5g$ N acting at angle of $60°$ to the slope, causing the particle to accelerate up the slope at 0.5 ms^{-2}.

Show that the mass of the particle is $\left(\frac{5g}{1+g}\right)$ kg

Shown

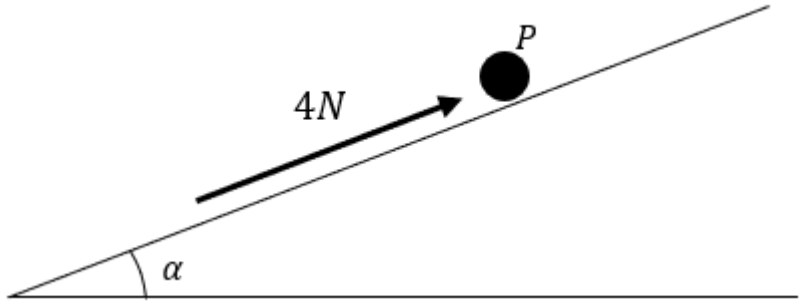
Worked example

A particle P of mass 4kg is moving on a smooth slope and is being acted on by a force of 8N that acts parallel to the slope, as shown.
The slop is inclined at an angle α to the horizontal, where $\tan\alpha = \frac{5}{12}$.
Work out the acceleration of the particle.



Your turn

A particle P of mass 2kg is moving on a smooth slope and is being acted on by a force of 4N that acts parallel to the slope, as shown.
The slop is inclined at an angle α to the horizontal, where $\tan\alpha = \frac{3}{4}$.
Work out the acceleration of the particle.



$3.9\ ms^{-2}$ down the slope

5.3) Friction

Chapter CONTENTS

Worked example

A particle of mass 10kg is pulled along a rough horizontal surface by a horizontal force of magnitude 40N. The coefficient of friction between the particle and the floor is 0.4. Calculate:

(a) the magnitude of frictional force

(b) the acceleration of the particle.

Your turn

A particle of mass 5kg is pulled along a rough horizontal surface by a horizontal force of magnitude 20N. The coefficient of friction between the particle and the floor is 0.2. Calculate:

(a) the magnitude of frictional force

(b) the acceleration of the particle.

a) $9.8\ N$ (2 sf)

b) $2.0\ ms^{-2}$ (2 sf)

Worked example

A block of mass $10\ kg$ lies on rough horizontal ground.
The coefficient of friction between the block and the ground is 0.4.
A horizontal force P is applied to the block.
Find the magnitude of the frictional force acting on the block and the acceleration of the block when the magnitude of P is:
a) $30\ N$
b) $39.2\ N$
c) $90\ N$

Your turn

A block of mass $5\ kg$ lies on rough horizontal ground.
The coefficient of friction between the block and the ground is 0.4.
A horizontal force P is applied to the block.
Find the magnitude of the frictional force acting on the block and the acceleration of the block when the magnitude of P is:
a) $10\ N$
b) $19.6\ N$
c) $30\ N$

a) $10\ N$; $0\ ms^{-2}$; Block at rest in equilibrium
b) $19.6\ N$; $0\ ms^{-2}$; Block at rest in limiting equilibrium
c) $19.6\ N$; $2.1\ ms^{-2}$ in the direction of P

Worked example

A particle of mass 4kg is sliding down a rough slope that is inclined at $60°$ to the horizontal.
Given that the acceleration of the particle is 2 ms^{-2}, find the coefficient of friction μ between the particle and the slope.

Your turn

A particle of mass 2kg is sliding down a rough slope that is inclined at $30°$ to the horizontal.
Given that the acceleration of the particle is 1 ms^{-2}, find the coefficient of friction μ between the particle and the slope.

0.46 (2 sf)

Worked example

A box of mass $4\ kg$ is held in equilibrium on a fixed rough inclined plane by a rope.
The rope lies in a vertical plane containing a line of greatest slope of the inclined plane.
The ropes is inclined to the plane at an angle α, where $\tan\alpha = \frac{5}{12}$, and the plane is at an angle of $45°$ to the horizontal.
The coefficient of friction between the box and the inclined plane is $\frac{1}{4}$ and the box is on the point of slipping up the plane.
By modelling the box as a particle and the rope as a light inextensible string, find the tension in the rope.

Your turn

A box of mass $2\ kg$ is held in equilibrium on a fixed rough inclined plane by a rope.
The rope lies in a vertical plane containing a line of greatest slope of the inclined plane.
The ropes is inclined to the plane at an angle α, where $\tan\alpha = \frac{3}{4}$, and the plane is at an angle of $30°$ to the horizontal.
The coefficient of friction between the box and the inclined plane is $\frac{1}{3}$ and the box is on the point of slipping up the plane.
By modelling the box as a particle and the rope as a light inextensible string, find the tension in the rope.

$15\ N$ (2 sf)