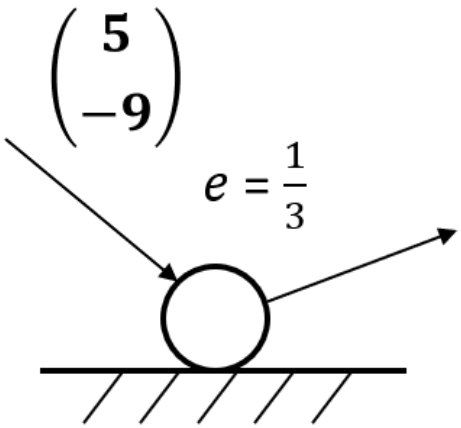
5) Elastic collisions in two dimensions

5.1) Oblique impact with a fixed surface

Chapter CONTENTS

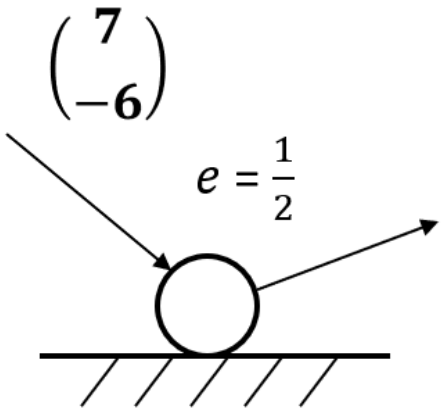
Worked example

A smooth sphere hits a smooth horizontal plane. Find the velocity after the collision.



Your turn

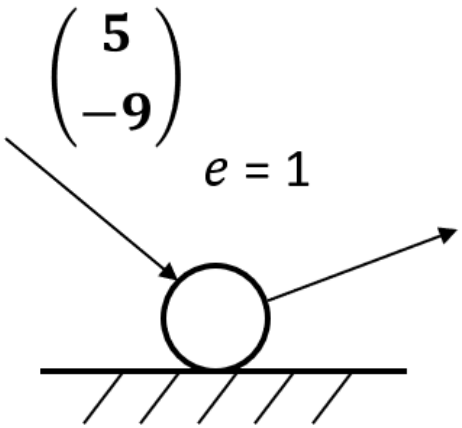
A smooth sphere hits a smooth horizontal plane. Find the velocity after the collision.



$\begin{pmatrix} 7 \\ 3 \end{pmatrix}$

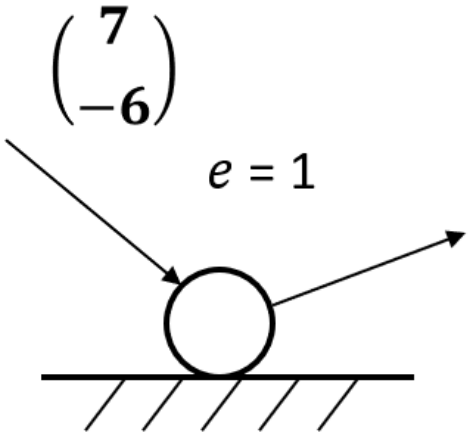
Worked example

A smooth sphere hits a smooth horizontal plane. Find the velocity after the collision.



Your turn

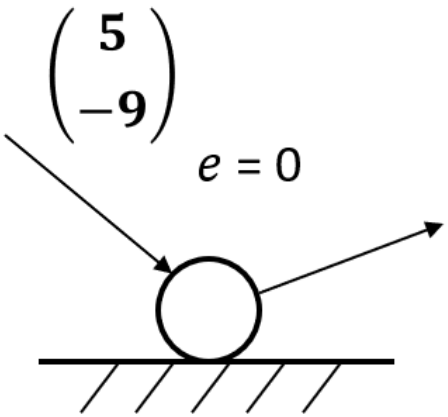
A smooth sphere hits a smooth horizontal plane. Find the velocity after the collision.



$\begin{pmatrix} 7 \\ 6 \end{pmatrix}$

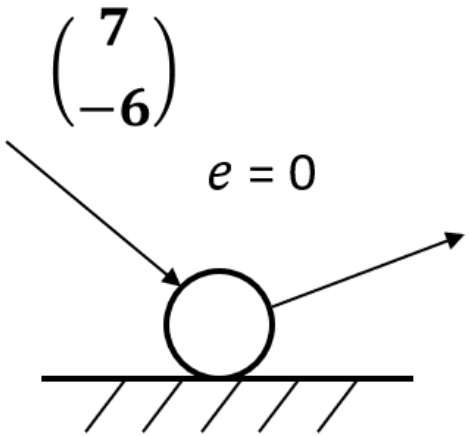
Worked example

A smooth sphere hits a smooth horizontal plane. Find the velocity after the collision.



Your turn

A smooth sphere hits a smooth horizontal plane. Find the velocity after the collision.



$\begin{pmatrix} 7 \\ 0 \end{pmatrix}$

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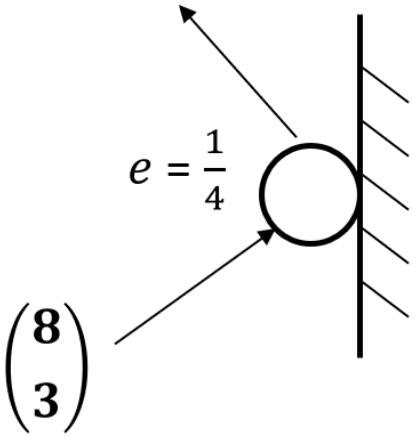
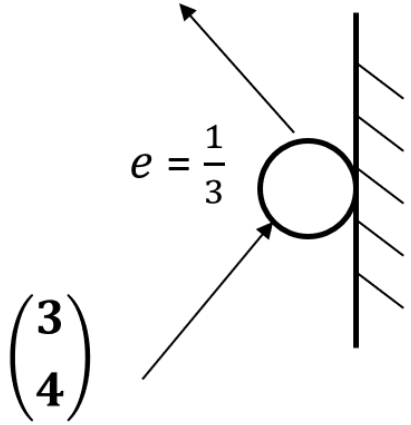
Worked example

A smooth sphere hits a smooth vertical plane. Find the velocity after the collision.



Your turn

A smooth sphere hits a smooth horizontal plane. Find the velocity after the collision.



$\begin{pmatrix} -1 \\ 3 \end{pmatrix}$

Worked example

A smooth sphere S is moving on a smooth horizontal plane with speed u when it collides with a smooth fixed vertical wall.
At the instant of collision the direction of motion of S makes an angle of 30° with the wall. The coefficient of restitution between S and the wall is $\frac{1}{2}$. Find:

a) The speed of S immediately after the collision
b) The angle of deflection of S

Your turn

A smooth sphere S is moving on a smooth horizontal plane with speed u when it collides with a smooth fixed vertical wall.
At the instant of collision the direction of motion of S makes an angle of 60° with the wall. The coefficient of restitution between S and the wall is $\frac{1}{4}$. Find:

a) The speed of S immediately after the collision
b) The angle of deflection of S

a) $\frac{\sqrt{19}}{8}u$
b) $83.4°$ (3 sf)

Worked example

A small smooth ball is falling vertically. The ball strikes a smooth plane which is inclined at an angle α to the horizontal, where $\tan\alpha = \frac{1}{3}$. Immediately before striking the plane the ball has speed $10\ ms^{-1}$. The coefficient of restitution between the ball and the plane is $\frac{1}{3}$.

Find the speed of the ball immediately after impact.

Your turn

A small smooth ball is falling vertically. The ball strikes a smooth plane which is inclined at an angle α to the horizontal, where $\tan\alpha = \frac{1}{2}$. Immediately before striking the plane the ball has speed $5\ ms^{-1}$. The coefficient of restitution between the ball and the plane is $\frac{1}{2}$.

Find the speed of the ball immediately after impact.

$3.16\ ms^{-1}$ (3 sf)

Worked example

A small smooth ball of mass $4\ kg$ is moving in the xy-plane and collides with a smooth fixed vertical wall which contains the y-axis. The velocity of the ball just before impact is $(-12\boldsymbol{i} - 8\boldsymbol{j})\ ms^{-1}$. The coefficient of restitution between the sphere and the wall is $\frac{1}{2}$. Find:

a) The velocity of the ball immediately after the impact.
b) The kinetic energy lost as a result of the impact.
c) The angle of deflection of the ball

Your turn

A small smooth ball of mass $2\ kg$ is moving in the xy-plane and collides with a smooth fixed vertical wall which contains the y-axis. The velocity of the ball just before impact is $(-6\boldsymbol{i} - 4\boldsymbol{j})\ ms^{-1}$. The coefficient of restitution between the sphere and the wall is $\frac{1}{3}$. Find:

a) The velocity of the ball immediately after the impact.
b) The kinetic energy lost as a result of the impact.
c) The angle of deflection of the ball

a) $(2\boldsymbol{i} - 4\boldsymbol{j})\ ms^{-1}$
b) $32\ J$
c) $82.9°$ (3 sf)

Worked example

A smooth sphere S, of mass m, is moving with velocity $7\boldsymbol{i} + 2\boldsymbol{j}$ when it collides with a smooth fixed vertical wall. After the collision the velocity of the sphere, S, is $3\boldsymbol{i} - 3\boldsymbol{j}$

a) The impulse exerted by the wall on the ball.

b) Use the scalar product to find the coefficient of restitution between the sphere and the wall.

Your turn

A smooth sphere S, of mass m, is moving with velocity $2\boldsymbol{i} + 7\boldsymbol{j}$ when it collides with a smooth fixed vertical wall. After the collision the velocity of the sphere, S, is $\boldsymbol{i} - 3\boldsymbol{j}$

a) The impulse exerted by the wall on the ball.

b) Use the scalar product to find the coefficient of restitution between the sphere and the wall.

a) $m(-\boldsymbol{i} - 10\boldsymbol{j})$

b) $\frac{29}{72}$

5.2) Successive oblique impacts

Chapter CONTENTS

Worked example

Two vertical walls meet at right angles. A smooth sphere slides across a smooth, horizontal floor, bouncing off each wall in turn. Just before the first impact the sphere is moving with speed 8 ms^{-1} at an angle of 60°. The coefficient of restitution between the sphere and both walls is $\frac{1}{4}$. Find

a) The direction of motion and speed of the sphere after the first collision
b) The direction of motion and speed of the sphere after the second collision.

Your turn

Two vertical walls meet at right angles. A smooth sphere slides across a smooth, horizontal floor, bouncing off each wall in turn. Just before the first impact the sphere is moving with speed $4ms^{-1}$ at an angle of 30°. The coefficient of restitution between the sphere and both walls is $\frac{3}{4}$. Find

a) The direction of motion and speed of the sphere after the first collision
b) The direction of motion and speed of the sphere after the second collision.

a) Angle of 23.4° (3 sf) to the first wall ; Speed 3.77 ms^{-1} (3 sf)
b) Angle of 60° to the second wall ; Speed 3 ms^{-1}

Worked example

Two cushions of a snooker table W_1 and W_2 meet at right angles. A snooker ball travels across the table and collides with W_1 then W_2. The cushions are modelled as smooth. Just before the first impact the ball is moving with speed $u\ ms^{-1}$ at an angle of $40°$ to W_1. The coefficients of restitution between the ball and the cushions W_1 and W_2 are $\frac{1}{3}$ and $\frac{2}{7}$ respectively.
Find the percentage of the ball's original kinetic energy that is lost in the collision

Your turn

Two cushions of a snooker table W_1 and W_2 meet at right angles. A snooker ball travels across the table and collides with W_1 then W_2. The cushions are modelled as smooth. Just before the first impact the ball is moving with speed $u\ ms^{-1}$ at an angle of $20°$ to W_1. The coefficients of restitution between the ball and the cushions W_1 and W_2 are $\frac{1}{2}$ and $\frac{2}{5}$ respectively.
Find the percentage of the ball's original kinetic energy that is lost in the collision

83%

Worked example

Two smooth vertical walls stand on a smooth horizontal surface and intersect at an angle of $30°$. A smooth sphere is projected across the surface with speed $2\ ms^{-1}$ at an angle of $40°$ to one of the walls and towards the intersection of the walls. The coefficient of restitution between the sphere and the walls is 0.4. Work out the speed and direction of motion of the sphere after:

a) The first collision

b) The second collision

Your turn

Two smooth vertical walls stand on a smooth horizontal surface and intersect at an angle of $60°$. A smooth sphere is projected across the surface with speed $1\ ms^{-1}$ at an angle of $20°$ to one of the walls and towards the intersection of the walls. The coefficient of restitution between the sphere and the walls is 0.4. Work out the speed and direction of motion of the sphere after:

a) The first collision

b) The second collision

a) Angle of $8.28°$ (3 sf) to the first wall ;
Speed $0.950\ ms^{-1}$ (3 sf)

b) Angle of $45.1°$ (3 sf) to the second wall ;
Speed $0.498\ ms^{-1}$ (3 sf)

Worked example

AB and BC are smooth vertical walls stood on a smooth floor. The angle between AB and BC is 150°. A ball is projected along the floor towards AB with speed $u\ m^{-1}$ on a path at an angle of 30° to AB. The ball hits AB and then hits BC. The ball is modelled as a particle. The coefficient of restitution between the ball and each wall is $\frac{1}{4}$.

a) Find the speed of the ball immediately after it has hit AB.

b) The speed of the ball immediately after it has hit *BC* is *w* m s^{-1}. Find *w* in terms of *u*.

Your turn

AB and BC are smooth vertical walls stood on a smooth floor. The angle between AB and BC is 120°. A ball is projected along the floor towards AB with speed $u\ m^{-1}$ on a path at an angle of 60° to AB. The ball hits AB and then hits BC. The ball is modelled as a particle. The coefficient of restitution between the ball and each wall is $\frac{1}{2}$.

a) Find the speed of the ball immediately after it has hit AB.

b) The speed of the ball immediately after it has hit *BC* is *w* m s^{-1}. Find *w* in terms of *u*.

a) $\frac{\sqrt{7}}{4}u$

b) $0.634u$

Worked example

Two smooth vertical walls stand on a smooth horizontal floor and intersect at an acute angle θ. A small smooth particle is projected along the floor at right angles to one of the walls and away from it. After one impact with each wall the particle is moving parallel to the first wall it struck. Given that the coefficient of restitution between the particle and each wall is e show that:

$$(1 + 2e)\tan^2\theta = e^2$$

Your turn

Two smooth vertical walls stand on a smooth horizontal floor and intersect at an acute angle θ. A small smooth particle is projected along the floor at right angles to one of the walls and away from it. After one impact with each wall the particle is moving parallel to the first wall it struck. Given that the coefficient of restitution between the particle and each wall is e show that:

$$(1 + 2e)\tan^2\theta = e^2$$

Shown

5.3) Oblique impact of smooth spheres

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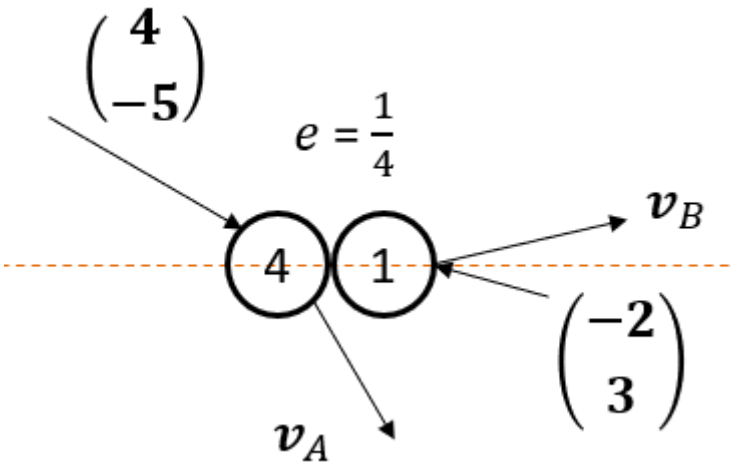
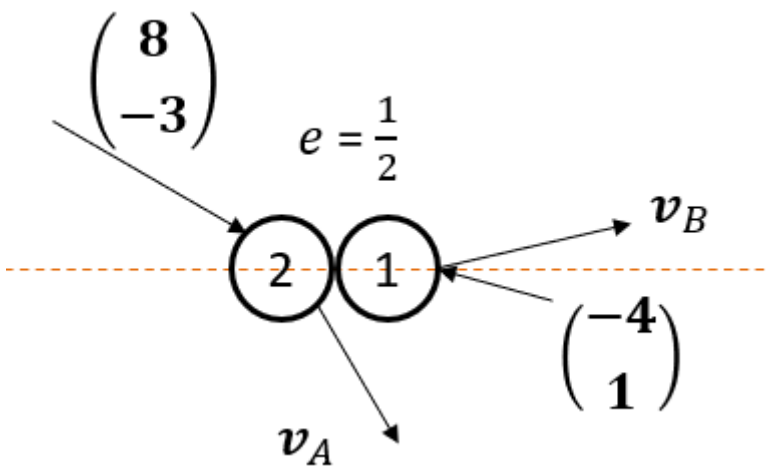
Worked example

Two smooth spheres collide obliquely.
Find the velocity of each sphere immediately after impact.



Your turn

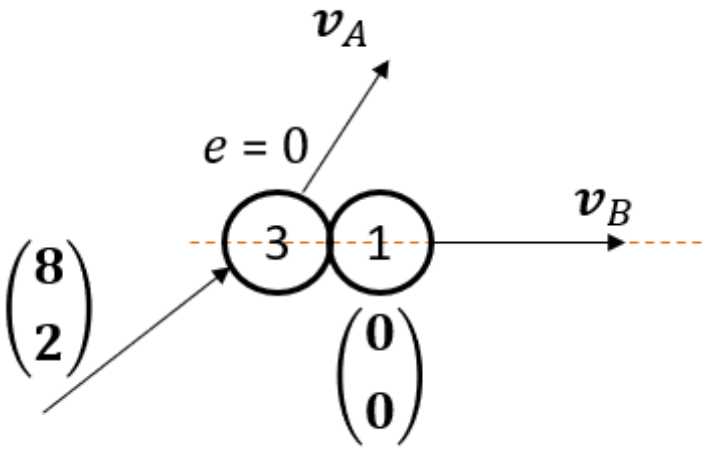
Two smooth spheres collide obliquely.
Find the velocity of each sphere immediately after impact.



$v_A = \begin{pmatrix} 2 \\ -3 \end{pmatrix}, v_B = \begin{pmatrix} 8 \\ 1 \end{pmatrix}$

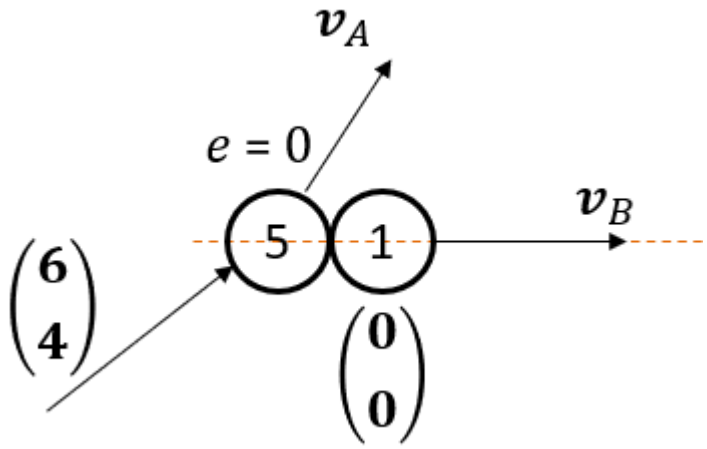
Worked example

Two smooth spheres collide obliquely.
Find the velocity of each sphere immediately after impact.



Your turn

Two smooth spheres collide obliquely.
Find the velocity of each sphere immediately after impact.



$v_A = \begin{pmatrix} 5 \\ 4 \end{pmatrix}, v_B = \begin{pmatrix} 5 \\ 0 \end{pmatrix}$

Worked example

A smooth sphere A, of mass $4\ kg$ and moving with speed $12\ ms^{-1}$, collides obliquely with a smooth sphere B of mass $8\ kg$. Just before the impact B is stationary and the velocity of A makes an angle of $30°$ with the lines of centres of the two spheres. The coefficient of restitution between the spheres is $\frac{1}{2}$. Find the magnitudes and directions of the velocities of A and B immediately after the impact.

Your turn

A smooth sphere A, of mass $2\ kg$ and moving with speed $6\ ms^{-1}$, collides obliquely with a smooth sphere B of mass $4\ kg$. Just before the impact B is stationary and the velocity of A makes an angle of $60°$ with the lines of centres of the two spheres. The coefficient of restitution between the spheres is $\frac{1}{4}$. Find the magnitudes and directions of the velocities of A and B immediately after the impact.

A: Speed $5.22\ ms^{-1}$ (3 sf) at angle of $84.5°$ (3 sf) to the line of centres

B: Speed $1.25\ ms^{-1}$ along the line of centres

Worked example

A small smooth sphere A of mass 1kg collides with a small smooth sphere B of mass 4kg. Just before the impact A is moving with a speed of 8 ms^{-1} in a direction at 30° to the line of centres and B is moving with a speed $2ms^{-1}$ at 45° to the line of centres.
The coefficient of restitution between the spheres is $\frac{1}{4}$. Find:

a) The kinetic energy lost in the impact
b) The magnitude of the impulse exerted by A on B

Your turn

A small smooth sphere A of mass 1kg collides with a small smooth sphere B of mass 2kg. Just before the impact A is moving with a speed of $4ms^{-1}$ in a direction at 45° to the line of centres and B is moving with a speed $3ms^{-1}$ at 60° to the line of centres.
The coefficient of restitution between the spheres is $\frac{3}{4}$. Find:

a) The kinetic energy lost in the impact
b) The magnitude of the impulse exerted by A on B

a) 2.73 J (3 sf)
b) 5.05 Ns (3 sf)

Worked example

A smooth sphere A of mass $10\ kg$ is moving on a smooth horizontal surface with velocity $(4\boldsymbol{i} + 6\boldsymbol{j})\ ms^{-1}$.
Another smooth sphere B of mass $6\ kg$ and the same radius as A is moving on the same surface with velocity $(8\boldsymbol{i} - 4\boldsymbol{j})\ ms^{-1}$.
The spheres collide when their line of centres is parallel to $\boldsymbol{j}$.
The coefficient of restitution between the spheres is $\frac{2}{5}$. Find the velocities of both spheres after the impact.

Your turn

A smooth sphere A of mass $5\ kg$ is moving on a smooth horizontal surface with velocity $(2\boldsymbol{i} + 3\boldsymbol{j})\ ms^{-1}$.
Another smooth sphere B of mass $3\ kg$ and the same radius as A is moving on the same surface with velocity $(4\boldsymbol{i} - 2\boldsymbol{j})\ ms^{-1}$.
The spheres collide when their line of centres is parallel to $\boldsymbol{j}$.
The coefficient of restitution between the spheres is $\frac{3}{5}$. Find the velocities of both spheres after the impact.

$$v_A = 2\boldsymbol{i}\ ms^{-1}\ ;\ v_B = (4\boldsymbol{i} + 3\boldsymbol{j})\ ms^{-1}$$

Worked example

Two small smooth spheres A and B have equal radii. The mass of A is $4m\ kg$ and the mass of B is $5m\ kg$. The spheres are moving on a smooth horizontal plane and they collide.

Immediately before the collision the velocity of A is $10\boldsymbol{j}\ ms^{-1}$ and the velocity of B is $(6\boldsymbol{i} - 2\boldsymbol{j})\ ms^{-1}$. Immediately after the collision the velocity of A is $(6\boldsymbol{i} + 4\boldsymbol{j})\ ms^{-1}$.

Find:

a) The speed of B immediately after the collision

b) A unit vector parallel to the line of centres of the spheres at the instant of the collision

Your turn

Two small smooth spheres A and B have equal radii. The mass of A is $2m\ kg$ and the mass of B is $3m\ kg$. The spheres are moving on a smooth horizontal plane and they collide.

Immediately before the collision the velocity of A is $5\boldsymbol{j}\ ms^{-1}$ and the velocity of B is $(3\boldsymbol{i} - \boldsymbol{j})\ ms^{-1}$. Immediately after the collision the velocity of A is $(3\boldsymbol{i} + 2\boldsymbol{j})\ ms^{-1}$. Find:

a) The speed of B immediately after the collision

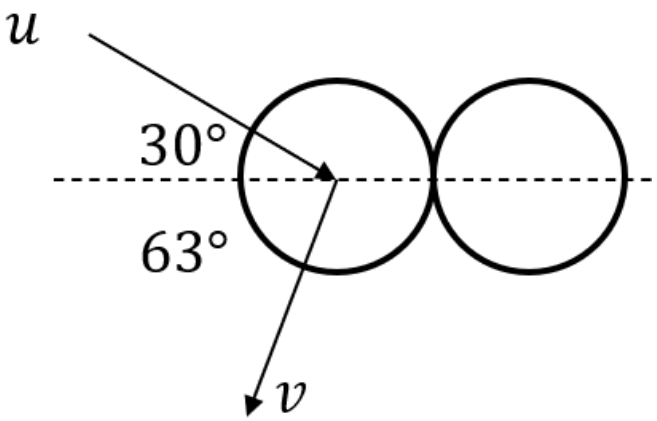
b) A unit vector parallel to the line of centres of the spheres at the instant of the collision

a) $1.41\ ms^{-1}$ (3 sf)

b) $\frac{1}{\sqrt{2}}(\boldsymbol{i} - \boldsymbol{j})$

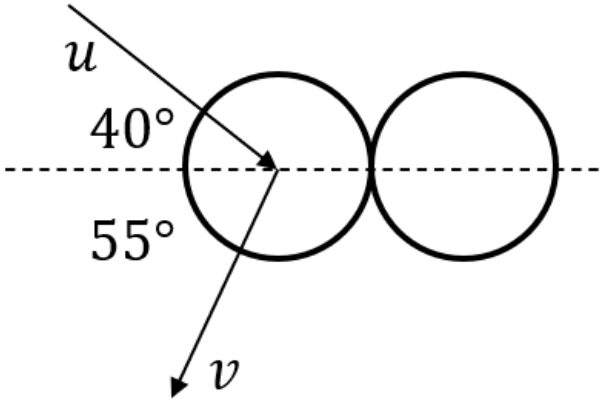
Worked example

Find the angle of deflection



Your turn

Find the angle of deflection



85°

Worked example

A smooth uniform sphere S, of mass m, is moving on a smooth horizontal plane when it collides obliquely with another smooth uniform sphere T, of the same radius as S but of mass $4m$, which is at rest on the plane. Immediately before the collision the velocity of S makes an angle α, where $\tan\alpha = \frac{5}{12}$, with the line joining the centres of the spheres. Immediately after the collision the speed of T is V. The coefficient of restitution between the two spheres is $\frac{1}{4}$.

a) Find, in terms of V, the speed of S

i) Immediately before the collision

ii) Immediately after the collision

b) Find the angle through which the direction of motion of S is deflected as a result of the collision

Your turn

A smooth uniform sphere S, of mass m, is moving on a smooth horizontal plane when it collides obliquely with another smooth uniform sphere T, of the same radius as S but of mass $2m$, which is at rest on the plane. Immediately before the collision the velocity of S makes an angle α, where $\tan\alpha = \frac{3}{4}$, with the line joining the centres of the spheres. Immediately after the collision the speed of T is V. The coefficient of restitution between the two spheres is $\frac{3}{4}$.

a) Find, in terms of V, the speed of S

i) Immediately before the collision

ii) Immediately after the collision

b) Find the angle through which the direction of motion of S is deflected as a result of the collision

a) i) $-\frac{2V}{7}$

ii) $\frac{V\sqrt{85}}{7}$

b) $65.7°$ (3 sf)

Worked example

Two small smooth spheres A and B have equal radii. The mass of A is $m$kg and the mass of B is $10m$kg. The spheres are moving on a smooth horizontal plane and they collide.

Immediately before the collision the velocity of A is $(4\boldsymbol{i} + 2\boldsymbol{j})\, ms^{-1}$ and B is stationary.

Immediately after the collision the velocity of A is $4\boldsymbol{j}\, ms^{-1}$. Find:

a) The velocity of B after the collision

b) The coefficient of restitution between the two spheres

Your turn

Two small smooth spheres A and B have equal radii. The mass of A is $2m$kg and the mass of B is $20m$kg. The spheres are moving on a smooth horizontal plane and they collide.

Immediately before the collision the velocity of A is $(2\boldsymbol{i} + \boldsymbol{j})\, ms^{-1}$ and B is stationary.

Immediately after the collision the velocity of A is $2\boldsymbol{j}\, ms^{-1}$. Find:

a) The velocity of B after the collision

b) The coefficient of restitution between the two spheres

a) $(0.2\boldsymbol{i} - 0.1\boldsymbol{j})\, ms^{-1}$

b) $e = \frac{5}{6}$