Example 11 **Questions involving 3 balls**

Three spheres A, B and C have masses m, $2m$ and $3m$ respectively. The spheres move along the same straight line on a horizontal plane with A following B, which is following C. Initially the speeds of A, B and C are $7\,\text{m s}^{-1}$, $3\,\text{m s}^{-1}$ and $1\,\text{m s}^{-1}$ respectively, in the direction ABC. Sphere A collides with sphere B and then sphere B collides with sphere C. The coefficient of restitution between A and B is $\frac{1}{2}$ and the coefficient of restitution between B and C is $\frac{1}{4}$

a Find the velocities of the three spheres after the second collision.

b Explain how you can predict that there will be a further collision between A and B.

Example 12 **Questions involving several collisions**

A uniform smooth sphere P of mass $3m$ is moving in a straight line with speed u on a smooth horizontal table. Another uniform smooth sphere Q of mass m is moving with speed $2u$ in the same straight line as P, but in the opposite direction. The sphere P collides with the sphere Q directly. The velocities of P and Q after the collision are v and w respectively, measured in the direction of motion of P before the collision. The coefficient of restitution between P and Q is e.

a Find expressions for v and w in terms of u and e.

b Show that, if the direction of motion of P is changed by the collision, then $e > \frac{1}{3}$

Following the collision with P, the sphere Q then collides with and rebounds from a vertical wall, which is perpendicular to the direction of motion of Q. The coefficient of restitution between Q and the wall is e'.

c Given that $e = \frac{5}{9}$ and that P and Q collide again in the subsequent motion, show that $e' > \frac{1}{9}$

20 A particle A of mass $2m$, moving with speed $2u$ in a straight line on a smooth horizontal table, collides with a particle B of mass $3m$, moving with speed u in the same direction as A. The coefficient of restitution between A and B is e.

a Show that the speed of B after the collision is

$$\tfrac{1}{5}u(7 + 2e).$$

b Find the speed of A after the collision, in terms of u and e.

The speed of A after the collision is $\frac{11}{10}u$.

c Show that $e = \frac{1}{2}$.

At the instant of collision, A and B are at a distance d from a vertical barrier fixed to the surface at right-angles to their direction of motion. Given that B hits the barrier, and that the coefficient of restitution between B and the barrier is $\frac{11}{16}$,

d find the distance of A from the barrier at the instant that B hits the barrier,

e show that, after B rebounds from the barrier, it collides with A again at a distance $\frac{5}{32}d$ from the barrier. **E**

11 A mass of 2 kg moving at $35\,\mathrm{m\,s^{-1}}$ catches up and collides with a mass of 10 kg moving in the same direction at $20\,\mathrm{m\,s^{-1}}$. Five seconds after the impact the 10 kg mass encounters a fixed barrier which reduces it to rest. Assuming the coefficient of restitution between the masses is $\frac{3}{5}$, find the further time that will elapse before the 2 kg mass strikes the 10 kg mass again. You may assume that the masses are moving on a smooth surface and have constant velocity between collisions.

Example 13 **Questions involving multiple vertical bounces**

A tennis ball, which may be modelled as a particle, is dropped from rest at a height of 90 cm onto a smooth horizontal plane. The coefficient of restitution between the ball and the plane is 0.5. Assume that there is no air resistance and that the ball falls under gravity and hits the plane at right angles.

a Find the height to which the ball rebounds after the first bounce.

b Find the height to which the ball rebounds after the second bounce.

c Find the total distance travelled by the ball before it comes to rest, according to this model.

d Criticise this model with respect to the motion of the ball as it continues to bounce.

Use the formula for the sum of an infinite geometric series:

$S = \frac{a}{1-r}$