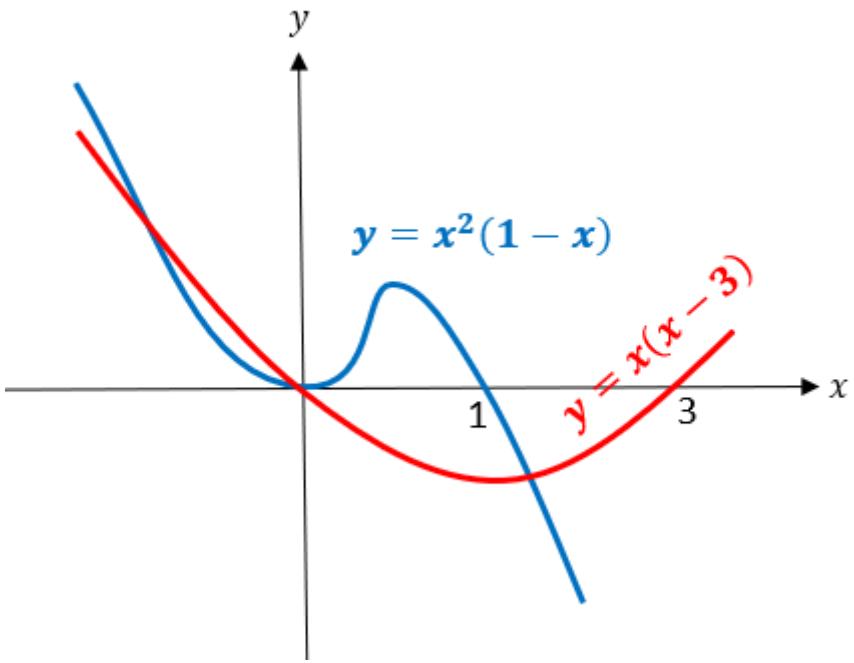
4.4) Points of intersection

Worked example

On the same diagram sketch the curves with equations $y = x(x-2)$ and $y = x^2(1-x)$. Find the coordinates of their points of intersection.

Your turn

On the same diagram sketch the curves with equations $y = x(x-3)$ and $y = x^2(1-x)$. Find the coordinates of their points of intersection.



$(-\sqrt{3}, 3+3\sqrt{3})$,
$(0,0)$,
$(\sqrt{3}, 3-3\sqrt{3})$

Worked example

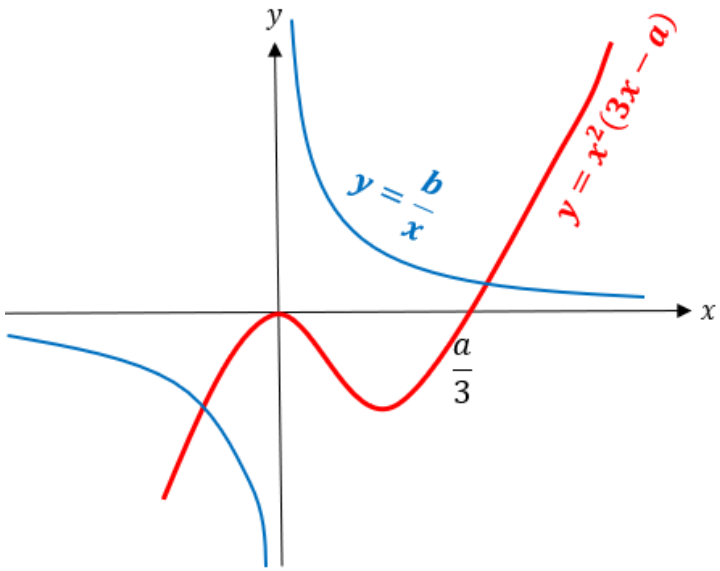
On the same diagram sketch the curves with equations $y = -x^2(5x - a)$ and $y = -\frac{b}{x}$, where a, b are positive constants.

State, giving a reason, the number of real solutions to the equation $x^2(5x - a) + \frac{b}{x} = 0$

Your turn

On the same diagram sketch the curves with equations $y = x^2(3x - a)$ and $y = \frac{b}{x}$, where a, b are positive constants.

State, giving a reason, the number of real solutions to the equation $x^2(3x - a) - \frac{b}{x} = 0$



2 points of intersection where

$$x^2(3x - a) = \frac{b}{x}$$

$$x^2(3x - a) - \frac{b}{x} = 0$$

$\therefore$ 2 solutions

Worked example

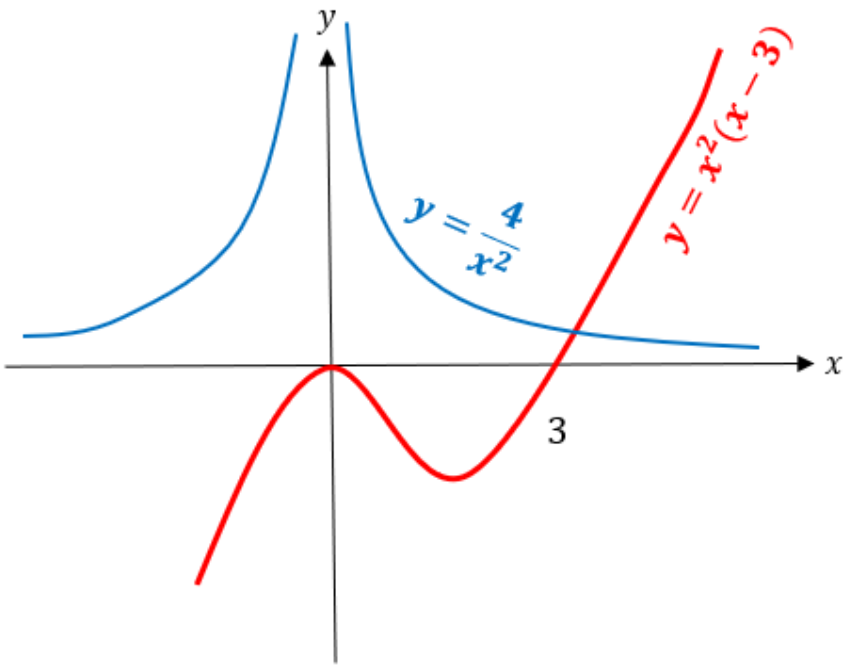
On the same diagram sketch the curves with equations $y = \frac{3}{x^2}$ and $y = x^2(x-4)$.

State, giving a reason, the number of real solutions to the equation $x^4(x-4) - 3 = 0$

Your turn

On the same diagram sketch the curves with equations $y = \frac{4}{x^2}$ and $y = x^2(x-3)$.

State, giving a reason, the number of real solutions to the equation $x^4(x-3) - 4 = 0$



1 point of intersection where

$$x^2(x-3) = \frac{4}{x^2}$$

$$x^4(x-3) = 4$$

$$x^4(x-3) - 4 = 0$$

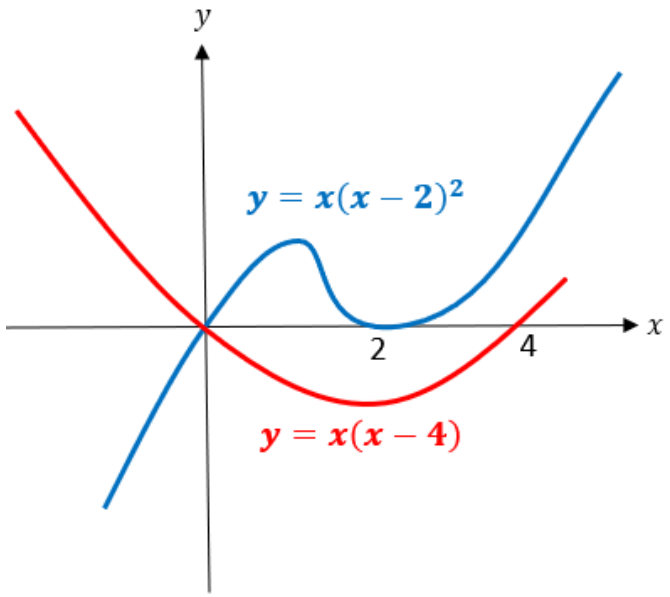
$\therefore$ 1 real solution

Worked example

On the same diagram sketch the curves with equations $y = x(x-5)$ and $y = x(x-3)^2$, and hence find the coordinates of any points of intersection.

Your turn

On the same diagram sketch the curves with equations $y = x(x-4)$ and $y = x(x-2)^2$, and hence find the coordinates of any points of intersection.



$(0, 0)$ only as:

$x(x-2)^2 = x(x-4)$

$x(x^2 - 4x + 4) = x^2 - 4x$

$x^3 - 4x^2 + 4x = x^2 - 4x$

$x^3 - 5x^2 + 8x = 0$

$x(x^2 - 5x + 8) = 0$

Discriminant of $x^2 - 5x + 8 = -7 < 0$

Worked example

Work out the range of values of a such that the graphs of $y = x^2 + a$ and $3y = x - 2$ have two points of intersection

Your turn

Work out the range of values of a such that the graphs of $y = x^2 + a$ and $4y = x - 3$ have two points of intersection

$$a < -\frac{47}{72}$$