## 4) Moments

## 4.1) Moments

4.2) Resultant moments
4.3) Equilibrium
4.4) Centres of mass
4.5) Tilting
4.1) Moments

Worked example

## Your turn

Calculate the moment of the force about the point $P$


Calculate the moment of the force about the point $P$


200 Nm clockwise


0 Nm

Calculate the moment of the force about the point $P$


Calculate the moment of the force about the point $P$


$$
50 \sin 30^{\circ} \mathrm{Nm}=25 \mathrm{Nm}
$$

## Your turn

Calculate the moment of the force about the point $P$


Calculate the moment of the force about the point $P$


> 33.6 Nm (3 sf) clockwise

Two forces act on a lamina. Find the moment of each of the forces about the point $P$.


Two forces act on a lamina. Find the moment of each of the forces about the point $P$.


Moment of 5 N force $=10 \mathrm{Nm}$ clockwise Moment of 8 N force
$=12.3 \mathrm{Nm}$ anticlockwise (3 sf)

## 4.2) Resultant moments

## Your turn

Calculate the resultant moment acting about
Calculate the resultant moment acting about P


50 Nm anticlockwise

## Your turn

The rod is light. Calculate the resultant moment acting about $P$.


The rod is light. Calculate the resultant moment acting about $P$.


8 Nm clockwise

Find the resultant moment acting about $P$


Find the resultant moment acting about $P$

$40 \mathrm{~g} \mathrm{Nm}=392 \mathrm{Nm}$ clockwise

## Your turn

The rod is light. Calculate the resultant moment acting about $P$.


The rod is light. Calculate the resultant moment acting about $P$.

$30 g(x-4)$ Nm clockwise

The rod is light. Calculate the resultant moment acting about $P$.


The rod is light. Calculate the resultant moment acting about $P$.

6.02 Nm anticlockwise (3 sf)

## Your turn

The rod is light. Calculate the resultant moment acting about $P$


Two forces act on a lamina. Calculate the resultant moment about the point $P$.

5.31 Nm clockwise (3 sf)

A set of forces act on a light rod. The resultant moment of P is 26 Nm clockwise. Find the value of $x$


A set of forces act on a light rod. The resultant moment of P is 48 Nm clockwise. Find the value of $x$


## Your turn

Person A and Person B are on opposite ends of a uniform seesaw of mass 30 kg .
A weighs 60 kg and is 5 m from the pivot.
$B$ is 4 m from the pivot.
The seesaw remains horizontal. Determine:
a) The reaction force at the pivot of the seesaw
b) The mass of $B$

Person $A$ and Person $B$ are on opposite ends of $a$ uniform seesaw of mass 20 kg .
A weighs 70 kg and is 10 m from the pivot.
$B$ is 8 m from the pivot.
The seesaw remains horizontal. Determine:
a) The reaction force at the pivot of the seesaw
b) The mass of $B$
a) 1764 N
b) 90 kg

## Worked example

## Your turn

A uniform beam $A B$, of mass 20 kg and length 10m, rests horizontally on supports at $C$ and $D$, where $A C=D B=2 \mathrm{~m}$.
When a man of mass 60kg stands on the beam at $E$ the magnitude of the reaction at $D$ is three times the magnitude of the reaction at $C$.
By modelling the beam as a rod and the man as a particle, find the distance $A E$.

A uniform beam $A B$, of mass 40 kg and length 5 m , rests horizontally on supports at $C$ and $D$, where $A C=D B=1 \mathrm{~m}$.
When a man of mass 80kg stands on the beam at $E$ the magnitude of the reaction at $D$ is twice the magnitude of the reaction at $C$.
By modelling the beam as a rod and the man as a particle, find the distance $A E$.

## Worked example

## Your turn

A uniform rod $A B$ has length 5 m and mass 20 kg . The rod is in equilibrium in a horizontal position, resting on two smooth supports at $C$ and $D$, where $A C=0.4$ metres and $D B=x$ metres. Given that the magnitude of the reaction on the rod at D is three times the magnitude of the reaction on the rod at $C$, find the value of $x$

A uniform rod $A B$ has length 2 m and mass 50 kg . The rod is in equilibrium in a horizontal position, resting on two smooth supports at $C$ and $D$, where $A C=0.2$ metres and $D B=x$ metres.
Given that the magnitude of the reaction on the rod at D is twice the magnitude of the reaction on the rod at C , find the value of $x$

$$
x=0.6
$$

## Your turn

A uniform ladder, $A B$, is leaning against a smooth vertical wall on rough horizontal ground at an angle of $50^{\circ}$ to the horizontal. The ladder has length 6 m and is held in equilibrium by a frictional force of magnitude 40 N acting horizontally at $B$ which is the end of the ladder on the ground. Find the mass of the ladder.

A uniform ladder, $A B$, is leaning against a smooth vertical wall on rough horizontal ground at an angle of $60^{\circ}$ to the horizontal. The ladder has length 5 m and is held in equilibrium by a frictional force of magnitude 80 N acting horizontally at $B$ which is the end of the ladder on the ground. Find the mass of the ladder.

$$
28.3 \mathrm{~kg} \text { (3 sf) }
$$

## Your turn

Sam and Tamsin are sitting on a non-uniform plan $A B$ of mass 45 kg and length 2 m .
The plank is pivoted at $M$, the midpoint of $A B$. The centre of mass of $A B$ is at $C$ where $A C$ is 0.8 . Sam has mass 70 kg . Tamsin has mass 50 kg and sits at $A$. Where must Sam sit for the plank to be horizontal?

Sam and Tamsin are sitting on a non-uniform plan $A B$ of mass 25 kg and length 4 m .
The plank is pivoted at $M$, the midpoint of $A B$.
The centre of mass of $A B$ is at $C$ where $A C$ is 1.8 m . Sam has mass 35 kg .
Tamsin has mass 25 kg and sits at $A$.
Where must Sam sit for the plank to be horizontal?

### 3.57 m from end $A$

## Worked example

## Your turn

A non-uniform rod $A B$ is 6 m long and has weight 40 N .
It is in a horizontal position resting on supports at points $C$ and $D$, where $A C=0.5 \mathrm{~m}$ and $A D=5 \mathrm{~m}$. The magnitude of the reaction at $C$ is four times the magnitude of the reaction at D .
Find the distance of the centre of mass of the rod from $A$

A non-uniform rod $A B$ is 3 m long and has weight 20 N . It is in a horizontal position resting on supports at points $C$ and $D$, where $A C=1 \mathrm{~m}$ and $A D=2.5 \mathrm{~m}$.
The magnitude of the reaction at $C$ is three times the magnitude of the reaction at $D$.
Find the distance of the centre of mass of the rod from $A$

$$
1.38 \mathrm{~m}(3 \mathrm{sf})
$$

## 4.5) Tilting

## Your turn

A uniform beam beam $A B$, of mass 54 kg and length 8 m , rests horizontally on supports $C$ and $D$ where $A C=2 \mathrm{~m}$ and $C D=7 \mathrm{~m}$.
When an object is placed at $A$, the beam is on the point of tilting about $C$.
Determine the mass of the object.

A uniform beam beam $A B$, of mass 45 kg and length 16 m , rests horizontally on supports $C$ and $D$ where $A C=5 \mathrm{~m}$ and $C D=9 \mathrm{~m}$.
When an object is placed at $A$, the beam is on the point of tilting about $C$.
Determine the mass of the object.
27 kg

## Worked example

## Your turn

A non-uniform rod $A B$, of length 5 m and weight 80 N , is suspended from a pair of light cables attached to $C$ and $D$ where $A C=2 \mathrm{~m}$ and $B D=1$ m.

When a weight of 50 N is hung from $A$ the rod is on the point of rotating.
Find the distance of the centre of mass of the rod from $A$.

A non-uniform rod $A B$, of length 10 m and weight 40 N , is suspended from a pair of light cables
attached to $C$ and $D$ where $A C=3 \mathrm{~m}$ and $B D=2$ m.

When a weight of 25 N is hung from $A$ the rod is on the point of rotating.
Find the distance of the centre of mass of the rod from $A$.
4.875 m

## Your turn

A beam $A B$ has length 25 m . The beam rests horizontally in equilibrium on two smooth supports at the points $P$ and $Q$, where $A P=4 \mathrm{~m}$ and $Q B=5 \mathrm{~m}$.
When an adult of mass 60 kg stands on the beam at $A$, the beam remains in equilibrium and is on the point of tilting about $P$.
When the same child stands on the beam at $B$, the beam remains in equilibrium and is on the point of tilting about $Q$.
The child is modelled as a particle and the beam is modelled as a non-uniform rod.
a) Find the mass of the beam
b) Find the distance of the centre of mass of the beam from $A$

A beam $A B$ has length 15 m . The beam rests horizontally in equilibrium on two smooth supports at the points $P$ and $Q$, where $A P=2 \mathrm{~m}$ and $Q B=3 \mathrm{~m}$.
When a child of mass 50 kg stands on the beam at $A$, the beam remains in equilibrium and is on the point of tilting about $P$.
When the same child stands on the beam at $B$, the beam remains in equilibrium and is on the point of tilting about $Q$.
The child is modelled as a particle and the beam is modelled as a non-uniform rod.
a) Find the mass of the beam
b) Find the distance of the centre of mass of the beam from $A$
a) 25 kg
b) 6 m

