4) Binomial expansion

4.1) Expanding $(1 + x)^n$
4.2) Expanding $(a + bx)^n$
4.3) Using partial fractions

4.1) Expanding $(1 + x)^n$

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Worked example

Find the binomial expansion of:

$(1 + x)^4$

$(1 - 2x)^5$

Your turn

Find the binomial expansion of:

$(1 + 3x)^6$

$1 + 18x + 135x^2 + 540x^3 + 1215x^4 + 1458x^5 + 729x^6$

Worked example

Find the first four terms in the binomial expansion of:

$$\frac{1}{1+x}$$

$$\frac{1}{(1-x)^3}$$

Your turn

Find the first four terms in the binomial expansion of:

$$\frac{1}{(1+x)^2}$$

$$1-2x+3x^2-4x^3+\cdots$$

Worked example

Find the first four terms in the binomial expansion of:

$$\frac{1}{1+2x}$$

$$\frac{1}{(1-3x)^4}$$

Your turn

Find the first four terms in the binomial expansion of:

$$\frac{1}{(1+4x)^3}$$

$1 - 12x + 96x^2 - 640x^3 + \cdots$

Worked example

Find the first four terms in the binomial expansion of:

$$\sqrt{1+x}$$

$$\sqrt[4]{1-x}$$

Your turn

Find the first four terms in the binomial expansion of:

$$\sqrt[3]{1-x}$$

$$1-\frac{1}{3}x-\frac{1}{9}x^2-\frac{5}{81}x^3+\cdots$$

Worked example

Find the first four terms in the binomial expansion of:

$$\sqrt{1+6x}$$

$$\sqrt{1-4x}$$

Your turn

Find the first four terms in the binomial expansion of:

$$\sqrt{1-2x}$$

$$1 - x - \frac{x^2}{2} - \frac{x^3}{2} + \cdots$$

Worked example

State when the binomial expansion is valid:

$$\frac{1}{1+2x}$$

$$\frac{1}{(1-3x)^4}$$

$$(1+5x)^{\frac{3}{2}}$$

$$\frac{1}{\sqrt{1+\frac{x}{7}}}$$

Your turn

State when the binomial expansion is valid:

$$\frac{1}{(1+4x)^3}$$

$|x| < \frac{1}{4}$ or $-\frac{1}{4} < x < \frac{1}{4}$

$$\sqrt{1-\frac{x}{5}}$$

$|x| < 5$ or $-5 < x < 5$

Worked example

State when the binomial expansion is valid:

$$\frac{2-x}{\sqrt{1+3x}}$$

$$\frac{5+x}{(1-2x)^4}$$

$$(1+5x)^{\frac{3}{2}}\sqrt{1-\frac{x}{4}}$$

Your turn

State when the binomial expansion is valid:

$$\frac{2+x}{\sqrt{1+5x}}$$

$|x| < \frac{1}{5}$ or $-\frac{1}{5} < x < \frac{1}{5}$

$$(1-3x)^{\frac{5}{3}}\sqrt[4]{1+\frac{x}{2}}$$

$|x| < \frac{1}{3}$ or $-\frac{1}{3} < x < \frac{1}{3}$

Worked example

By substituting $x = 0.07$ into the binomial expansion for $\sqrt{1-4x}$, find a decimal approximation to $\sqrt{2}$

Your turn

By substituting $x = 0.01$ into the binomial expansion for $\sqrt{1-2x}$, find a decimal approximation to $\sqrt{2}$

$1.414213571\ldots$

Worked example

By substituting $x = 0.04$ into the binomial expansion for $\sqrt{1-4x}$, find a decimal approximation to $\sqrt{21}$ to 5 decimal places

Your turn

By substituting $x = 0.01$ into the binomial expansion for $\sqrt{1-8x}$, find a decimal approximation to $\sqrt{23}$ to 5 decimal places

4.79584

Worked example

Find the series expansion, in ascending powers of x, up to and including the x^3 term, of $\sqrt{1+7x}$

Find the percentage error in using $x = 0.01$ in this series expansion to estimate $\sqrt{107}$

Your turn

Find the series expansion, in ascending powers of x, up to and including the x^3 term, of $\sqrt{1+9x}$

Find the percentage error in using $x = 0.01$ in this series expansion to estimate $\sqrt{109}$

0.003% (3 dp)

Worked example

Find the x^2 term in the series expansion of:

$$\frac{5-x}{\sqrt{1-3x}}$$

Your turn

Find the x^2 term in the series expansion of:

$$\frac{2+x}{\sqrt{1+5x}}$$

$$\frac{65}{4}x^2$$

Worked example

Find the first three terms in the series expansion of:

$$\sqrt{\frac{1+2x}{1-3x}}$$

Your turn

Find the first three terms in the series expansion of:

$$\sqrt{\frac{1+x}{1-x}}, |x| < 1$$

$$1 + x + \frac{1}{2}x^2$$

Worked example

Find the series expansion, in ascending powers of x, up to and including the x^2 term for:

$$\frac{6}{1-3x} - \frac{4}{1-2x}$$

Your turn

Find the series expansion, in ascending powers of x, up to and including the x^2 term for:

$$\frac{6}{1+3x} - \frac{4}{1-5x}$$

$2 - 38x - 46x^2$

Worked example

In the expansion of $(1+kx)^{-3}$ the coefficient of x is 12. Find k

In the expansion of $(1+kx)^{-3}$ the coefficient of x^2 is 4 and $k>0$. Find k

Your turn

In the expansion of $(1+kx)^{-4}$ the coefficient of x is 20. Find k

$k=-5$

4.2) Expanding $(a + bx)^n$

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Worked example

Find first four terms in the binomial expansion of $\sqrt{2+x}$

State the values of x for which the expansion is valid.

Your turn

Find first four terms in the binomial expansion of $\sqrt{4+x}$

State the values of x for which the expansion is valid.

$$2+\frac{1}{4}x-\frac{1}{64}x^2+\frac{1}{512}x^3-\cdots$$

Valid for $|x|<4$

Worked example

Find first four terms in the binomial expansion of $\frac{1}{(3+2x)^4}$

State the values of x for which the expansion is valid.

Your turn

Find first four terms in the binomial expansion of $\frac{1}{(2+3x)^2}$

State the values of x for which the expansion is valid.

$$\frac{1}{4} - \frac{3}{4}x + \frac{27}{16}x^2 - \frac{27}{8}x^3 + \cdots$$

Valid for $|x| < \frac{2}{3}$

Worked example

Find first three terms in ascending powers of x of the series expansion of $\frac{3x+4}{\sqrt{2-5x}}$

State the values of x for which the expansion is valid.

Your turn

Find first three terms in ascending powers of x of the series expansion of $\frac{3x-4}{\sqrt{5+2x}}$

State the values of x for which the expansion is valid.

$$-\frac{4\sqrt{5}}{5}+\frac{19\sqrt{5}}{25}x+\frac{9\sqrt{5}}{25}x^2$$

Valid for $|x|<\frac{5}{2}$

Worked example

Use the binomial expansion of $\sqrt{8+9x}$ up to the x^2 term to estimate $\sqrt{11}$, giving your answer as a single fraction

Your turn

Use the binomial expansion of $\sqrt{9+8x}$ up to the x^2 term to estimate $\sqrt{11}$, giving your answer as a single fraction

$$\frac{179}{54}$$

Worked example

Find the series expansion, in ascending powers of x, up to and including the x^2 term for:

$$\frac{6}{2-3x} - \frac{4}{5+2x}$$

Your turn

Find the series expansion, in ascending powers of x, up to and including the x^2 term for:

$$\frac{6}{2+5x} - \frac{4}{3-2x}$$

$$\frac{5}{3} - \frac{151}{18}x + \frac{1961}{108}x^2$$

Worked example

Find the percentage error in approximating $\sqrt{53}$ using $x = \frac{1}{9}$ in the series expansion of $\sqrt{6 - x}$ up to and including the x^2 term.

Your turn

Find the percentage error in approximating $\sqrt{35}$ using $x = \frac{1}{9}$ in the series expansion of $\sqrt{4 - x}$ up to and including the x^2 term.

0.000138%

Worked example

State when the binomial expansion is valid:

$(2 + x)^{-3}$

$(9 + 2x)^{\frac{1}{2}}$

$(8 - x)^{\frac{1}{3}}$

$(5 - 2x)^{-3}$

$(16 + 3x)^{-\frac{1}{2}}$

Your turn

State when the binomial expansion is valid:

$(25 - 2x)^{-\frac{3}{4}}$

$|x| < \frac{25}{2}$

4.3) Using partial fractions

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Worked example

Find the cubic approximation of

$$\frac{4+5x}{(1-x)(2+x)}$$

and state the range of values of x for which the expansion is valid

Your turn

Find the cubic approximation of

$$\frac{4-5x}{(1+x)(2-x)}$$

and state the range of values of x for which the expansion is valid

$$2-\frac{7}{2}x+\frac{11}{4}x^2-\frac{25}{8}x^3$$

Valid for $|x|<1$

Worked example

Find the quadratic approximation of

$$\frac{2x^2 - 5x - 10}{x^2 - x - 2}$$

Your turn

Find the quadratic approximation of

$$\frac{2x^2 + 5x - 10}{x^2 + x - 2}$$

$$5 + \frac{3}{2}x^2 + \cdots$$

Worked example

Find the quadratic approximation of

$$\frac{40x^2 - 37x + 9}{(4x-1)^2(x+2)}$$

Your turn

Find the quadratic approximation of

$$\frac{8x^2 - 13x + 6}{(2x-1)^2(x+1)}$$

$6 + 5x + 23x^2$