3.6) Approximating a binomial distribution

Worked example	Your turn
A biased coin has $P(tails) = 0.47$. The coin is tossed 200 times and the number of tails is recorded. a) Write a binomial model for X b) Show that X can be approximated with a normal distribution $Y \sim N(\mu, \sigma^2)$ and find the values of μ and σ	A biased coin has $P(tails) = 0.53$. The coin is tossed 100 times and the number of tails is recorded. a) Write a binomial model for X b) Show that X can be approximated with a normal distribution $Y \sim N(\mu, \sigma^2)$ and find the values of μ and σ a) $X \sim (100, 0.53)$ b) $Y \sim N(53, 4.99^2)$

Worked example	Your turn
Convert these discrete probabilities, where $X \sim B(n, p)$ to continuous probabilities, where $Y \sim N(np, np(1-p))$ $P(X = 5)$ $P(X = 4)$	Convert these discrete probabilities, where $X \sim B(n, p)$ to continuous probabilities, where $Y \sim N(np, np(1-p))$ $P(X = 3)$ $\approx P(2.5 \leq Y \leq 3.5)$

Worked example	Your turn
Convert these discrete probabilities, where $X \sim B(n, p)$ to continuous probabilities, where $Y \sim N(np, np(1-p))$	Convert these discrete probabilities, where $X \sim B(n, p)$ to continuous probabilities, where $Y \sim N(np, np(1-p))$
$P(X \leq 5)$	$P(X \leq 3)$ $\approx P(Y \leq 3.5)$
$P(X \leq 4)$	

Worked example	Your turn
Convert these discrete probabilities, where $X \sim B(n,p)$ to continuous probabilities, where $Y \sim N(np, np(1-p))$	Convert these discrete probabilities, where $X \sim B(n,p)$ to continuous probabilities, where $Y \sim N(np, np(1-p))$
$P(X < 5)$	$P(X < 3)$
	$\approx P(Y \leq 2.5)$
$P(X < 4)$	

Worked example	Your turn
Convert these discrete probabilities, where $X \sim B(n, p)$ to continuous probabilities, where $Y \sim N(np, np(1-p))$	Convert these discrete probabilities, where $X \sim B(n, p)$ to continuous probabilities, where $Y \sim N(np, np(1-p))$
$P(X \geq 5)$	$P(X \geq 3)$
	$\approx P(Y \geq 2.5)$
$P(X \geq 4)$	

Worked example	Your turn
Convert these discrete probabilities, where $X \sim B(n, p)$ to continuous probabilities, where $Y \sim N(np, np(1-p))$	Convert these discrete probabilities, where $X \sim B(n, p)$ to continuous probabilities, where $Y \sim N(np, np(1-p))$
$P(X > 5)$	$P(X > 3)$
	$\approx P(Y \geq 3.5)$
$P(X > 4)$	

Worked example	Your turn
Convert these discrete probabilities, where $X \sim B(n,p)$ to continuous probabilities, where $Y \sim N(np, np(1-p))$	Convert these discrete probabilities, where $X \sim B(n,p)$ to continuous probabilities, where $Y \sim N(np, np(1-p))$
$P(5 \leq X \leq 8)$	$P(3 < X < 6)$ $\approx P(3.5 \leq Y \leq 5.5)$
$P(4 < X < 7)$	

Worked example

Convert these discrete probabilities, where

$$X \sim B(n, p)$$

to continuous probabilities, where

$$Y \sim N(np, np(1-p))$$

$$P(5 < X \leq 8)$$

$$P(4 \leq X < 7)$$

Your turn

Convert these discrete probabilities, where

$$X \sim B(n, p)$$

to continuous probabilities, where

$$Y \sim N(np, np(1-p))$$

$$P(3 < X \leq 6)$$

$$\approx P(3.5 \leq Y \leq 6.5)$$

Worked example	Your turn
For a particular type of flower bulbs, 44% will produce red flowers. A random sample of 160 bulbs is planted. (a) Calculate the actual probability that there are exactly 70 red flowers. (b) Use a normal approximation to find a estimate that there are exactly 70 red flowers. (c) Hence determine the percentage error of the normal approximation for 70 red flowers.	For a particular type of flower bulbs, 55% will produce yellow flowers. A random sample of 80 bulbs is planted. (a) Calculate the actual probability that there are exactly 50 yellow flowers. (b) Use a normal approximation to find a estimate that there are exactly 50 yellow flowers. (c) Hence determine the percentage error of the normal approximation for 50 yellow flowers. a) 0.0365 b) $0.0362\ (4dp)$ c) 0.82% (2 dp)

Worked example	Your turn
The random variable $X \sim B(100, 0.51)$ Use a suitable approximation to estimate: $P(X = 45)$	The random variable $X \sim B(200, 0.47)$ Use a suitable approximation to estimate: $P(X = 87)$ 0.0346 (4 dp)
$P(X > 54)$	$P(X > 102)$ 0.1142 (4 dp)
$P(X \leq 43)$	$P(X \leq 91)$ 0.3616 (4 dp)
$P(47 < X \leq 51)$	$P(89 \leq X < 98)$ 0.4721 (4 dp)