

## 3.5) Sum to infinity

## Worked example

Comment on the sum of these series up to infinity:

$$1 + 2 + 4 + 8 + 16 + 32 + \dots$$

$$1 - 2 + 3 - 4 + 5 - 6 + 7 - 8 \dots$$

$$1 + 0.5 + 0.25 + 0.125 + 0.0625 + \dots$$

## Your turn

Comment on the sum of these series up to infinity:

$$1 + 3 + 9 + 27 + 81 + 243 + \dots$$

**Divergent. The sum tends towards infinity**

$$1 - 3 + 5 - 7 + 9 - 11 + 13 \dots$$

**Divergent. The partial sum alternates between positive and negative but the magnitude gets further from 0**

$$1 + 0.25 + 0.0625 + 0.015625 + 0.00390625 \dots$$

**Convergent. The sum tends towards a fixed value, in this case  $\frac{4}{3}$**

## Worked example

Find the sum to infinity of the series:

$$1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81} + \dots$$

$$32 - 16 + 8 - 4 + 2 \dots$$

$$p + p^3 + p^5 + p^7 + p^9 + \dots$$

$$k + \frac{1}{k} + \frac{1}{k^3} + \frac{1}{k^5} + \frac{1}{k^7} + \dots$$

## Your turn

Find the sum to infinity of the series:

$$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots$$

$$27 - 9 + 3 - 1 + \frac{1}{3} \dots$$

$$k + k^2 + k^3 + k^4 + k^5 + \dots$$

$$p + 1 + \frac{1}{p} + \frac{1}{p^2} + \frac{1}{p^3} + \dots$$

## Worked example

Given that the geometric series

$1 - 4x + 16x^2 - 64x^3 + \dots$  is convergent:

- Find the range of possible values of  $x$
- Find an expression for  $S_\infty$

## Your turn

Given that the geometric series

$1 - 3x + 9x^2 - 27x^3 + \dots$  is convergent:

- Find the range of possible values of  $x$
- Find an expression for  $S_\infty$

a)  $-\frac{1}{3} < x < \frac{1}{3}$

b)  $S_\infty = \frac{1}{1-3x}$

## Worked example

The third term of a geometric series is 1.5 and the eighth term is 0.046875.

- a) Show that this series is convergent.
- b) Find the sum to infinity of this series.

## Your turn

The fourth term of a geometric series is 1.08 and the seventh term is 0.23328.

- a) Show that this series is convergent.
- b) Find the sum to infinity of this series.

a)  $|r| = 0.6 < 1$

b) 12.5

## Worked example

For a geometric series with first term  $a$  and common ratio  $r$ ,  $S_4 = 12.75$  and  $S_\infty = 12.8$ .

- Find the possible values of  $r$ .
- Given that the terms in the series alternate between positive and negative values, find the value of  $a$

## Your turn

For a geometric series with first term  $a$  and common ratio  $r$ ,  $S_4 = 15$  and  $S_\infty = 16$ .

- Find the possible values of  $r$ .
- Given that all the terms in the series are positive, find the value of  $a$ .

a)  $r = \pm \frac{1}{2}$

b) 8