

# The Law of Conservation of Energy (expanded)

$$\text{Initial Energy} = \text{Final Energy}$$

work done by engine +

$$\text{initial EPE} + \text{initial GPE} + \text{initial KE} = \text{final EPE} + \text{final GPE} + \text{final KE} + \text{w.d. against friction}$$

Potential energy  
released/energy going in

=

Potential energy  
stored/energy going out

Consider the energy it has at the beginning - I tend to think of this as the energy it has in the bank (a bit like money).

Some of this energy is 'spent' in various ways - it is either spent and converted into another type of energy - or it is spent on having to overcome friction/resistance. Some energy is not spent, but is instead increased (e.g. the KE may increase if it gets faster because GPE is converted to KE)

If there is a force/engine doing work, then there is more energy 'in the bank' to be converted. This is why it is on the LHS of the equation.

A light elastic string, of natural length 1.6m and modulus of elasticity 10N, has one end fixed at a point A on a smooth horizontal table. A particle of mass 2kg is attached to the other end of the string. The particle is held at the point A and projected horizontally along the table with speed  $2\text{ms}^{-1}$ . Find how far it travels before first coming to instantaneous rest.

A particle of mass  $0.5\text{kg}$  is attached to one end of an elastic string, of natural length  $2\text{m}$  and modulus of elasticity  $19.6\text{N}$ . The other end of the elastic string is attached to a point  $O$ . The particle is released from the point  $O$ . Find the greatest distance it will reach below  $O$ .

A light elastic spring, of natural length 1m and modulus of elasticity 10N, has one end attached to a fixed point  $A$ . A particle of mass 2kg is attached to the other end of the spring and is held at a point  $B$  which is 0.8m vertically below  $A$ . The particle is projected vertically downwards from  $B$  with speed  $2\text{ms}^{-1}$ . Find the distance it travels before first coming to rest.

A light elastic spring, of natural length  $0.5\text{m}$  and modulus of elasticity  $10\text{N}$ , has one end attached to a point  $A$  on a rough horizontal plane. The other end is attached to a particle  $P$  of mass  $0.8\text{kg}$ . The coefficient of friction between the particle and the plane is  $0.4$ . The particle initially lies on the plane with  $AP = 0.5\text{m}$  and is then projected with speed  $2\text{ms}^{-1}$  away from  $A$  along the plane. Find the distance moved by  $P$  before it first comes to rest.

One end  $A$  of a light elastic string  $AB$ , of modulus of elasticity  $mg$  and natural length  $a$ , is fixed to a point on a rough plane inclined at an angle  $\theta$  to the horizontal. The other end  $B$  of the string is attached to a particle of mass  $m$  which is held at rest on the plane. The string  $AB$  lies along a line of greatest slope of the plane, with  $B$  lower than  $A$  and  $AB = a$ . The coefficient of friction between the particle and the plane is  $\mu$ , where  $\mu < \tan \theta$ . The particle is released from rest.

(a) Show that when the particle comes to rest it has moved a distance  $2a(\sin \theta - \mu \cos \theta)$  down the plane. (6)

(b) Given that there is no further motion, show that  $\mu \geq \frac{1}{3} \tan \theta$ . (5)

Ex 3D

9 Two points  $A$  and  $B$  are on the same horizontal level with  $AB = 3a$ . A particle  $P$  of mass  $m$  is joined to  $A$  by a light inextensible string of length  $4a$  and is joined to  $B$  by a light elastic string, of natural length  $a$  and modulus of elasticity  $\frac{mg}{4}$ . The particle  $P$  is held at a point  $C$ , such that  $BC = a$  and both strings are taut. The particle  $P$  is released from rest.

a Show that when  $AP$  is vertical the speed of  $P$  is  $2\sqrt{ga}$ . (6 marks)

b Find the tension in the elastic string in this position. (4 marks)