

3.4) The standard normal distribution

Worked example

Z is the number of standard deviations above the mean.

Assume $X \sim N(100, 15^2)$

Find z if

$$X = 100$$

$$X = 130$$

$$X = 62.5$$

Your turn

Z is the number of standard deviations above the mean.

Assume $X \sim N(100, 15^2)$

Find Z if

$$X = 85$$

$$Z = -1$$

$$X = 165$$

$$Z = 4.3333 \dots$$

Worked example

The random variable $X \sim N(40, 5^2)$.
Write in terms of $\Phi(z)$ for some value of z .

(a) $P(X \leq 45)$

(b) $P(X > 43)$

Your turn

The random variable $X \sim N(50, 4^2)$. Write in terms of $\Phi(z)$ for some value of z .

(a) $P(X < 53)$

(b) $P(X \geq 55)$

a) $\Phi(0.75)$

b) $1 - \Phi(1.25)$

Worked example

If $X \sim N(100, 15^2)$, determine, in terms of Φ :

- (a) $P(X > 70)$
- (b) $P(88 < X < 122.5)$

Your turn

If $X \sim N(100, 15^2)$, determine, in terms of Φ :

- (a) $P(X > 115)$
 - (b) $P(77.5 < X < 112)$
- a) $1 - \Phi(1)$
b) $\Phi(0.8) + \Phi(1.5) - 1$

Worked example

The systolic blood pressure of an adult population, S mmHg, is modelled as a normal distribution with mean 721 and standard deviation 4.

A medical research wants to study adults with blood pressures higher than the 90th percentile.

Find the minimum blood pressure for an adult included in her study.

Your turn

The systolic blood pressure of an adult population, S mmHg, is modelled as a normal distribution with mean 127 and standard deviation 16.

A medical researcher wants to study adults with blood pressures higher than the 95th percentile.

Find the minimum blood pressure for an adult included in her study.

$$s = 153 \text{ (3 sf)}$$

Worked example

Determine:

$$P(Z > -1.7)$$

$$P(Z \leq -1.5)$$

Your turn

Determine:

$$P(Z > -1.3)$$

0.9032 (4 dp)

Worked example

Determine:

$$P(-1 < Z < 0)$$

$$P(-1.5 < Z < 0.5)$$

Your turn

Determine:

$$P(-2 < Z < 1)$$

0.8185 (4 dp)

Worked example

Determine a such that:

$$P(Z > a) = 0.3$$

$$P(Z < a) = 0.4$$

Your turn

Determine a such that:

$$P(Z > a) = 0.7$$

$$a = -0.5244 \text{ (4 dp)}$$

Worked example

Determine a such that:

$$P(-a < Z < a) = 0.4$$

$$P(-a < Z < a) = 0.5$$

Your turn

Determine a such that:

$$P(-a < Z < a) = 0.6$$

$$a = -0.8416 \text{ (4 dp)}$$

Worked example

Use the percentage points table to find values of z which correspond to the 10% to 80% interpercentile range.

Your turn

Use the percentage points table to find values of z which correspond to the 20% to 90% interpercentile range.

$$-0.8416 < z < 1.2816$$