

## 3.3) Geometric sequences

## Worked example

Identify the common ratio:

1, 3, 9, 27, 81, 243, ...

8, 12, 18, 27, ...

80, 20, 5, 1.25, ...

4, -4, 4, -4, 4, -4, ...

$x, -4x^3, 16x^5, \dots$

$p^4, p^3, p^2, p, \dots$

4, -2, 1, -0.5, ...

## Your turn

Identify the common ratio:

1, 2, 4, 8, 16, 32, ...

$$r = 2$$

27, 18, 12, 8, ...

$$r = \frac{2}{3}$$

10, 5, 2.5, 1.25, ...

$$r = \frac{1}{2}$$

5, -5, 5, -5, 5, -5, ...

$$r = -1$$

$x, -2x^2, 4x^3, \dots$

$$r = -2x$$

$1, p, p^2, p^3, \dots$

$$r = p$$

4, -1, 0.25, -0.0625, ...

$$r = -\frac{1}{4}$$

## Worked example

Find the  $n^{\text{th}}$  term of the sequence:

3, 6, 12, 24, 48, ...

4, 20, 100, 500, 2500, ...

## Your turn

Find the  $n^{\text{th}}$  term of the sequence:

2, 6, 18, 54, 162, ...

$$u_n = 2 \times 3^{n-1}$$

## Worked example

Find the 10<sup>th</sup> term of the sequence:

3, 6, 12, 24, 48, ...

4, 20, 100, 500, 2500, ...

## Your turn

Find the 10<sup>th</sup> term of the sequence:

2, 6, 18, 54, 162, ...

**39366**

## Worked example

Find the 10<sup>th</sup> and n<sup>th</sup> term of the sequence:

80, -20, 5, -1.25, 0.3125, ...

## Your turn

Find the 10<sup>th</sup> and n<sup>th</sup> term of the sequence:

40, -20, 10, -5, 2.5, ...

$$u_{10} = -\frac{5}{64}$$

$$u_n = (-1)^{n-1} \times \frac{5}{2^{n-4}}$$

## Worked example

The second term of a geometric sequence is 6 and the 4<sup>th</sup> term is 18. The common ratio is positive. Find the exact values of:

- a) The common ratio.
- b) The first term.
- c) The 20<sup>th</sup> term.

## Your turn

The second term of a geometric sequence is 4 and the 4<sup>th</sup> term is 8. The common ratio is positive. Find the exact values of:

- a) The common ratio.
- b) The first term.
- c) The 10<sup>th</sup> term.

a)  $r = \sqrt{2}$

b)  $a = 2\sqrt{2}$

c)  $u_{10} = ar^9 = 64$

## Worked example

The numbers 2,  $x$  and  $x + 12$  form the first three terms of a positive geometric sequence. Find:

- a) The value of  $x$ .
- b) The 20<sup>th</sup> term in the sequence.

## Your turn

The numbers 3,  $x$  and  $x + 6$  form the first three terms of a positive geometric sequence. Find:

- a) The value of  $x$ .
- b) The 10<sup>th</sup> term in the sequence.

a)  $x = 6$

b) 1536

## Worked example

What is the first term in the geometric progression 2, 6, 18, 54, ... to exceed 1 million?

## Your turn

What is the first term in the geometric progression 3, 6, 12, 24, ... to exceed 1 million?

$$n = 20 ; u_{20} = 1572864$$



## Worked example

The second, third and fourth term of a geometric sequence are the following:

$$x, \quad x + 4, \quad 10x - 2$$

Given the common ratio is positive, find the common ratio and the first term of the sequence

## Your turn

The second, third and fourth term of a geometric sequence are the following:

$$x, \quad x + 6, \quad 5x - 6$$

Given the common ratio is positive, find the common ratio and the first term of the sequence

$$r = 2$$

$$a = 3$$

## Worked example

The second, third and fourth term of a geometric sequence are the following:

$$x, \quad x - 8, \quad 10x - 2$$

Given the common ratio is negative, find the common ratio and the first term of the sequence

## Your turn

The second, third and fourth term of a geometric sequence are the following:

$$x, \quad x - 9, \quad 5x - 3$$

Given the common ratio is negative, find the common ratio and the first term of the sequence

$$r = -2$$

$$a = 3$$

## Worked example

The first three terms of a geometric sequence are:  
16, 144, 1296

Determine whether 944784 is in the sequence

## Your turn

The first three terms of a geometric sequence are:  
4, 36, 324

Determine whether 2125764 is in the sequence

$$n = 7$$

$n$  is an integer

2125764 is in the sequence