3.2) Hooke's law and dynamics problems

Worked example

One end of a light elastic string, of natural length 1m and modulus of elasticity 40N, is attached to a fixed point A.
The other end is attached to a particle of mass 4kg.
The particle is held at a point which is 3m below A and released from rest. Find:

a) The initial acceleration of the particle
b) The length of the string when the particle reaches its maximum speed.

Your turn

One end of a light elastic string, of natural length 0.5m and modulus of elasticity 20N, is attached to a fixed point A.
The other end is attached to a particle of mass 2kg.
The particle is held at a point which is 1.5m below A and released from rest. Find:

a) The initial acceleration of the particle
b) The length of the string when the particle reaches its maximum speed.

a) $10.2\ ms^{-2}$
b) $0.99\ m$

Worked example

A particle of mass $1\ kg$ is attached to one end of a light elastic spring of natural length $3\ m$ and modulus of elasticity $39.2\ N$.
The other end of the spring is attached to a fixed point O on a rough plane inclined to the horizontal at an angle α, where $\tan\alpha = \frac{5}{12}$.
The coefficient of friction between the particle and the plane is 0.4.
The particle is held at rest on the plane at a point which is $2\ m$ from O down a line of greatest slope of the plane.
The particle is released from rest and moves down the slope. Find its initial acceleration.

Your turn

A particle of mass $0.5\ kg$ is attached to one end of a light elastic spring of natural length $1.5\ m$ and modulus of elasticity $19.6\ N$.
The other end of the spring is attached to a fixed point O on a rough plane inclined to the horizontal at an angle α, where $\tan\alpha = \frac{3}{4}$.
The coefficient of friction between the particle and the plane is 0.2.
The particle is held at rest on the plane at a point which is $1\ m$ from O down a line of greatest slope of the plane.
The particle is released from rest and moves down the slope. Find its initial acceleration.

$17.4\ ms^{-2}$ (3 sf)

Worked example

A particle *P* of mass 3 kg is attached to the mid-point of a light elastic string of natural length 0.60 m and modulus of elasticity λ newtons.
The ends of the string are attached to two fixed points *A* and *B*, where *AB* is horizontal and *AB* = 0.96 m.
Initially *P* is held at rest at the mid-point, *M*, of the line *AB* and the tension in the string is 480 N.

a) Find λ

The particle is now held at rest at the point C, which is 0.14 m vertically below M. The particle is released from rest at C.

b) Find the magnitude of the initial acceleration of P

Your turn

A particle *P* of mass 1.5 kg is attached to the mid-point of a light elastic string of natural length 0.30 m and modulus of elasticity λ newtons.
The ends of the string are attached to two fixed points *A* and *B*, where *AB* is horizontal and *AB* = 0.48 m.
Initially *P* is held at rest at the mid-point, *M*, of the line *AB* and the tension in the string is 240 N.

a) Find λ

The particle is now held at rest at the point C, which is 0.07 m vertically below M. The particle is released from rest at C.

b) Find the magnitude of the initial acceleration of P

a) $\lambda = 400$

b) $89.8\ ms^{-2}$ (3 sf)