

3.2) Arithmetic series

Worked example

Find the sum of the first 50 terms
the sequence which begins:

7, 10, 13, 16, ...

Your turn

Find the sum of the first 80 terms
the sequence which begins:

2, 6, 10, 14, ...

12800

Worked example

Find the sum of the first 50 terms
the sequence which begins:

$10, 7, 4, 1, -2, \dots$

Your turn

Find the sum of the first 80 terms
the sequence which begins:

$18, 14, 10, 6, 2, \dots$

-11200

Worked example

Find the sum of the first 50 terms
the sequence which begins:

$$p, 3p, 5p, 7p, 9p, \dots$$

Your turn

Find the sum of the first 80 terms
the sequence which begins:

$$k, 4k, 7k, 10k, 13k, \dots$$

$$9560k$$

Worked example

Find the least number of terms for the sum of $6 + 11 + 16 + \dots$ to exceed 2000.

Your turn

Find the least number of terms for the sum of $4 + 9 + 14 + \dots$ to exceed 2000.

28

Worked example

A company offers two salary schemes for a 10-year period, Year 1 to Year 10 inclusive.

Scheme 1: Salary in Year 1 is $\pounds(P + 900)$

Salary increases by $\pounds(T)$ each year, forming an arithmetic sequence.

Scheme 2: Salary in Year 1 is $\pounds P$

Salary increases by $\pounds 2T$ each year, forming an arithmetic sequence.

For the 10-year period, the total earned is the same for both salary schemes.

- Find the value of T
- For this value of T , the salary in Year 10 under Scheme 1 is $\pounds 25890$. Find the value of P

Your turn

A company offers two salary schemes for a 10-year period, Year 1 to Year 10 inclusive.

Scheme 1: Salary in Year 1 is $\pounds P$

Salary increases by $\pounds(2T)$ each year, forming an arithmetic sequence.

Scheme 2: Salary in Year 1 is $\pounds(P + 1800)$

Salary increases by $\pounds T$ each year, forming an arithmetic sequence.

For the 10-year period, the total earned is the same for both salary schemes.

- Find the value of T
- For this value of T , the salary in Year 10 under Scheme 2 is $\pounds 29850$. Find the value of P

a) $T = 400$

b) $P = 24450$

Worked example

Prove that the sum of the first 200 natural numbers is 20100

Your turn

Prove that the sum of the first 100 natural numbers is 5050

Proof

Worked example

Find the sum of the first 50 odd numbers

Your turn

Find the sum of the first 50 even numbers
2550

Worked example

Prove that the sum of the first n even numbers is $n^2 + n$

Your turn

Prove that the sum of the first n odd numbers is n^2

Proof

Worked example

An arithmetic series is given by
 $(k + 1) + (2k + 5) + (3k + 9) + \dots + 217$

Given that the sum of the series is 2250,
find the value of k

Your turn

An arithmetic series is given by
 $(k + 1) + (2k + 3) + (3k + 5) + \dots + 303$

Given that the sum of the series is 2568,
find the value of k

$$k = 17$$

Worked example

The common difference of an arithmetic sequence is 4. The sum of the first 60 terms of this sequence is 7380. Find the first term.

Your turn

The common difference of an arithmetic sequence is 5. The sum of the first 40 terms of this sequence is 4020. Find the first term.

3

Worked example

The common difference of an arithmetic sequence is 4. The sum of the first 60 terms of this sequence is -240 . Find the first term.

Your turn

The common difference of an arithmetic sequence is 5. The sum of the first 40 terms of this sequence is 2620. Find the first term.

-32

Worked example

The first term of an arithmetic sequence is 3. The sum of the first 50 terms is 2600. Work out the common difference of the sequence.

Your turn

The first term of an arithmetic sequence is 4. The sum of the first 40 terms is 2500. Work out the common difference of the sequence. **3**

Worked example

The eighth term of an arithmetic sequence is 11. The fifth term of the same arithmetic sequence is 2. Find the sum of the first 50 terms of this arithmetic sequence.

Your turn

The tenth term of an arithmetic sequence is 35. The fourth term of the same arithmetic sequence is 17. Find the sum of the first 50 terms of this arithmetic sequence. **4075**