<u>Elastic Strings and Springs (Chapter 3)</u>

Hooke's Law: When an elastic string or spring is stretched, the tension, T, produced is proportional to the extension, x.

or compression

$$T = \frac{\lambda}{l} x$$

- λ is known as 'the modulus of elasticity' (see note)
- l is the 'natural (unstretched) length' of the string/spring.

Note about λ:

- In A level Maths 'the modulus of elasticity' refers to the constant λ in the equation $T=\frac{\lambda}{l}x$.
- λ can be understood as the force needed to double the length of a string/spring: hence, the units of λ are Newtons (N)
- λ should not be confused with 'Young's modulus' – a constant that does not feature in A level maths, but which you might have come across in physics (and which is confusingly sometimes referred to as the 'elastic modulus').

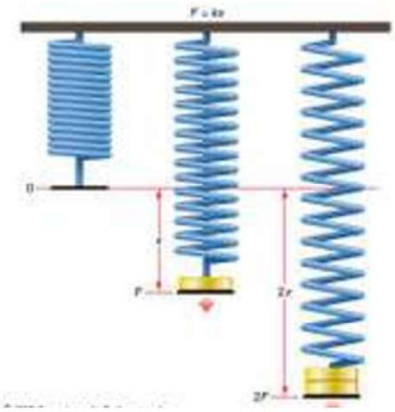
Hooke's Law: When it Applies

Many springs, strings, wires and solids obey Hooke's Law in a limited range. Hooke's law only ever applies up to a maximum force known as the *elastic limit* which varies for each spring etc. Under very large forces springs deform and break.

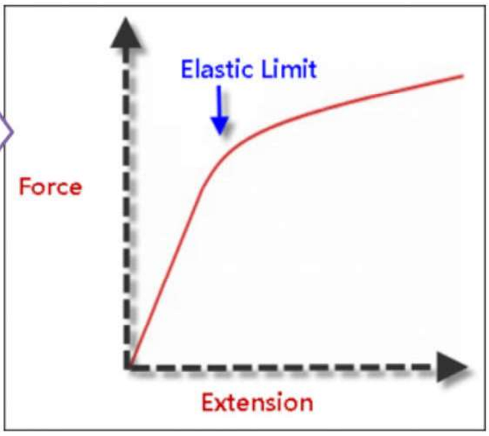
If a string or spring obeys Hooke's Law it is called *elastic*.

An elastic spring can also be compressed (producing thrust instead of tension). However, elastic strings don't resist compression (think about how string behaves).

All string/springs (in A Level Maths) are light - they don't extend under their own weight.



Graph showing the extension of a Spring as the weight hung from it, and hence the tension in the spring, increases.
The variables are in direct proportion (obeying Hooke's Law) up to the 'elastic limit' beyond which the law doesn't hold.



An elastic string of natural length 2m and modulus of elasticity 29.4N has one end fixed. A particle of mass 4kg is attached to the other end and hangs at rest. Find the extension of the string.

An elastic spring of natural length 1.5m has one end attached to a fixed point. A horizontal force of magnitude 6N is applied to the other end and compresses the spring to a length of 1m.

Find the modulus of elasticity of the spring.

A string of natural length l and modulus of elasticity 123N is stretched to a length $2l$. What is the Tension in the string?

A string of natural length l and modulus of elasticity 123N is stretched to a length $3l$. What is the Tension in the string?

A string of natural length l and modulus of elasticity 123N is extended by a distance $2l$. What is the Tension in the string?

A spring of natural length 3m is stretched to a length of 6m by applying a force to one end of 99N. What is the modulus of elasticity of the spring?

A spring of natural length l and $\lambda = 40N$ is compressed to a length $\frac{3}{4}l$. What is the compressive force?

A string of natural length l and $\lambda = 40N$ is compressed by a force 30N. What is x?

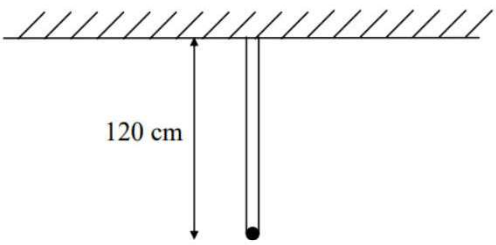
Combining strings/springs

The elastic springs PQ and QR are joined together at Q to form one long spring. The spring PQ has natural length 1.6m and modulus of elasticity 20N. The spring QR has natural length 1.4m and modulus of elasticity 28N. The ends P and R, of the long string are attached to two fixed points which are 4m apart on the same horizontal plane. [*Assume Q is at rest and in equilibrium*].
Find the tension in the combined spring.

Two identical elastic springs PQ and QR each have natural length *l* and modulus of elasticity $2mg$.
The springs are joined together at Q. Their other ends, P and R, are attached to fixed points, with P being $4l$ vertically above R.
A particle of mass *m* is attached at Q and hangs at rest in equilibrium.
Find the distance of the particle below P.

Your Turn

Figure 1



A particle of mass 5 kg is attached to one end of two light elastic strings. The other ends of the strings are attached to a hook on a beam. The particle hangs in equilibrium at a distance 120 cm below the hook with both strings vertical, as shown in Fig. 1. One string has natural length 100 cm and modulus of elasticity 175 N. The other string has natural length 90 cm and modulus of elasticity λ newtons.

Find the value of λ.

(5)

Ex 3A Q1-7
Skip 5

Slopes, Friction, Moments, Resolving Forces, etc.

Elastic Strings and Springs can be introduced into similar problems to those encountered in A level Mechanics. e.g. slopes, friction, **moments (not yet studied)**

An elastic string has natural length 2m and modulus of elasticity 98N.
One end of the string is attached to a fixed point *O* and the other end is attached to a particle P of mass 4kg.
The particle is held in equilibrium by a horizontal force of magnitude 28N, with *OP* making an angle α with the vertical.
Find:
a) the value of α
b) The length *OP*

An elastic string of natural length $2l$ and modulus of elasticity $4mg$ is stretched between two points A and B.
The points A and B are on the same horizontal level and AB =$2l$.
A particle P is attached to the midpoint of the string and hangs in equilibrium with both parts of the string making an angle of 30° with the line AB.
Find, in terms of m, the mass of the particle.

we'll need a different m... use M instead

A particle P, of mass m, rests in equilibrium on a rough plane inclined at 30° to the horizontal. The coefficient of friction between the particle and the plane is $\frac{\sqrt{3}}{3}$. P is attached to a fixed point A on the plane by a light elastic spring with natural length a and modulus of elasticity $3mg$. P is free to move only in a straight line below A down the line of greatest slope. Write an inequality for the length AP.

One end, *A*, of a light elastic string *AB*, of natural length 0.6m and modulus of elasticity 10N, is fixed to a point on a fixed rough plane inclined at an angle θ to the horizontal, where $\sin \theta = 0.8$. A ball of mass 3kg is attached to the end, *B*, of the string. The coefficient of friction, μ, between the ball and the plane is 1/3. The ball rests in limiting equilibrium, on the point of sliding down the plane, with *AB* along the line of greatest slope.

Find:

a) The tension in the string
b) The length of the string
c) If $\mu > 1/3$, without doing any further calculation, state how your answer to b) would change

Ex 3A
9, 10, 11