3) Equations and inequalities

3.1) Linear simultaneous equations
3.2) Quadratic simultaneous equations
3.3) Simultaneous equations on graphs
3.4) Linear inequalities
3.5) Quadratic inequalities
3.6) Inequalities on graphs
3.7) Regions

3.1) Linear simultaneous equations

Chapter CONTENTS

Worked example	Your turn
Solve: $-5x - 2y = -26$ $-5x - 4y = -32$ $-2x - 3y = -17$ $-5x - 3y = -20$	Solve: $-3x - 2y = -12$ $-7x - 2y = -20$ $x = 2, y = 3$

Worked example	Your turn
Solve: $5x + 4y = 23$ $-5x + 2y = 19$ $5x + 3y = 23$ $2x - 3y = 5$	Solve: $6x + 8y = 22$ $-6x + 2y = 28$ $x = -3\,, y = 5$

Worked example	Your turn
Solve: $2x + 3y = 11$ $3x + y = 13$ $3x + 2y = 9$ $5x + 7y = 4$	Solve: $2x + 3y = 9$ $5x + 7y = 23$ $x = 6\,, y = -1$

Worked example	Your turn
Solve: $y = 4x - 9$ $5y - 3x = 23$	Solve: $y = 2x - 3$ $9y - 4x = 1$ $x = 2, y = 1$

Worked example	Your turn
Solve: $y = 4x - 9$ $-5y - 3x = -1$	Solve: $y = 2x - 3$ $-9y - 4x = -17$ $x = 2, y = 1$

3.2) Quadratic simultaneous equations

Chapter CONTENTS

Worked example	Your turn
Solve: $y = x^2 + x - 2$ $y = 2x + 4$	Solve: $y = x^2 + 7x - 2$ $y = 2x + 4$ $x = 1, y = 6$ $x = -6, y = -8$

Worked example	Your turn
Solve: $x + y = 3$ $x^2 + y^2 = 9$	Solve: $x^2 + y^2 = 4$ $x + y = 2$ $x = 0, y = 2$ $x = 2, y = 0$

Worked example	Your turn
Solve: $y = 2x + 1$ $x^2 + y^2 = 29$	Solve: $y = 3x - 1$ $x^2 + y^2 = 73$ $x = 3, y = 8$ $x = -\frac{12}{5}, y = -\frac{41}{5}$

Worked example	Your turn
Solve: $5x^2 + y^2 = 49$ $y = x - 1$	Solve: $3x^2 + y^2 = 21$ $y = x + 1$ $x = -\frac{5}{2}, y = -\frac{3}{2}$ $x = 2, y = 3$

Worked example	Your turn
Solve: $3y^2 - 2x^2 = 10$ $y + x = 13$	Solve: $4y^2 - 3x^2 = -12$ $y + x = 7$ $x = 4, y = 3$ $x = 52, y = -45$

Worked example	Your turn
Solve: $3y^2 - 2x^2 = 19$ $2y + 3x = 15$	Solve: $2y^2 - 3x^2 = 38$ $3y + 2x = 19$ $x = 2, y = 5$ $x = -10, y = 13$

Worked example	Your turn
Solve: $xy = 12$ $y = x + 4$ $xy = 12$ $x = y - 2$	Solve: $xy = 12$ $y = x + 11$ $x = 1, y = 12$ $x = -12, y = -1$

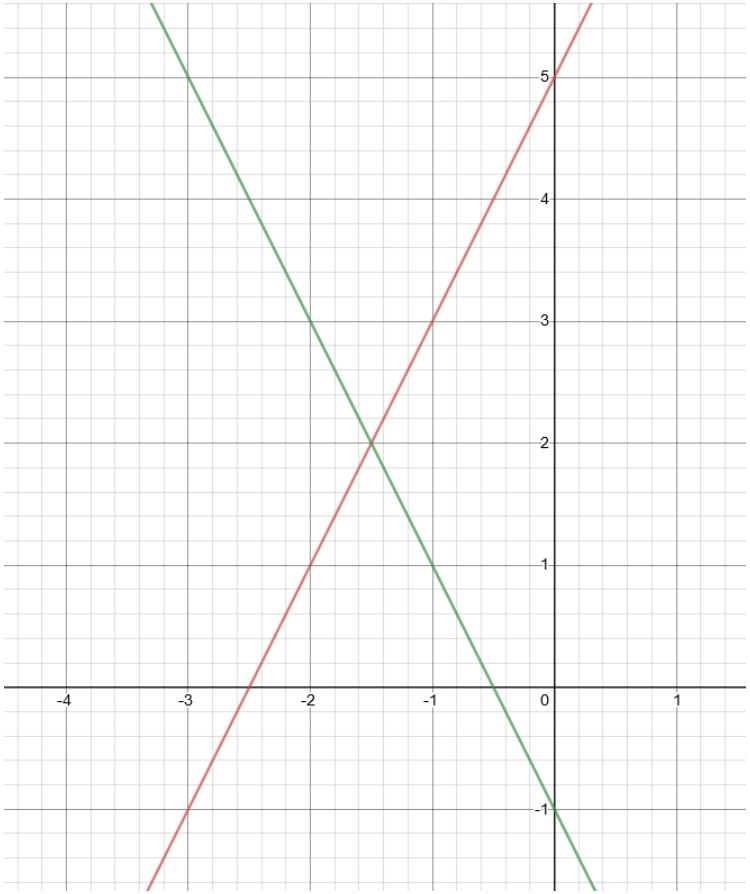
3.3) Simultaneous equations on graphs

Chapter CONTENTS

Worked example

Solve:

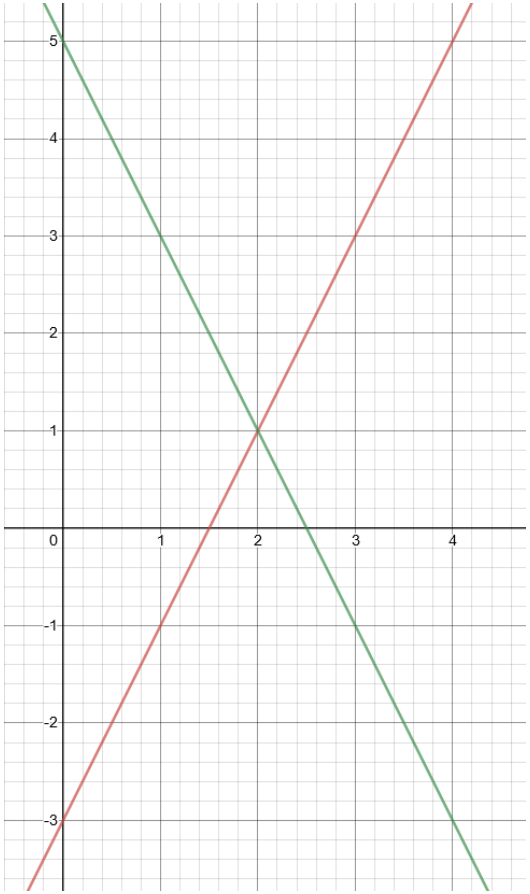
$$y = 2x + 5$$
$$y = -2x - 1$$



Your turn

Solve:

$$y = 2x - 3$$
$$y = -2x + 5$$

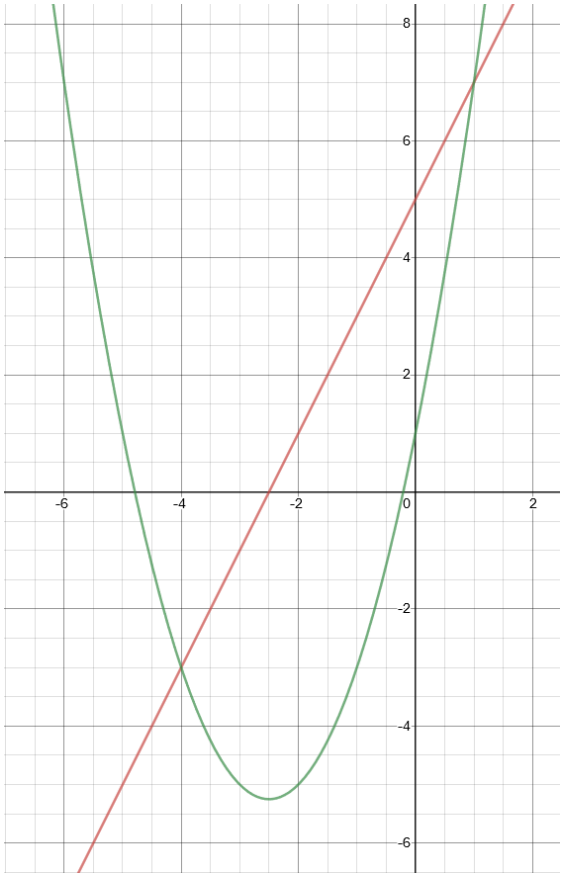


$x = 2, y = 1$

Worked example

Solve:

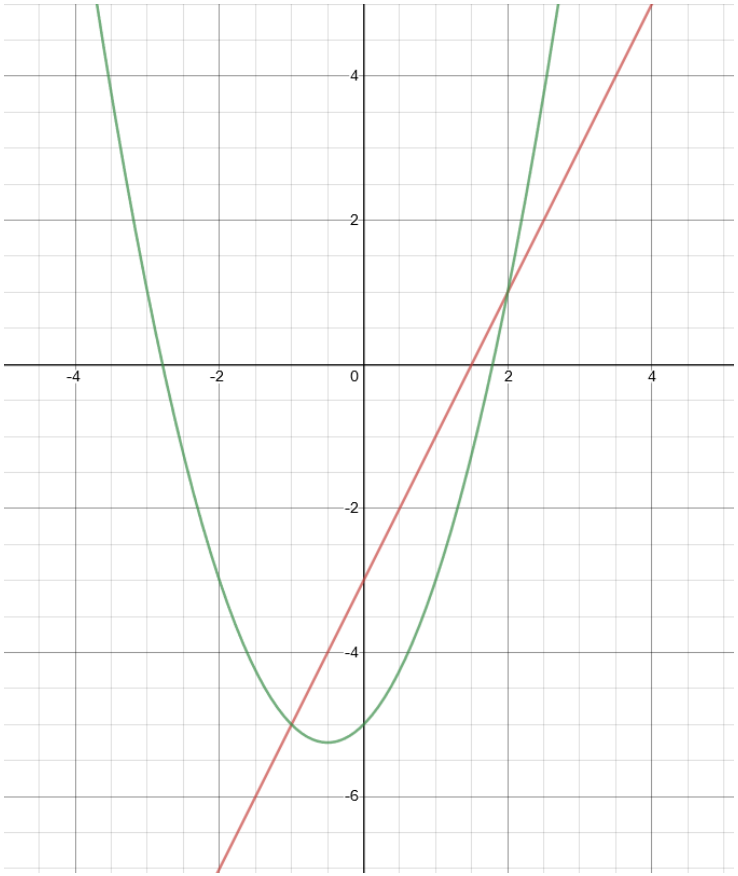
$$y = 2x + 5$$
$$y = x^2 + 5x + 1$$



Your turn

Solve:

$$y = 2x - 3$$
$$y = x^2 + x - 5$$



$$x = 2, y = 1$$
$$x = -1, y = -5$$

Worked example

By using the discriminant of a subsequent equation, show that the graphs of $4x + y = 3$ and $y = x^2 - 3x + 1$ have two points of intersection

Your turn

By using the discriminant of a subsequent equation, show that the graphs of $2x + y = 3$ and $y = x^2 - 3x + 1$ have two points of intersection

$$x^2 - x - 2 = 0$$

$$\text{Discriminant} = 9 > 0$$

Worked example

Prove algebraically, and show graphically, that the lines never meet:

$$y = 3x - 3$$
$$y = x^2 + 5x + 4$$

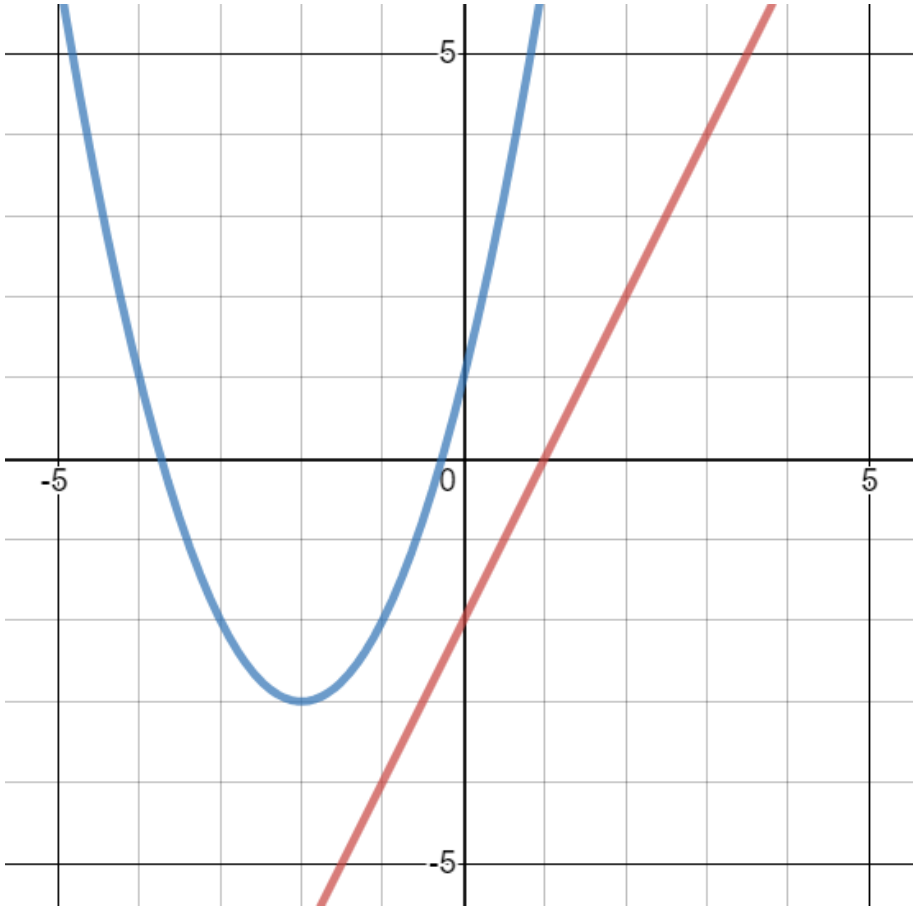
Your turn

Prove algebraically, and show graphically, that the lines never meet:

$$y = 2x - 2$$
$$y = x^2 + 4x + 1$$

$$x^2 + 2x + 3 = 0$$

Discriminant $= -8 < 0$



Worked example

The line with equation $y = 3x + 4$ meets the curve with equation $kx^2 + 2y + (k - 8) = 0$ at exactly one point. Given that k is a positive constant:

a) Find the value of k.

b) For this value of k, find the coordinates of this point of intersection.

Your turn

The line with equation $y = 2x + 1$ meets the curve with equation $kx^2 + 2y + (k - 2) = 0$ at exactly one point. Given that k is a positive constant:

a) Find the value of k.

b) For this value of k, find the coordinates of this point of intersection.

a) $k = 2$

b) $(-1, -1)$

3.4) Linear inequalities

Chapter CONTENTS

Worked example	Your turn
Solve:	Solve:
$3x < 12$	$15 \geq 3x - 6$
	$x \leq 7$
$4x - 1 > 15$	
$11 \leq 2x - 5$	

Worked example	Your turn
Solve: $5x + 2 < 3x - 4$	Solve: $4x - 3 \geq 2 - x$
	$x \geq 1$
$3x + 2 \leq 5x - 4$	
$3x + 2 > 4 - 5x$	

Worked example

Solve:

$$-x < 2$$

$$-x \geq -3$$

Your turn

Solve:

$$-x \leq -4$$

$$x \geq 4$$

Worked example	Your turn
Solve: $-x < 12$	Solve: $-4x + 5 \leq 17$
$12 < -2x$	$x \geq -3$
$16 \geq -3x + 4$	

Worked example

If $x < 3$ and $2 \leq x < 4$, what is the combined solution set?

Your turn

If $x < 3$ and $2 \leq x < 4$, what is the combined solution set?

$2 \leq x < 3$

Worked example

Use set notation to describe the set of values for which:

$$10(9x + 8) < 7 \text{ or } 6(5x - 4) \geq \frac{3-2x}{4}$$

Your turn

Use set notation to describe the set of values for which:

$$2(3x + 4) < 5 \text{ or } 6(7x - 8) \geq \frac{9-10x}{2}$$

$$\left\{x : x < -\frac{1}{2}\right\} \cup \left\{x : x \geq \frac{105}{94}\right\}$$

3.5) Quadratic inequalities

Chapter CONTENTS

Worked example	Your turn
Solve: $x^2 - 5x + 6 = 0$	Solve: $x^2 - 4x + 3 < 0$
$x^2 - 5x + 6 < 0$	$1 < x < 3$
$x^2 - 5x + 6 \leq 0$	

Worked example	Your turn
Solve: $x^2 - 5x + 6 = 0$	Solve: $x^2 - 4x + 3 > 0$
$x^2 - 5x + 6 > 0$	$x < 1 \cup x > 3$
$x^2 - 5x + 6 \geq 0$	

Worked example	Your turn
Solve: $2x^2 - 7x + 3 \leq 0$	Solve: $2x^2 - 7x + 6 \leq 0$
	$\frac{3}{2} \leq x \leq 2$
$2x^2 - 3x - 5 < 0$	

Worked example	Your turn
Solve: $2x^2 + x - 6 \geq 0$	Solve: $3x^2 + x - 2 \geq 0$
	$x \leq -1 \cup x \geq \frac{2}{3}$
$2x^2 + x - 6 > 0$	

Worked example

Find the set of values of x for which:

$$3 + 5x - 2x^2 < 0$$

Your turn

Find the set of values of x for which:

$$3 - 5x - 2x^2 < 0$$

$x < -3$ or $x > \frac{1}{2}$

Worked example	Your turn
Solve: $x^2 + 5x + 23 \leq -3x + 8$ $x^2 - 14x + 57 > 2x - 3$	Solve: $x^2 + 7x + 38 < -7x - 2$ $-10 < x < -4$

Worked example	Your turn
Solve: $x^2 < 9$	Solve: $x^2 < 16$ $-4 < x < 4$
$2x^2 \leq 8$	

Worked example	Your turn
Solve: $x^2 > 25$	Solve: $x^2 > 36$ $x < -6 \cup x > 6$
$2x^2 \geq 2$	

Worked example

Find the set of values for which both are true:
$2(x-3) < 7-5x$ and $(3x-4)(5+x) < 0$

Your turn

Find the set of values for which both are true:
$3(x-2) < 8-2x$ and $(2x-7)(1+x) < 0$

$$-1 < x < \frac{14}{5}$$

Worked example

Find the set of values for which $\frac{10}{x} > 5, x \neq 0$

Your turn

Find the set of values for which $\frac{6}{x} > 2, x \neq 0$

$0 < x < 3$

Worked example

Find the set of values for which $\frac{5}{x-3} < 2$

Your turn

Find the set of values for which $\frac{5}{x-2} < 3$

$x < 2$ or $x > \frac{11}{3}$

Worked example

The equation $kx^2 - 5kx + 50 = 0$, where k is a constant, has no real roots.
Prove that k satisfies the inequality $0 \leq k < 8$

Your turn

The equation $kx^2 - 3kx + 9 = 0$, where k is a constant, has no real roots.
Prove that k satisfies the inequality $0 \leq k < 4$

Proof

3.6) Inequalities on graphs

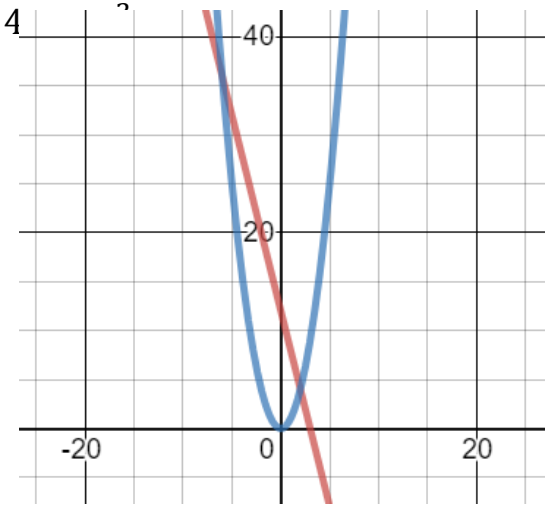
Chapter CONTENTS

Worked example

L_1 has equation $y = 12 - 4x$.
L_2 has equation $y = x^2$.
The diagram shows a sketch of L_1 and L_2 on the same axes.

a) Find the coordinates of the points of intersection.

b) Hence write down the solution to the inequality $12 - 4$

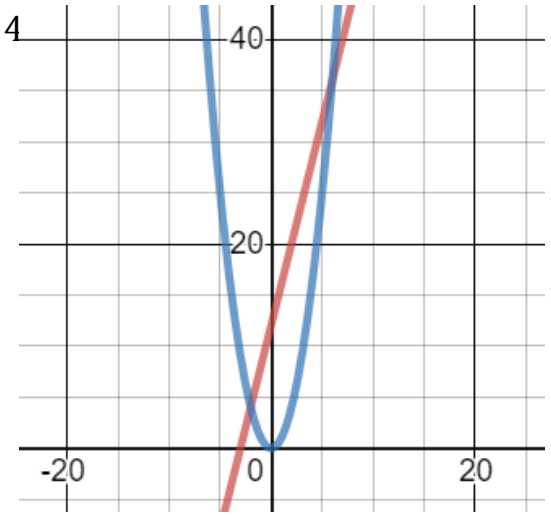


Your turn

L_1 has equation $y = 12 + 4x$.
L_2 has equation $y = x^2$.
The diagram shows a sketch of L_1 and L_2 on the same axes.

a) Find the coordinates of the points of intersection.

b) Hence write down the solution to the inequality $12 + 4$



a) $(6, 36)$ and $(-2, 4)$

b) $-2 < x < 6$

3.7) Regions

Chapter CONTENTS

Worked example

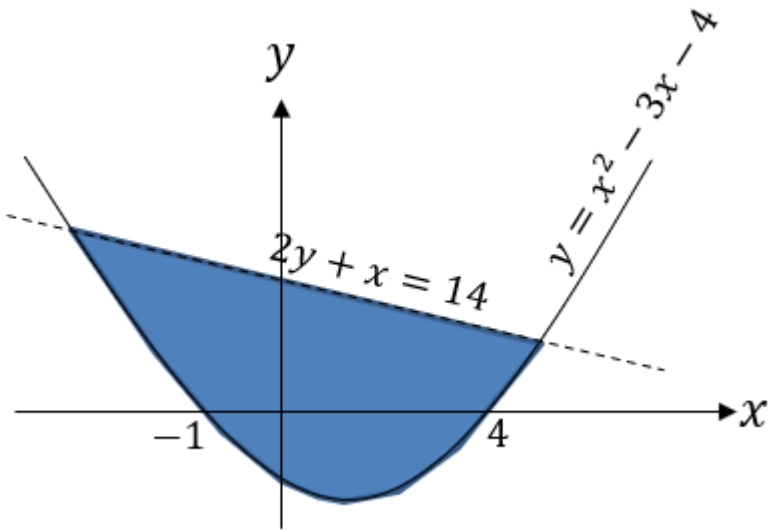
Shade the region that satisfies the inequalities:

$$4y + x \leq 12$$
$$y > x^2 - 5x - 6$$

Your turn

Shade the region that satisfies the inequalities:

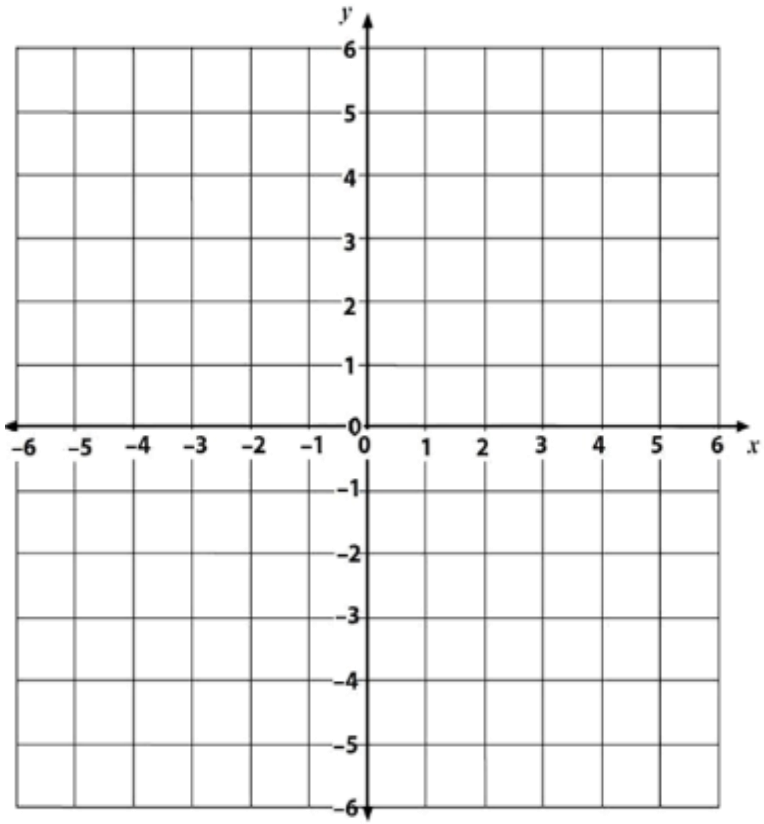
$$2y + x < 14$$
$$y > x^2 - 3x - 4$$



Worked example

Shade the region which satisfies the inequalities. Label it R.

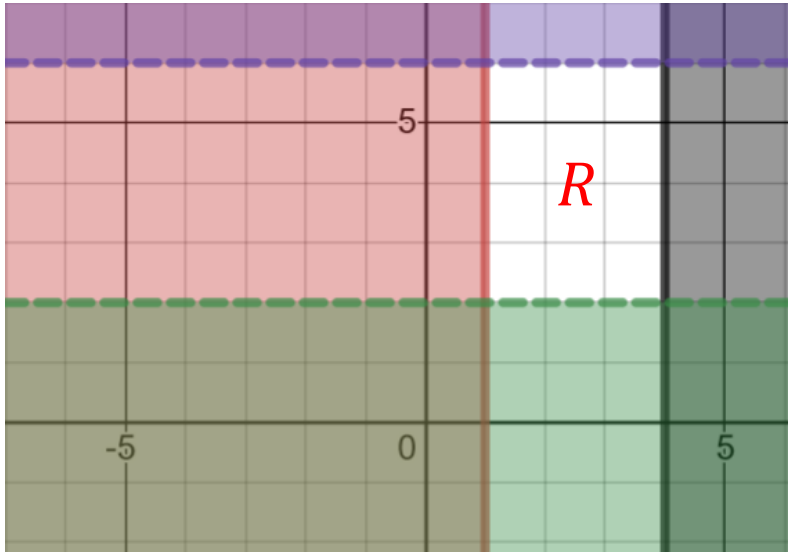
$2 \leq x \leq 5$ and $1 < y < 3$



Your turn

Shade the region which satisfies the inequalities. Label it R

$1 \leq x \leq 4$ and $2 < y < 6$



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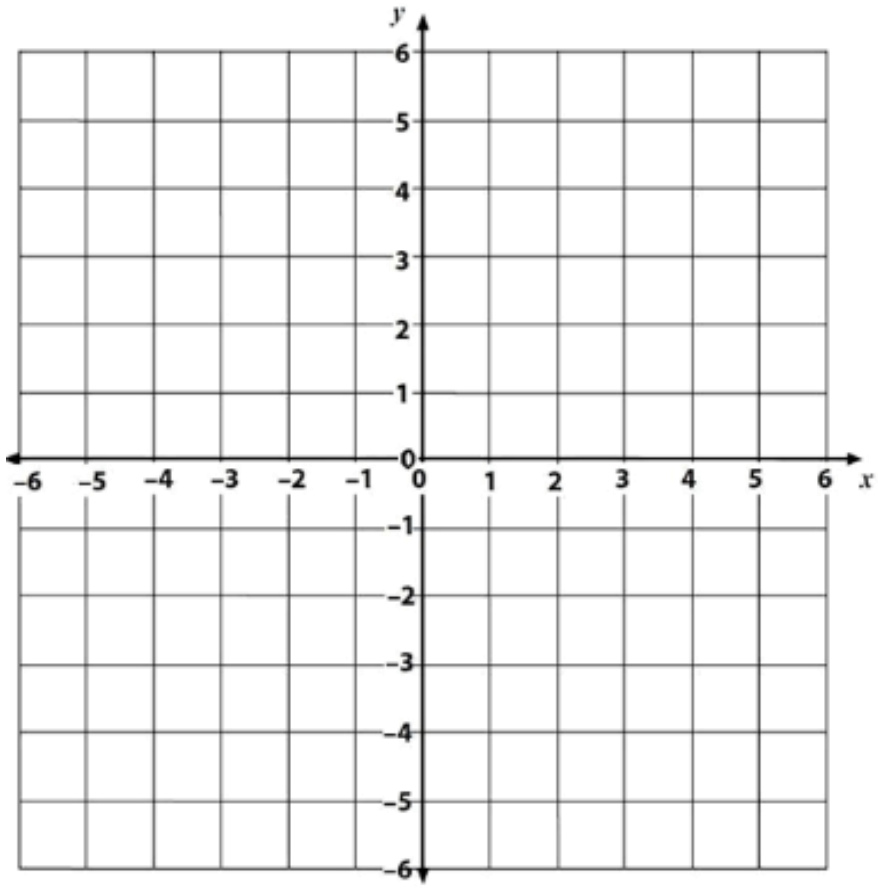
Worked example

Shade the region which satisfies the inequalities. Label it R

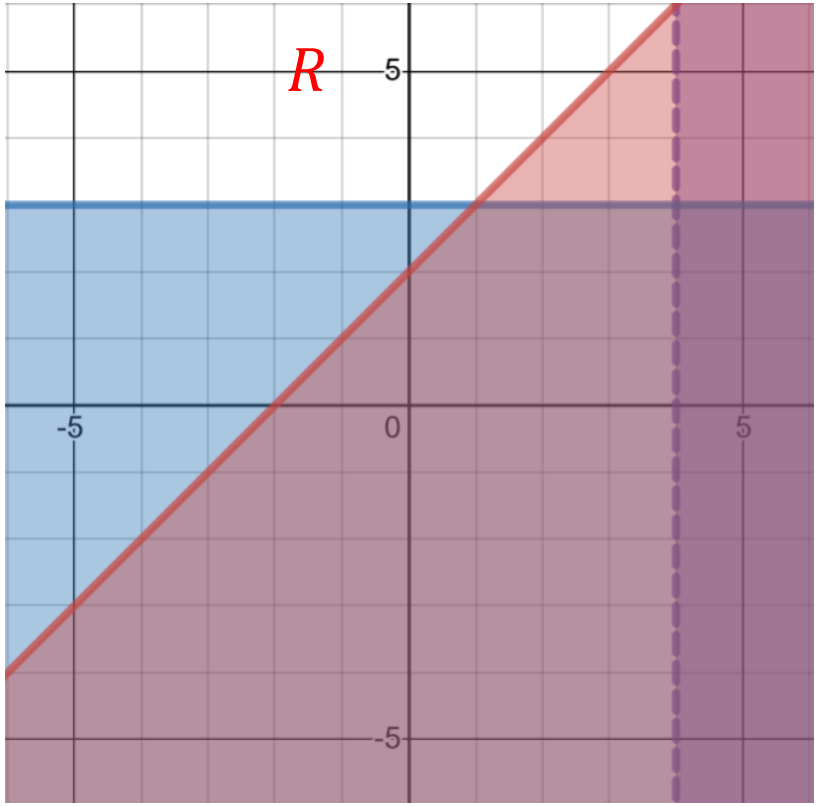
$x \leq 3, y > 1$ and $y \geq x + 3$



Your turn

Shade the region which satisfies the inequalities. Label it R.

$x < 4, y \geq 3, y \geq x + 2$



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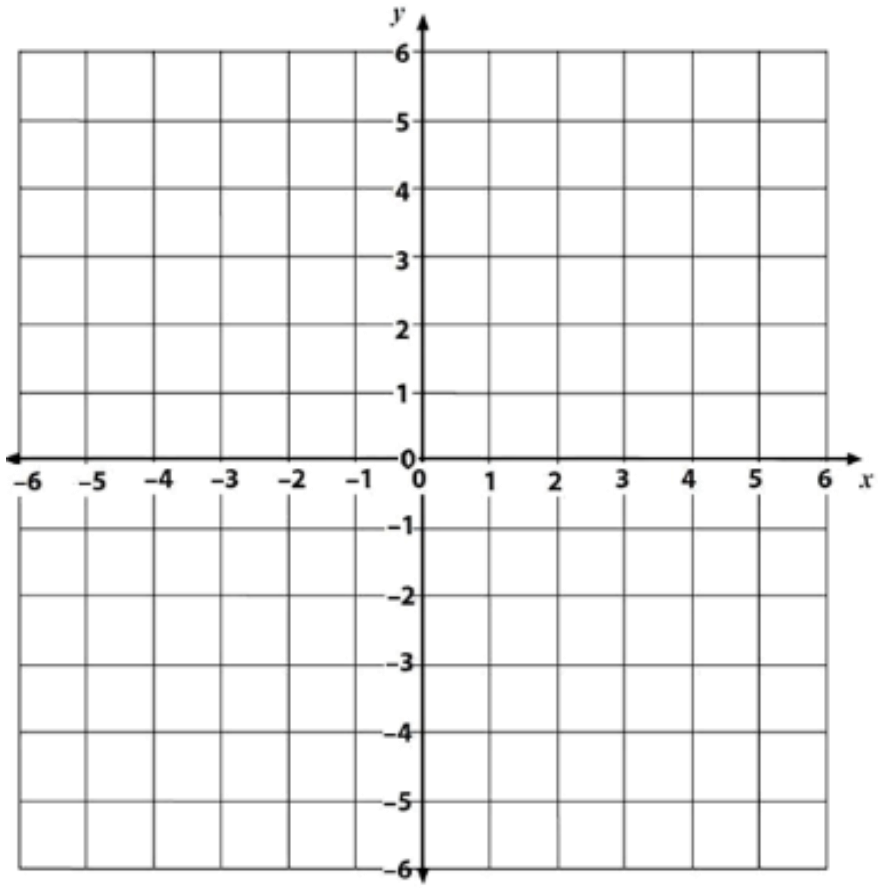
Worked example

Shade the region which satisfies the inequalities:

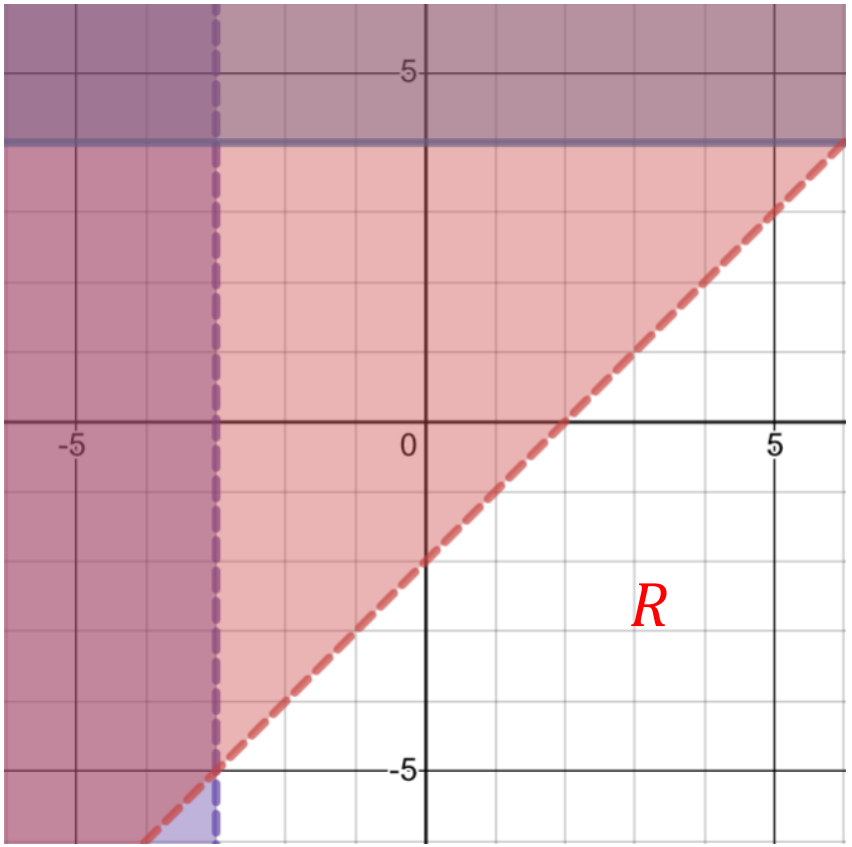
$x \geq -2, y < 1$ and $y < x - 1$



Your turn

Shade the region which satisfies the inequalities. Label it R.

$x > -3, y \leq 4$ and $y < x - 2$



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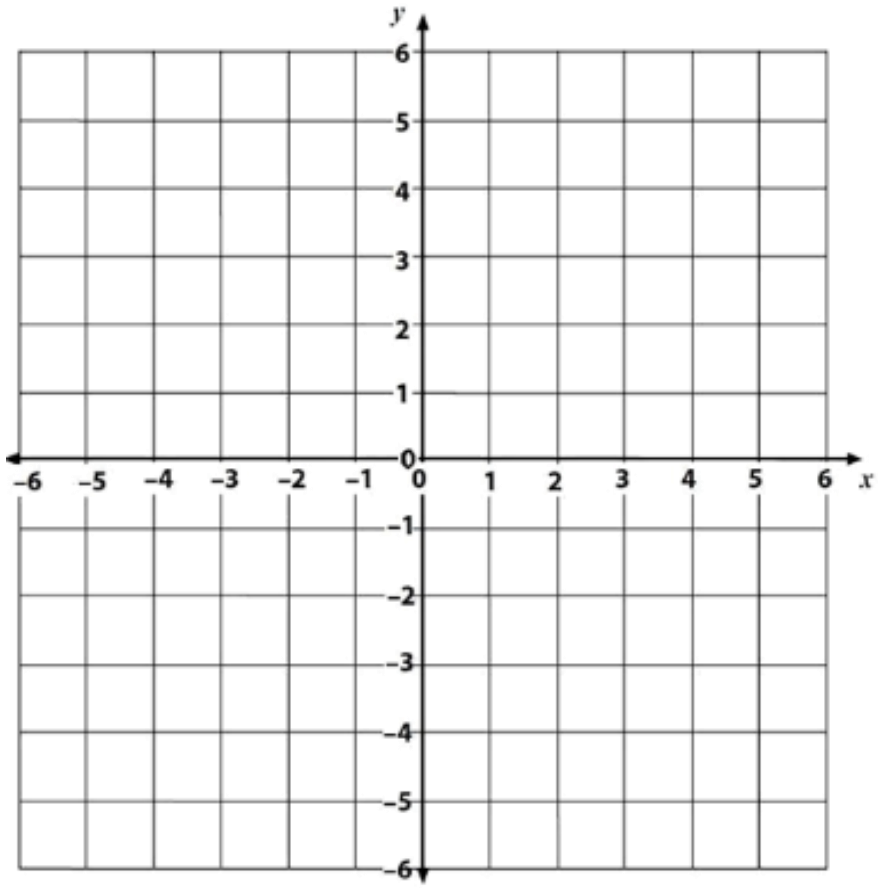
Worked example

Shade the region which satisfies the inequalities:

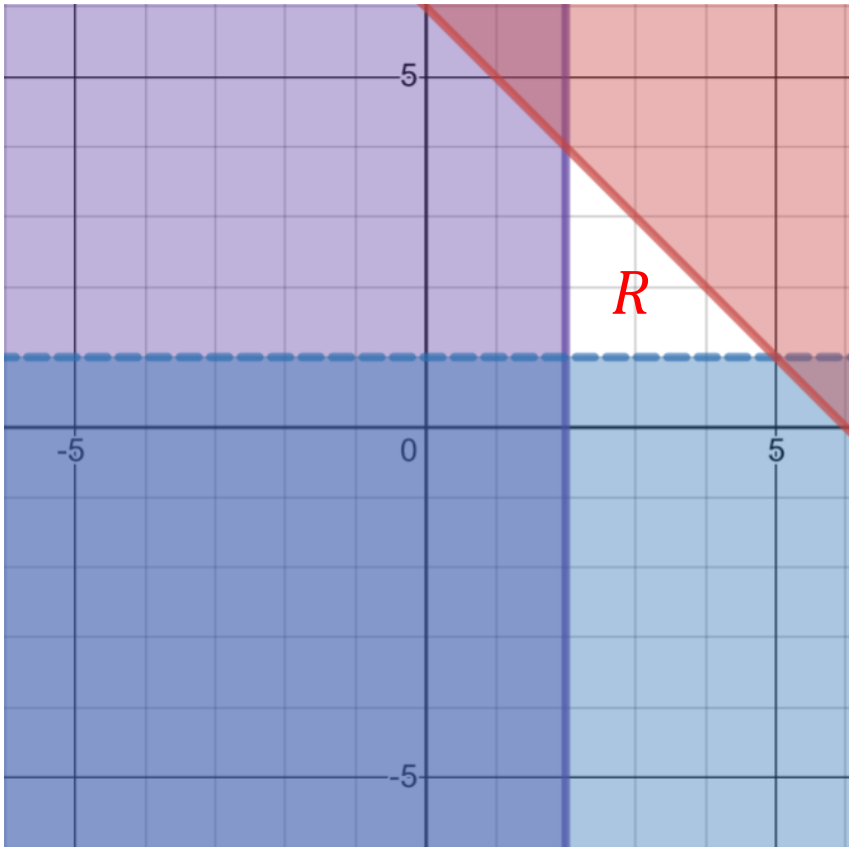
$x \geq 2, y > -1$ and $x + y \leq 5$



Your turn

Shade the region which satisfies the inequalities. Label it R.

$x \geq 2, y > 1$ and $x + y \leq 6$



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