

2.5) The discriminant

Worked example

How many distinct real solutions do these equations have?

$$x^2 + 6x + 8 = 0$$

$$x^2 + 6x + 9 = 0$$

$$x^2 + 6x + 10 = 0$$

Your turn

How many distinct real solutions do these equations have?

$$x^2 + 8x + 12 = 0$$

2

$$x^2 + 8x + 16 = 0$$

1 (equal roots)

$$x^2 + 8x + 17 = 0$$

0

Worked example

Find the value of the discriminant:

$$x^2 + 5x + 6 = 0$$

$$x^2 - 5x + 6.25 = 0$$

$$x^2 - 5x + 7 = 0$$

Your turn

Find the value of the discriminant:

$$x^2 + 3x + 2 = 0$$

1

$$x^2 - 3x + 2.25 = 0$$

0

$$x^2 - 3x + 4 = 0$$

-7

Worked example

Find the value of the discriminant:

$$6x^2 - 3x - 2 = 0$$

$$3x^2 - 2x - 6 = 0$$

Your turn

Find the value of the discriminant:

$$2x^2 - 6x - 3 = 0$$

60

Worked example

Find the value of the discriminant:

$$4 + 3x - x^2$$

$$4 - 3x - 2x^2$$

$$4 - x^2$$

Your turn

Find the value of the discriminant:

$$9 - 5x - x^2$$

61

$$9 - 5x - 3x^2$$

133

$$9 - x^2$$

36

Worked example

Find the range of values of k for which $f(x) = x^2 + kx + 25$ has equal roots

Your turn

Find the range of values of k for which $f(x) = x^2 + kx + 9$ has equal roots

$$k = \pm 6$$

Worked example

Find the range of values of k for which $x^2 + 6x + k = 0$ has two distinct real solutions

Your turn

Find the range of values of k for which $x^2 + 4x + k = 0$ has two distinct real solutions

$$k < 4$$

Worked example

The equation $x^2 + 4px + (11p + 3) = 0$, where p is a positive constant, has equal roots.

- a) Find the value of p
- b) For this value of p solve the equation

Your turn

The equation $x^2 + 2px + (3p + 4) = 0$, where p is a positive constant, has equal roots.

- a) Find the value of p
- b) For this value of p solve the equation

a) $p = 4$

b) $x = -4$

Worked example

$$x^2 + 3kx + (6k + 12) = 0$$

where k is a negative constant.

Given that this equation has equal roots, determine the value of k .

Your turn

$$x^2 + 5kx + (10k + 5) = 0$$

where k is a negative constant.

Given that this equation has equal roots, determine the value of k .

$$k = -\frac{2}{5}$$

Worked example

Find the range of values of k for which $5x^2 - 3x + k = 0$ has no real solutions.

Your turn

Find the range of values of k for which $3x^2 - 5x + k = 0$ has no real solutions.

$$k > \frac{25}{12}$$

Worked example

Prove that the function

$$f(x) = 4x^2 + (k + 8)x - k$$

has two distinct real roots for all values of k

Your turn

Prove that the function

$$f(x) = 3x^2 + (k + 6)x + k$$

has two distinct real roots for all values of k

Proof