

2.4) Inverse functions

Worked example

Find the inverse functions:

$$f(x) = 4x + 3, \quad x \in \mathbb{R}$$

$$g(x) = 4 - 3x, \quad x \in \mathbb{R}$$

Your turn

Find the inverse function:

$$h(x) = 3 - 4x, \quad x \in \mathbb{R}$$

$$h^{-1}(x) = \frac{3 - x}{4}, \quad x \in \mathbb{R}$$

Worked example

Find the inverse functions:

$$f(x) = \frac{x - 2}{2x + 1}, \quad x \neq -\frac{1}{2}$$

$$g(x) = \frac{2x + 3}{4x - 5}, \quad x \neq \frac{5}{4}$$

Your turn

Find the inverse function:

$$h(x) = \frac{x + 2}{2x - 1}, \quad x \neq \frac{1}{2}$$

$$h^{-1}(x) = \frac{x + 2}{2x - 1}, \quad x \neq \frac{1}{2}$$

Worked example

Find the inverse functions:

$$f(x) = 3x^2 - 5, \quad x \geq 0$$

$$g(x) = 4x^2 + 6, \quad x \geq 0$$

Your turn

Find the inverse function:

$$h(x) = 2x^2 - 7, \quad x \geq 0$$

$$h^{-1}(x) = \sqrt{\frac{x+7}{2}}, \quad x \geq -7$$

Worked example

Find the inverse functions:

$$f(x) = x^2 + 4x + 3, \quad x \geq -2$$

$$g(x) = x^2 - 8x - 5, \quad x \geq 5$$

Your turn

Find the inverse function:

$$h(x) = x^2 - 6x - 5, \quad x \geq 3$$

$$h^{-1}(x) = 3 + \sqrt{x + 14}, \quad x \geq -14$$

Worked example

Find the inverse functions:

$$f(x) = \frac{2}{x-5}, \quad x \in \mathbb{R}, x \neq 5$$

$$g(x) = \frac{7}{x+2}, \quad x \in \mathbb{R}, x \neq -2$$

Your turn

Find the inverse function:

$$h(x) = \frac{3}{x-1}, \quad x \in \mathbb{R}, x \neq 1$$

$$h^{-1}(x) = \frac{3+x}{x}, \quad x \neq 0$$

Worked example

Find the inverse functions:

$$f(x) = e^x - 3, \quad x \in \mathbb{R}$$

$$g(x) = e^x + 4, \quad x \in \mathbb{R}$$

Your turn

Find the inverse function:

$$h(x) = e^x - 5, \quad x \in \mathbb{R}$$

$$h^{-1}(x) = \ln(x + 5), \quad x > -5$$

Worked example

Find the inverse functions:

$$f(x) = \ln x - 3, \quad x > 0$$

$$g(x) = \ln(x - 4), \quad x > 4$$

Your turn

Find the inverse function:

$$h(x) = \ln(x - 5), \quad x > 5$$

$$h^{-1}(x) = e^x + 5, \quad x \in \mathbb{R}$$

Worked example

$$f(x) = \sqrt{x-3} \{x \in \mathbb{R}, x \geq 3\}$$

- State the range of $f(x)$
- Find the function $f^{-1}(x)$ and state its domain and range
- Sketch $y = f(x)$, $y = f^{-1}(x)$ and $y = x$

Your turn

$$p(x) = \sqrt{x-2} \{x \in \mathbb{R}, x \geq 2\}$$

- State the range of $p(x)$
- Find the function $p^{-1}(x)$ and state its domain and range
- Sketch $y = p(x)$, $y = p^{-1}(x)$ and $y = x$

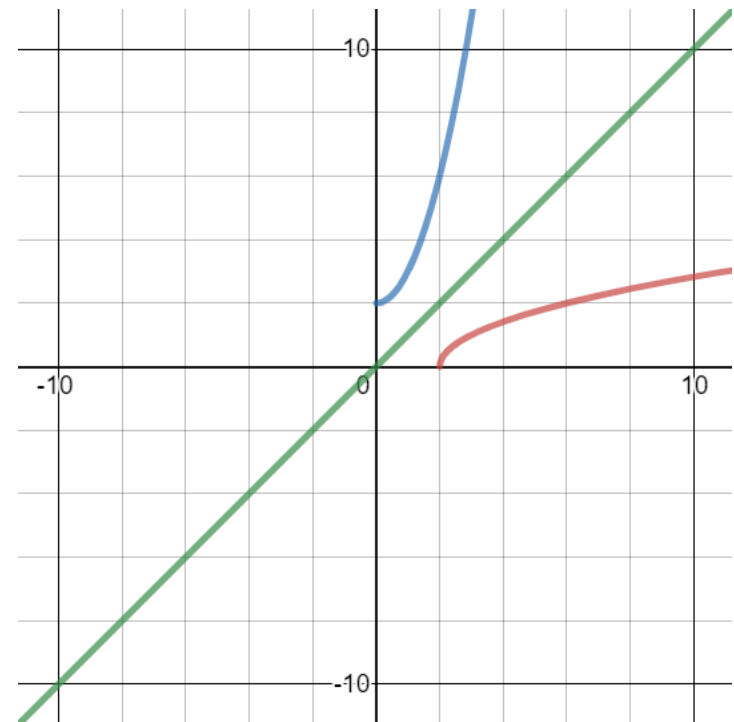
a) $p(x) \geq 0$

b) $p^{-1}(x) = x^2 + 2$

Domain: $x \in \mathbb{R}, x \geq 0$

Range: $p^{-1}(x) \geq 2$

c) Sketch



Worked example

$$f(x) = x^2 - 5, x \in \mathbb{R}, x \geq 0.$$

- State the range of $f(x)$
- Find the function $f^{-1}(x)$ and state its domain and range
- Sketch $y = f(x)$, $y = f^{-1}(x)$ and $y = x$
- Solve the equation $f(x) = f^{-1}(x)$.

Your turn

$$p(x) = x^2 - 3, x \in \mathbb{R}, x \geq 0.$$

- State the range of $p(x)$
- Find the function $p^{-1}(x)$ and state its domain and range
- Sketch $y = p(x)$, $y = p^{-1}(x)$ and $y = x$
- Solve the equation $p(x) = p^{-1}(x)$.

a) $p(x) \geq -3$

b) $p^{-1}(x) = \sqrt{x+3}$

Domain: $x \in \mathbb{R}, x \geq -3$

Range: $p^{-1}(x) \geq 0$

c) Sketch

d) $x = \frac{1+\sqrt{13}}{2}$

