

## **2.3) Conservation of mechanical energy and the work-energy principle**

## Worked example

A smooth plane is inclined at  $60^\circ$  to the horizontal. A particle of mass  $1 \text{ kg}$  slides down a line of greatest slope of the plane. The particle starts from rest at point A and passes point B with a speed of  $12 \text{ ms}^{-1}$ . Find the distance AB.

## Your turn

A smooth plane is inclined at  $30^\circ$  to the horizontal. A particle of mass  $0.5 \text{ kg}$  slides down a line of greatest slope of the plane. The particle starts from rest at point A and passes point B with a speed of  $6 \text{ ms}^{-1}$ . Find the distance AB.

$3.67 \text{ m (3 sf)}$

## Worked example

A particle of mass 4 kg is projected with speed  $16 \text{ ms}^{-1}$  up the line of greatest slope of a rough plane inclined at  $30^\circ$  to the horizontal. The coefficient of friction between the particle and the plane is 0.8. Calculate the distance the particle travels up the plane before coming to instantaneous rest.

## Your turn

A particle of mass 2 kg is projected with speed  $8 \text{ ms}^{-1}$  up the line of greatest slope of a rough plane inclined at  $45^\circ$  to the horizontal. The coefficient of friction between the particle and the plane is 0.4. Calculate the distance the particle travels up the plane before coming to instantaneous rest.

**3.30 m (3 sf)**

## Worked example

A skier is moving downhill and passes point A on a ski run at  $12 \text{ m s}^{-1}$ . After descending 100 m vertically the run begins to ascend. When the skier has ascended 50 m to the point B her speed is  $8 \text{ m s}^{-1}$ . The skier and her skis have a combined mass of 55 kg. The total distance she travels from A to B is 2800 m. The non-gravitational resistances to motion are constant and have a total magnitude of 24 N. Calculate the work done by the skier.

## Your turn

A skier is moving downhill and passes point A on a ski run at  $6 \text{ m s}^{-1}$ . After descending 50 m vertically the run begins to ascend. When the skier has ascended 25 m to the point B her speed is  $4 \text{ m s}^{-1}$ . The skier and her skis have a combined mass of 55 kg. The total distance she travels from A to B is 1400 m. The non-gravitational resistances to motion are constant and have a total magnitude of 12 N. Calculate the work done by the skier.

**2780 J (3 sf)**

## Worked example

Two particles, A and B, of mass  $m$  and  $4m$  respectively, are attached to the ends of a light inextensible string. The particle A lies on a rough plane inclined at an angle  $\alpha$  to the horizontal, where  $\tan \alpha = \frac{5}{12}$ . The string passes over a small light smooth pulley P fixed at the top of the plane. The particle B hangs freely below P. The particles are released from rest with the string taut and the section of the string from A to P parallel to a line of greatest slope of the plane. The coefficient of friction between A and the plane is  $\frac{3}{8}$ . When each particle has moved a distance  $h$ , B has not reached the ground and A has not reached P.

- Find an expression for the potential energy lost by the system when each particle has moved a distance  $h$
- When each particle has moved a distance  $h$ , they are moving with speed  $v$ . Using the work-energy principle, find an expression for  $v^2$  in the form  $kgh$  where  $k$  is a number.

## Your turn

Two particles, A and B, of mass  $m$  and  $2m$  respectively, are attached to the ends of a light inextensible string. The particle A lies on a rough plane inclined at an angle  $\alpha$  to the horizontal, where  $\tan \alpha = \frac{3}{4}$ . The string passes over a small light smooth pulley P fixed at the top of the plane. The particle B hangs freely below P. The particles are released from rest with the string taut and the section of the string from A to P parallel to a line of greatest slope of the plane. The coefficient of friction between A and the plane is  $\frac{5}{8}$ . When each particle has moved a distance  $h$ , B has not reached the ground and A has not reached P.

- Find an expression for the potential energy lost by the system when each particle has moved a distance  $h$
- When each particle has moved a distance  $h$ , they are moving with speed  $v$ . Using the work-energy principle, find an expression for  $v^2$  in the form  $kgh$  where  $k$  is a number.

a)  $\frac{7mgh}{5}$

b)  $v^2 = \frac{3}{5}gh$