2.3) Composite functions

Worked example	Your turn
$f(x) = 3x - 2$, and $g(x) = x^2 - 4$	$f(x) = 3x + 2$, and $g(x) = x^2 + 4$
Find:	Find:
$fg(x)$	$fg(x)$ $fg(x) = 3x^2 + 14$
$gf(x)$	$gf(x)$ $gf(x) = 9x^2 + 12x + 8$
$f^2(x)$	$f^2(x)$ $f^2(x) = 9x + 8$
$g^2(x)$	$g^2(x)$ $g^2(x) = x^4 + 8x^2 + 20$

Worked example

$f(x) = 3x - 2$, and $g(x) = x^2 - 4$

Find:

$fg(1)$

$gf(-2)$

$f^2(3)$

$g^2(-4)$

Your turn

$f(x) = 3x + 2$, and $g(x) = x^2 + 4$

Find:

$fg(4)$

62

$gf(-3)$

53

$f^2(2)$

26

$g^2(-1)$

29

Worked example

$f(x) = 3x - 2$, and $g(x) = x^2 - 4$

Solve:

$$fg(a) = 13$$

$$gf(b) = 12$$

Your turn

$f(x) = 3x + 2$, and $g(x) = x^2 + 4$

Find:

$$fg(a) = 62$$

$$a = \pm 4$$

$$gf(b) = 293$$

$$b = 5, b = -\frac{19}{3}$$

Worked example	Your turn
The functions f and g are defined by $$f: x \to \lvert 3x - 12 \rvert$$ $$g: x \to \frac{x+2}{3}$$ a) Find $fg(2)$ b) Solve $fg(x) = x$	The functions f and g are defined by $$f: x \to \lvert 2x - 8 \rvert$$ $$g: x \to \frac{x+1}{2}$$ a) Find $fg(3)$ b) Solve $fg(x) = x$ a) 4 b) $x = \frac{7}{2}$

Worked example

The function g is defined by

$$g: x \to 4 - 3x, \qquad x \in \mathbb{R}$$

Solve the equation

$$g^2(x) + [g(x)]^2 = 0$$

Your turn

The function g is defined by

$$g: x \to 3 - 4x, \qquad x \in \mathbb{R}$$

Solve the equation

$$g^2(x) + [g(x)]^2 = 0$$

$$x = 0, x = \frac{1}{2}$$

Worked example

The functions f and g are defined by

$$f : x \to e^{x} + 3, \qquad x \in \mathbb{R}$$
$$g : x \to \ln x, \qquad x > 0$$

Find $fg(x)$, giving your answer in its simplest form.

The functions f and g are defined by

$$f : x \to e^{3x} - 2, \qquad x \in \mathbb{R}$$
$$g : x \to 4\ln(x+1), \qquad x > -1$$

Find $fg(x)$, giving your answer in its simplest form.

Your turn

The functions f and g are defined by

$$f : x \to e^{2x} + 4, \qquad x \in \mathbb{R}$$
$$g : x \to 3\ln(x-1), \qquad x > 1$$

Find $fg(x)$, giving your answer in its simplest form

$$fg(x) = (x-1)^6 + 4$$

Worked example	Your turn
The functions f and g are defined by $f: x \to 2^x + 3, \quad x \in \mathbb{R}$ $g: x \to \log_2 x, \quad x > 0$ Find $fg(x)$, giving your answer in its simplest form. The functions f and g are defined by $f: x \to 3^{2x} - 1, \quad x \in \mathbb{R}$ $g: x \to 4\log_3(x+5), \quad x > -5$ Find $fg(x)$, giving your answer in its simplest form.	The functions f and g are defined by $f: x \to 2^{3x} + 4, \quad x \in \mathbb{R}$ $g: x \to 5\log_2(x-1), \quad x > 1$ Find $fg(x)$, giving your answer in its simplest form $fg(x) = (x-1)^{15} + 4$

Worked example

$$f(x) = \frac{1}{x-1}, x \neq 1$$

Find an expression for $f^2(x)$ and $f^3(x)$

Your turn

$$f(x) = \frac{1}{x+1}, x \neq -1$$

Find an expression for $f^2(x)$ and $f^3(x)$

$$f^2(x) = \frac{x+1}{x+2}, x \neq -1, x \neq -2$$

$$f^3(x) = \frac{x+2}{2x+3}, x \neq -1, x \neq -2, x \neq -\frac{3}{2}$$

Worked example	Your turn
A function f has domain $-3 \leq x \leq 12$ and is linear from $(-3, 9)$ to $(0, 6)$ and from $(0, 6)$ to $(12,10)$. Find the value of $f^2(0)$	A function f has domain $-4 \leq x \leq 13$ and is linear from $(-4, 9)$ to $(0, 5)$ and from $(0, 5)$ to $(13, 31)$. Find the value of $f^2(0)$ 15