2.2) Functions and mappings

Worked example

State whether:

- the mapping is one-to-one, many-to-one, or one-to-many
- the mapping is a function

$$f(x) = 2x - 3, \qquad x \in \mathbb{R}$$

$$g(x) = x^2, \qquad x \in \mathbb{R}$$

$$h(x) = \frac{1}{x}, \qquad x \in \mathbb{R}$$

$$i(x) = \sqrt{x}, \qquad x \in \mathbb{R}$$

Your turn

State whether:

- the mapping is one-to-one, many-to-one, or one-to-many
- the mapping is a function

$$p(x) = x^3, \qquad x \in \mathbb{R}$$

One-to-one: a function

$$q(x) = \left|\frac{1}{x}\right|, \qquad x \in \mathbb{R}$$

Many-to-one: Not a function

$$r(x) = \sqrt{x}, x \in \mathbb{R}, x \geq 0, \qquad x \in \mathbb{R}$$

One-to-one: a function

$$s(x) = \pm\sqrt{x}, x \in \mathbb{R}, x \geq 0$$

One-to-many: Not a function

Worked example	Your turn
Write down the largest possible domain for:	Write down the largest possible domain for:
$f(x) = \frac{1}{x-3}$	$p(x) = \frac{6}{x+4}$
	$x \neq -4$
$g(x) = \frac{2}{7x-21}$	$q(x) = \frac{7}{5x+20}$
	$x \neq -4$
$h(x) = \frac{3}{2x^2-x-3}$	$r(x) = \frac{8}{3x^2+10x-8}$
	$x \neq \frac{2}{3}, x \neq -4$
$i(x) = \frac{4x+5}{x^2-64}$	$s(x) = \frac{9x-10}{x^2-16}$
	$x \neq -4, x \neq 4$

Worked example

Write down the largest possible domain for:

$$f(x) = \sqrt{x-3}$$

$$g(x) = \sqrt{7x-21}$$

$$h(x) = \sqrt{7x+21}$$

$$i(x) = \sqrt{21-7x}$$

Your turn

Write down the largest possible domain for:

$$p(x) = \sqrt{x+4}$$

$$x \geq -4$$

$$q(x) = \sqrt{5x+20}$$

$$x \geq -4$$

$$r(x) = \sqrt{5x-20}$$

$$x \geq 4$$

$$s(x) = \sqrt{20-5x}$$

$$x \leq 4$$

Worked example	Your turn
Write down the largest possible domain for: $$f(x)=\frac{\sqrt{x+3}}{x^2-2x}$$ $$g(x)=\frac{x^3-2x^2}{\sqrt{x^2+5x+6}}$$	Write down the largest possible domain for: $$h(x)=\frac{\sqrt{x+4}}{x^4-25x^2}$$ $$x\geq -4, x\neq 0, x\neq 5$$

Worked example	Your turn
Find the range of the following functions:	Find the range of the following functions:
$f(x) = 2x - 3, \quad x = \{1, 2, 3, 4\}$	$p(x) = 3x - 2, \quad x = \{1, 2, 3, 4\}$
	$p(x) = \{1, 4, 7, 10\}$
$g(x) = 3 - 2x, \quad x \in \mathbb{R}, x \leq 0$	$q(x) = 2 - 3x, \quad x \in \mathbb{R}, x > 0$
	$q(x) < 2$
$h(x) = 3 - 2x, \quad x \in \mathbb{R}, 2 < x < 5$	$r(x) = 2 - 3x, \quad x \in \mathbb{R}, -3 < x \leq 4$
	$-10 \leq r(x) < 11$

Worked example

Find the range of the following functions:

$$f(x) = x^4, \qquad x = \{1, 2, 3, 4\}$$

$$g(x) = x^4, \qquad x \in \mathbb{R}, x \leq 0$$

$$h(x) = x^4, \qquad x \in \mathbb{R}, -2 \leq x < 5$$

Your turn

Find the range of the following functions:

$$p(x) = x^2, \qquad x = \{1, 2, 3, 4\}$$

$$p(x) = \{1, 4, 9, 16\}$$

$$q(x) = x^2, \qquad x \in \mathbb{R}, x > 0$$

$$q(x) > 0$$

$$r(x) = x^2, \qquad x \in \mathbb{R}, -3 < x \leq 4$$

$$0 \leq r(x) \leq 16$$

Worked example	Your turn
Find the range of the following functions:	Find the range of the following functions:
$f(x) = \frac{1}{x}, \qquad x = \{-1, -2, -3, -4\}$	$p(x) = \frac{1}{x}, \qquad x = \{1, 2, 3, 4\}$
	$p(x) = \left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}\right\}$
$g(x) = \frac{1}{x-2}, \qquad x \in \mathbb{R}, x \leq 1$	$q(x) = \frac{1}{x+2}, \qquad x \in \mathbb{R}, x > -1$
	$q(x) < 1$
$h(x) = \frac{1}{x+3}, \qquad x \in \mathbb{R}, -2 \leq x < 5$	$r(x) = \frac{1}{x-5}, \qquad x \in \mathbb{R}, -3 < x \leq 4$
	$-1 \leq r(x) < -\frac{1}{8}$

Worked example

Find the range of the following functions:

$$f(x) = \frac{1}{x}, \qquad x \in \mathbb{R}, x \neq 0$$

$$g(x) = \frac{1}{x} + 2, \qquad x \in \mathbb{R}, x \neq 0$$

Your turn

Find the range of the following functions:

$$h(x) = \frac{1}{x} - 3, \qquad x \in \mathbb{R}, x \neq 0$$

$$h(x) \in \mathbb{R}$$

Worked example	Your turn
Find the range of the following functions:	Find the range of the following functions:
$f(x) = e^x + 5, \quad x \in \mathbb{R}$	$p(x) = e^x + 8, \quad x \in \mathbb{R}$
	$p(x) > 8$
$g(x) = e^x - 4, \quad x \in \mathbb{R}, x > 0$	$q(x) = e^x - 7, \quad x \in \mathbb{R}, x < 0$
	$-7 < x < -6$
$h(x) = -e^x - 3, \quad x \in \mathbb{R}, x \leq 0$	$r(x) = -e^x - 6, \quad x \in \mathbb{R}, x \geq 0$
	$r(x) \leq -7$

Worked example

Find the range of the following functions:

$$f(x) = \ln x + 5, \qquad x \in \mathbb{R}, x > 0$$

$$g(x) = \ln x - 4, \qquad x \in \mathbb{R}, x > 0$$

Your turn

Find the range of the following functions:

$$h(x) = \ln x + 3, \qquad x \in \mathbb{R}, x > 0$$

$$h(x) \in \mathbb{R}$$

Worked example	Your turn
The function f is defined by $f: x \to x^2 - 8x + 3, \qquad x \in \mathbb{R}, 0 \leq x \leq 5$ Find the range of f.	The function h is defined by $h: x \to x^2 - 4x + 1, \qquad x \in \mathbb{R}, 0 \leq x < 5$ Find the range of h. $-3 \leq h(x) < 6$
The function f is defined by $g: x \to x^2 + 6x - 2, \qquad x \in \mathbb{R}, -5 < x \leq 0$ Find the range of f.	

Worked example	Your turn
The function f is defined by $f(x) = x^2 - 8x + 27$ and has domain $x \geq a$. Given that $f(x)$ is a one-to-one function, find the smallest possible value of the constant a	The function h is defined by $h(x) = x^2 - 6x + 20$ and has domain $x \geq a$. Given that $f(x)$ is a one-to-one function, find the smallest possible value of the constant a $a = 3$
The function g is defined by $g(x) = x^2 + 4x + 15$ and has domain $x \leq a$. Given that $g(x)$ is a one-to-one function, find the smallest possible value of the constant a	