

2.1) Solving quadratic equations

Worked example

Solve:

$$(2x - 3)^2 = 4$$

Your turn

Solve:

$$(3x - 5)^2 = 9$$

$$x = \frac{2}{3}, x = \frac{8}{3}$$

Worked example

Solve:

$$x - 8\sqrt{x} + 15 = 0$$

Your turn

Solve:

$$x - 6\sqrt{x} + 8 = 0$$

$$x = 4, x = 16$$

Worked example

Solve:

$$3x + 2\sqrt{x} - 8 = 0$$

Your turn

Solve:

$$2x + \sqrt{x} - 1 = 0$$

$$x = \frac{1}{4}$$

Worked example

Solve:

$$\sqrt{x + 6} = x - 6$$

Your turn

Solve:

$$\sqrt{x + 3} = x - 3$$

$$x = 6 \text{ only}$$

Worked example

Solve:

$$(2x - 3)^2 = 4$$

Your turn

Solve:

$$(3x - 5)^2 = 9$$

$$x = \frac{2}{3}, x = \frac{8}{3}$$

Worked example

Solve:

$$2x^2 - 1 = 17$$

$$45 + 2x^2 = 5x^2 - 3$$

$$1 - x^2 = -6x^2 + 6$$

Your turn

Solve:

$$3x^2 + 49 = 7x^2 + 13$$

$$x = \pm 3$$

Worked example

Solve:

$$2x^2 = 18$$

$$(2x)^2 = 4$$

Your turn

Solve:

$$3x^2 = 48$$

$$x = \pm 4$$

$$(3x)^2 = 36$$

$$x = \pm 2$$

Worked example

Solve:

$$2x^2 = \frac{32}{25}$$

$$3x^2 = \frac{27}{49}$$

Your turn

Solve:

$$5x^2 = \frac{45}{121}$$

$$x = \pm \frac{3}{11}$$

Worked example

Solve:

$$4^x - 12(2^x) + 32 = 0$$

$$16^x - 5(4^x) + 4 = 0$$

Your turn

Solve:

$$9^x - 10(3^x) + 9 = 0$$

$$x = 2, x = 0$$

Worked example

Solve:

$$x^4 - 5x^2 + 4 = 0$$

$$x^4 - 13x^2 + 36 = 0$$

Your turn

Solve:

$$x^4 - 17x^2 + 16 = 0$$

$$x = \pm 4, x = \pm 1$$

Worked example

Solve:

$$x^6 - 35x^3 + 215 = 0$$

Your turn

Solve:

$$x^6 - 9x^3 + 8 = 0$$

$$x = 2, x = 1$$

Worked example

Solve $6x^{\frac{2}{3}} + 5x^{\frac{1}{3}} - 4 = 0$

Your turn

Solve $3y^{\frac{2}{3}} + 2y^{\frac{1}{3}} - 1 = 0$

$$y = \frac{1}{27}, y = -1$$

Worked example

Solve:

$$x + \frac{2}{x} = 3$$

$$x - \frac{3}{x} = 5$$

Your turn

Solve:

$$x + \frac{4}{x} = 5$$

$$x = 4, x = 1$$

Worked example

Solve:

$$\frac{3}{x^2} + \frac{2}{x} = 1$$

$$\frac{2}{x^2} - \frac{7}{x} = 3$$

Your turn

Solve:

$$\frac{2}{x^2} - \frac{5}{x} = 3$$

$$x = \frac{1}{3}, x = -2$$

Worked example

Solve:

$$x^3 + 2x^2 - 8x = 0$$

Your turn

Solve:

$$x^3 - 3x^2 - 10x = 0$$

$$x = 0, x = 5, x = -2$$

Worked example

Solve by factorising:

$$x^2 + 6x + 9 = 0$$

$$x^2 + 8x + 16 = 0$$

$$x^2 + 10x + 25 = 0$$

$$x^2 + 2x + 1 = 0$$

Your turn

Solve by factorising:

$$x^2 + 12x + 36 = 0$$

$$x = -6$$

Worked example

Solve by factorising:

$$x^2 - 6x + 9 = 0$$

$$x^2 - 8x + 16 = 0$$

$$x^2 - 10x + 25 = 0$$

$$x^2 - 2x + 1 = 0$$

Your turn

Solve by factorising:

$$x^2 - 12x + 36 = 0$$

$$x = 6$$

Worked example

Solve by factorising:

$$x^2 + 17x + 16 = 0$$

$$x^2 + 10x + 16 = 0$$

$$x^2 + 8x + 16 = 0$$

$$x^2 - 8x + 16 = 0$$

Your turn

Solve by factorising:

$$x^2 + 37x + 36 = 0$$

$$x = -36, x = -1$$

$$x^2 + 20x + 36 = 0$$

$$x = -18, x = -2$$

$$x^2 + 15x + 36 = 0$$

$$x = -12, x = -3$$

$$x^2 + 13x + 36 = 0$$

$$x = -9, x = -4$$

$$x^2 + 12x + 36 = 0$$

$$x = -6$$

Worked example

Solve by factorising:

$$x^2 + 10x + 9 = 0$$

$$x^2 + 10x + 16 = 0$$

$$x^2 + 10x + 25 = 0$$

$$x^2 + 10x = 0$$

Your turn

Solve by factorising:

$$x^2 + 12x + 11 = 0$$

$$x = -11, x = -1$$

$$x^2 + 12x + 27 = 0$$

$$x = -9, x = -3$$

$$x^2 + 12x + 36 = 0$$

$$x = -6$$

$$x^2 + 12x = 0$$

$$x = 0, x = -12$$

Worked example

Solve by factorising:

$$3x^2 + 10x + 3 = 0$$

$$3x^2 + 10x + 8 = 0$$

$$3x^2 + 14x + 8 = 0$$

Your turn

Solve by factorising:

$$5x^2 + 8x + 3 = 0$$

$$x = -\frac{3}{5}, x = -1$$

$$5x^2 + 16x + 12 = 0$$

$$x = -\frac{6}{5}, x = -2$$

$$5x^2 + 32x + 12 = 0$$

$$x = -\frac{2}{5}, x = -6$$

Worked example

Solve by factorising:

$$2x^2 + 8x + 6 = 0$$

$$3x^2 + 21x + 30 = 0$$

$$5x^2 + 5x - 30 = 0$$

Your turn

Solve by factorising:

$$3x^2 + 15x - 42 = 0$$

$$x = -7, x = 2$$

Worked example

Solve by factorising:

$$6 + 5x - x^2 = 0$$

$$3 - 2x - x^2 = 0$$

Your turn

Solve by factorising:

$$12 - x - x^2 = 0$$

$$x = 3, x = -4$$

Worked example

Solve:

$$6 + 5x - x^2 = 0$$

$$-6 + 5x - x^2 = 0$$

Your turn

Solve:

$$6 - 5x - x^2 = 0$$

$$x = -6, x = 1$$

Worked example

Solve with the quadratic formula:

$$2x^2 + x - 3 = 0$$

$$3x^2 + x - 10 = 0$$

Your turn

Solve with the quadratic formula:

$$5x^2 + 13x - 6 = 0$$

$$x = \frac{2}{5}, x = -3$$

Worked example

Solve with the quadratic formula:

$$2x^2 + x - 4 = 0$$

$$3x^2 + x - 11 = 0$$

Your turn

Solve with the quadratic formula:

$$5x^2 + 13x - 7 = 0$$

$$x = \frac{-13 + \sqrt{309}}{10}, x = \frac{-13 - \sqrt{309}}{10}$$

Worked example

Solve with the quadratic formula:

$$x^2 + x - 11 = 0$$

$$-2x^2 + x + 3 = 0$$

Your turn

Solve with the quadratic formula:

$$-x^2 + 13x - 7 = 0$$

$$x = \frac{13 + \sqrt{141}}{2}, x = \frac{13 - \sqrt{141}}{2}$$

Worked example

The solutions to a quadratic equation are $x = \frac{5 \pm \sqrt{25+24}}{6}$
What is the quadratic equation?

Your turn

The solutions to a quadratic equation are $x = \frac{6 \pm \sqrt{36+8}}{2}$
What is the quadratic equation?

$$x^2 - 6x - 2 = 0$$

Worked example

Your turn

How many real roots are there to:

$$x^2 + 6x + 8 = 0$$

$$x^2 + 6x + 9 = 0$$

$$x^2 + 6x + 10 = 0$$

How many real roots are there to:

$$x^2 + 8x + 12 = 0$$

Two: $x = -6, x = -2$

$$x^2 + 8x + 16 = 0$$

One: $x = -4$

$$x^2 + 8x + 17 = 0$$

No real roots

Worked example

Solve:

$$x + \frac{2}{x} = 3$$

$$x - \frac{3}{x} = 5$$

Your turn

Solve:

$$x + \frac{4}{x} = 5$$

$$x = 4, x = 1$$

Worked example

Solve:

$$\frac{3}{x^2} + \frac{2}{x} = 1$$

$$\frac{2}{x^2} - \frac{7}{x} = 3$$

Your turn

Solve:

$$\frac{2}{x^2} - \frac{5}{x} = 3$$

$$x = \frac{1}{3}, x = -2$$

Worked example

Solve by completing the square:

$$x^2 + 8x + 3 = 0$$

$$x^2 + 10x - 4 = 0$$

Your turn

Solve by completing the square:

$$x^2 + 6x + 4 = 0$$

$$x = -3 + \sqrt{5}, x = -3 - \sqrt{5}$$

Worked example

Solve by completing the square:

$$2x^2 - 8x + 3 = 0$$

$$3x^2 - 10x - 4 = 0$$

Your turn

Solve by completing the square:

$$5x^2 - 6x - 2 = 0$$

$$x = \frac{3 + \sqrt{19}}{5}, x = \frac{3 - \sqrt{19}}{5}$$

Worked example

Solve using three methods:

$$x^2 + 6x + 8 = 0$$

$$x^2 + 6x + 8 = 0$$

$$x^2 + 6x + 8 = 0$$

Your turn

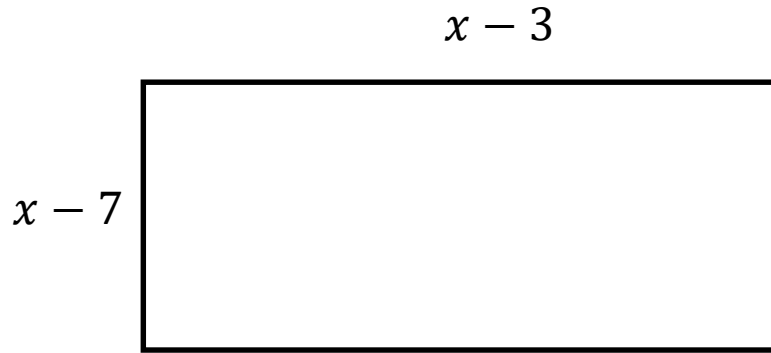
Solve using three methods:

$$x^2 + 6x + 5 = 0$$

$$x = -5, x = -1$$

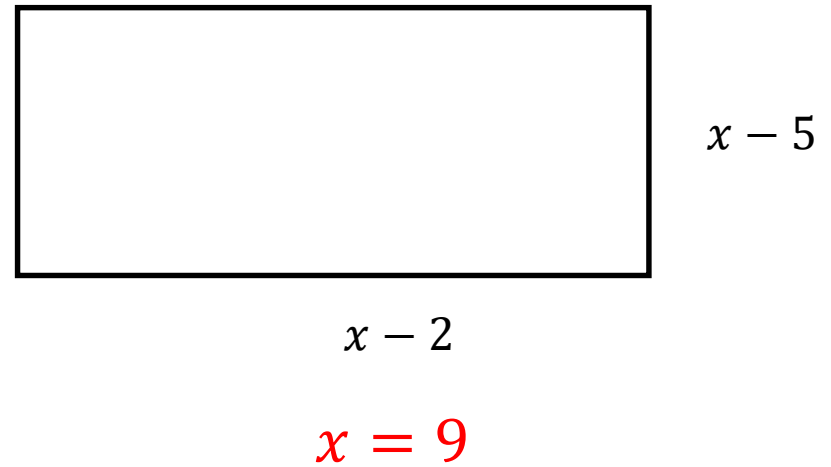
Worked example

The area of the rectangle is equal to 32 square units. Find x



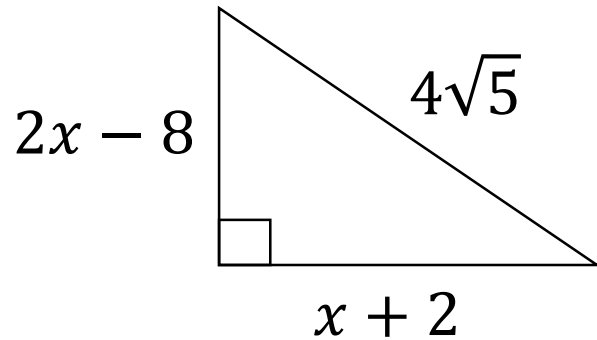
Your turn

The area of the rectangle is equal to 28 square units. Find x



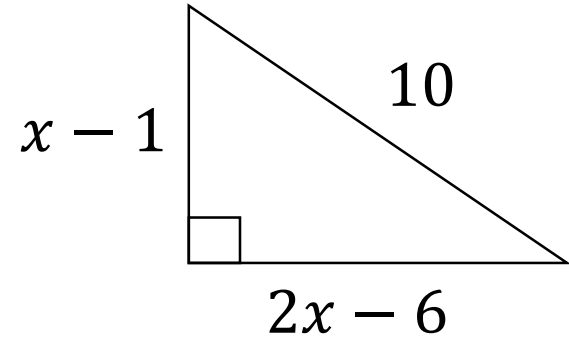
Worked example

The area of the triangle is equal to 16 square units. Find x



Your turn

The area of the triangle is equal to 24 square units. Find x



$$x = 7$$

Worked example

Two numbers have a difference of 3 and a product of 88. Find the two numbers.

Two numbers have a difference of 5 and a product of 14. Find the two numbers.

Your turn

Two numbers have a difference of 4 and a product of 12. Find the two numbers.

$$x = 6, y = 2$$
$$x = -2, y = -6$$