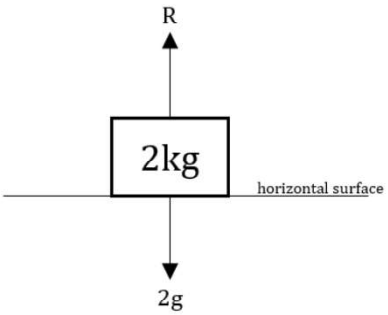
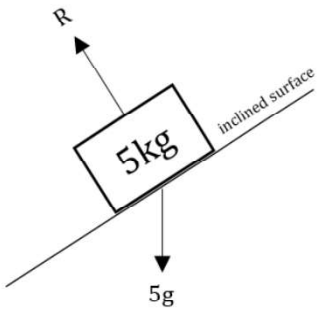
Mechanics essentials

weight = mass × g (where g is the acceleration due to gravity, $g = 9.8ms^{-2}$
$W = mg$
Weight acts vertically downwards (obviously)

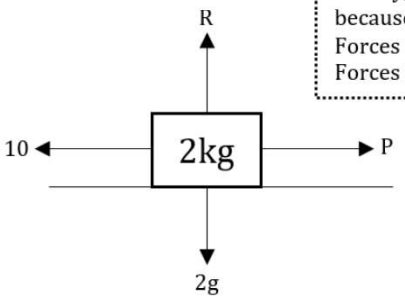
The normal reaction (sometimes called the contact force) is the force which acts on a box/particle from the surface that it is on.
It is called a **normal** reaction because it acts normal (perpendicular) to the surface.
It is called a normal **reaction** because it has reacted to the forces in the opposing direction.
For example, when you are sat on a chair, your weight acts down, but the chair (surface) has a reaction force upwards which stops you falling to the floor. This is the normal reaction.
We use the letter R for the normal reaction.





Note that the weight acts vertically downwards, but the normal reaction is perpendicular to the slope

Newton's First Law
"An object will remain at rest or will continue to move with constant velocity unless acted upon by an external force"
Essentially, this means that something will not move, or move with no acceleration if there is no overall resultant force. It means that all the forces are balanced.
We call this **equilibrium** (think of the word 'equal')



Given that this particle is rest, work out the value of P and R.
Clearly, R = 2g and P = 10, because it is in equilibrium.
Forces left = forces right.
Forces up = forces down

Newton's Second Law

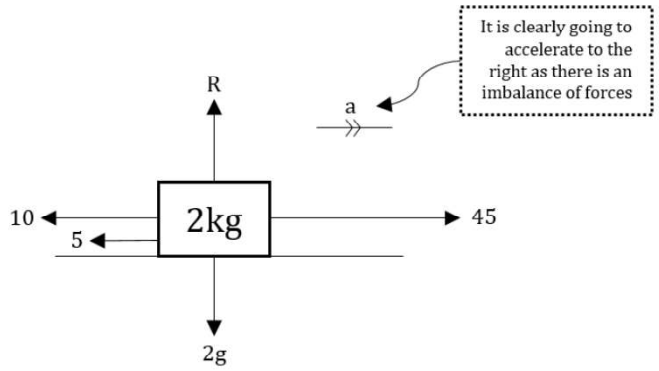
"An object will accelerate if there is an overall resultant force on the object. The acceleration is proportional to this force, and inversely proportional to its mass."

In other words

$$\mathbf{F = ma}$$

Where **F** = resultant force, **m** = mass, **a** = acceleration

The resultant force is found by finding the difference between the forces in one direction, and the forces in the opposing direction. This tells you the overall force in one direction.



Notice how we find the resultant force by doing the forces to the right minus the forces to the left

Work out the acceleration for the particle.

Using F = ma

$45 - 10 - 5 = 2a$

$30 = 2a$

$15 = a$

So the particle will accelerate at $15 ms^{-2}$ to the right

Maximum friction = coefficient of friction (μ) × normal reaction (R)

Maximum $F_r = \mu R$

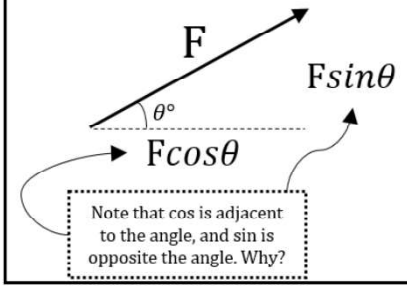
Note, the coefficient of friction is always greater than 0 and usually less than about 1.5.
The coefficient of friction is specific for the particle and the surface it is on.
A box on sandpaper would have a high coefficient of friction.
A box on ice would have a low coefficient of friction.
A box on a smooth surface would have a coefficient of friction = 0 (i.e. there is no friction)

Friction acts in the **opposite direction to its motion** (obviously)
Friction can be less than this maximum value **if the particle is not moving**.

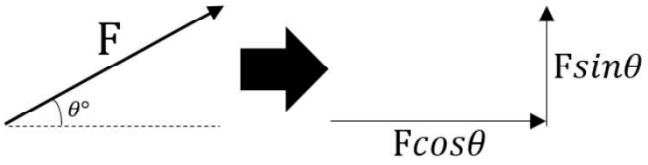
Forces at angles

To consider forces at angles, and boxes on slopes, we have to simplify the situation.
We do this by resolving forces into perpendicular components, and then dealing with the up/down forces, and the left/right forces separately.

To separate a force, given its angle:

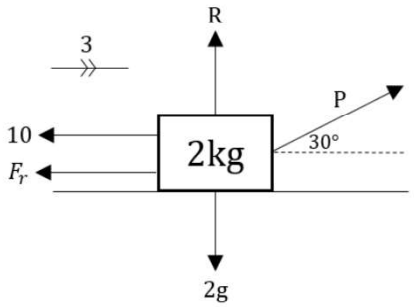


Note that cos is adjacent to the angle, and sin is opposite the angle. Why?



Hence a force of magnitude F at angle θ above the horizontal,
is equivalent to a force of magnitude **F$cos\theta$ to the right**, and **F$sin\theta$ up.**

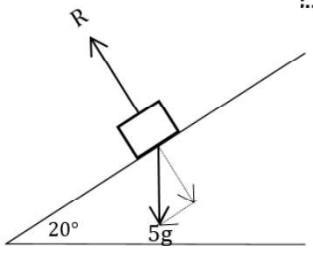
Given that the coefficient of friction is 0.5, work out the value of the pulling force, P.

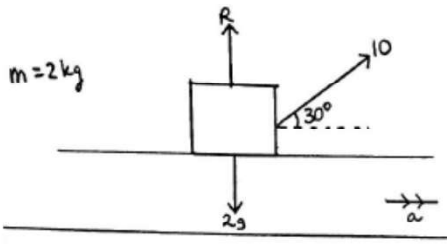
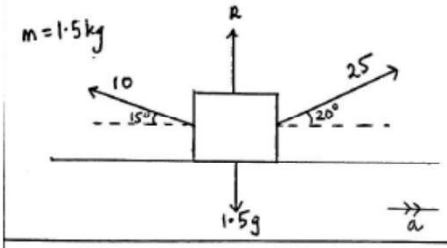
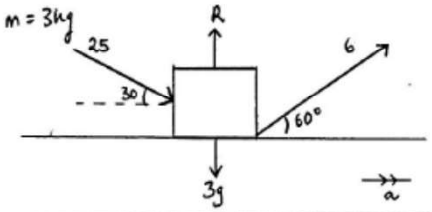
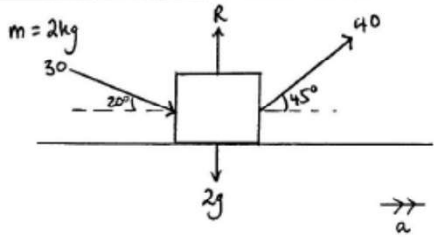


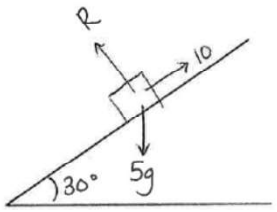
Working with slopes

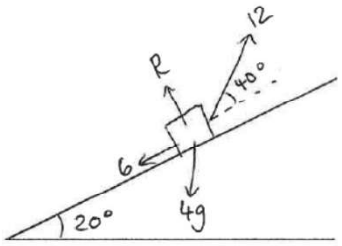
To consider forces on slopes, we resolve forces so that they are all either parallel to the slope, or perpendicular to the slope. Then we use the same strategies as before.

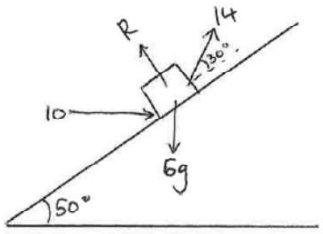
Given that the coefficient of friction is 0.1, work out the acceleration of the box down the slope.



	New Diagram	Find the value of R and a
		
		
		
		







Work

If an object has ***energy,*** it can do ***work***.
Work transfers energy from one place or form to another.
Work is done by a force when it moves an object.

Work (a bit like energy) = Force × distance

Force and distance must be in the same direction

Units:
Force is in Newtons (N) and distance in meters (m)
... **Work** is in **Joules (J)**

A horizontal force of 8N moves a box 5m across a horizontal floor. Calculate the work done by the force.

A bricklayer raises a load of bricks of total mass 30kg at a constant speed by attaching a cable to the bricks. Assuming the cable is vertical, calculate the work done when the bricks are raised a distance of 7m.

Note: Work done against gravity is always:
weight (mg) × distance moved in direction of weight (h)
= mgh

Ex 2A Q1-8

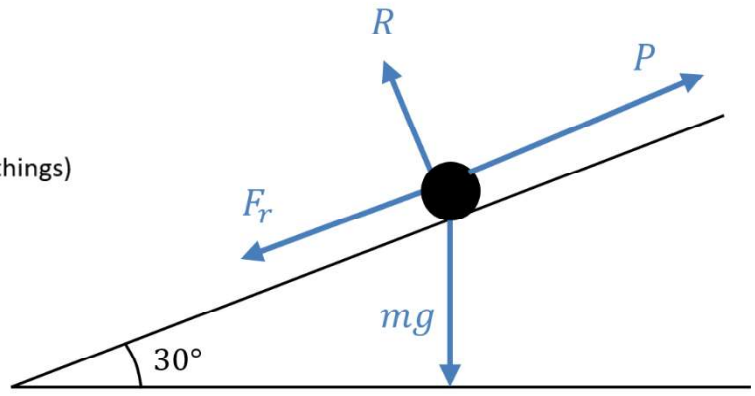
A box of mass 3kg is pushed across a rough horizontal floor.
The box moves at 3ms^{-1} and the coefficient of friction between the box and the floor is 0.45. Calculate the work done in 2 seconds.

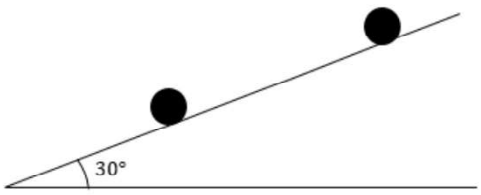
A boy of mass 40kg slides 3m down a slide, which is inclined at 25° to the horizontal. Modelling the slide as a smooth slope, calculate the work done by gravity.

Q9-13

What is the energy put into the system?

What does this energy get 'spent' on? (three things)





Note: Work done by P = Work done against friction + Work done against gravity + Kinetic energy gained.

A package of mass 2kg is pulled at a constant speed up a rough plane which is inclined at 30° to the horizontal. The coefficient of friction between the package and the surface is 0.35. The package is pulled 12m up a line of greatest slope of the plane. Calculate:

a) The work done against gravity
b) The work done against friction
c) The total work done by the pulling force.

A rough surface is inclined at 25° to the horizontal.
A box of mass 3kg is pulled at a constant speed up the surface by a force T acting along a line of greatest slope.
The coefficient of friction between the particle and the surface is 0.4.
Modelling the box as a particle, calculate the work done by T when the particle travels 8m up the slope.

A rough surface is inclined at an angle $\arcsin\left(\frac{3}{5}\right)$ to the horizontal.
A particle of mass 2kg is pulled 3m at a constant speed up the surface by a force acting along a line of greatest slope.
The only resistances to motion are those due to friction and gravity.
The work done by the force is 50J.
Calculate the coefficient of friction between the particle and the surface.

Q14-17

Energy

K.E. = $\frac{1}{2}mv^2$

Mass	Velocity	Kinetic Energy
10 kg	5 m/s	
2 tonnes	3 ms^{-1}	
4 kg	(3i – 4j) ms^{-1}	
20 kg	(-5i + 12j) m/s	

G.P.E. = mgh

Δ G.P.E. = work done against gravity = $mg(h_2 - h_1)$, if h_1=0 this gives us G.P.E. =mgh

Note: choose a 'zero level' of potential energy before calculating a particles gravitational potential energy

Ex 2B Q2, 3, 4, 5