2) Work, energy and power

2.1) Work done

Worked example	Your turn
A horizontal force of 16 N moves a box 2.5 m across a horizontal floor. Calculate the work done by the force.	A horizontal force of 8 N moves a box 5 m across a horizontal floor. Calculate the work done by the force. $40\ J$

Worked example	Your turn
An object is pulled across a horizontal floor by a horizontal rope. The object moves at a constant speed and there is a constant resistance to motion. When the case has moved a distance of $24\ m$ the work done is $96\ J$. Calculate the magnitude of the resistance.	An object is pulled across a horizontal floor by a horizontal rope. The object moves at a constant speed and there is a constant resistance to motion. When the case has moved a distance of $12\ m$ the work done is $96\ J$. Calculate the magnitude of the resistance. $8\ N$

Worked example	Your turn
An object of mass 15 kg is raised vertically at a constant speed by means of a vertical cable. Calculate the work done when the object is raised a distance of 14 m.	An object of mass 30 kg is raised vertically at a constant speed by means of a vertical cable. Calculate the work done when the object is raised a distance of 7 m. 2100 J (2 sf)

Worked example	Your turn
A package of mass 4 kg is pulled at a constant speed up a rough plane which is inclined at 60° to the horizontal. The coefficient of friction between the package and the surface is 0.7. The package is pulled 24 m up a line of greatest slope of the plane. Calculate: a) The work done against gravity b) The work done against friction c) The total work done by the pulling force.	A package of mass 2 kg is pulled at a constant speed up a rough plane which is inclined at 30° to the horizontal. The coefficient of friction between the package and the surface is 0.35. The package is pulled 12 m up a line of greatest slope of the plane. Calculate: a) The work done against gravity b) The work done against friction c) The total work done by the pulling force. a) 118 J (3 sf) b) 71.3 J (3 sf) c) 189 J (3 sf)

Worked example

An object is pulled 30 m across a smooth horizontal plane by a force of magnitude $54\ N$. The force is inclined at 50° to the horizontal.

By modelling the object as a particle calculate the work done by the force.

Your turn

An object is pulled 15 m across a smooth horizontal plane by a force of magnitude $27\ N$. The force is inclined at 25° to the horizontal.

By modelling the object as a particle calculate the work done by the force.

$367\ J$ (3 sf)

2.2) Kinetic and potential energy

Worked example	Your turn
A particle of mass 0.9 tonnes is moving at a speed of $3\ ms^{-1}$. Calculate its kinetic energy.	A particle of mass $0.3\ kg$ is moving at a speed of $9\ ms^{-1}$. Calculate its kinetic energy. $12.2\ J$ (3 sf)

Worked example	Your turn
A particle of mass $400\ g$ is moving at a velocity of $(3\boldsymbol{i} - 4\boldsymbol{j})\ ms^{-1}$. Calculate its kinetic energy.	A particle of mass 0.02 tonnes is moving at a velocity of $(-5\boldsymbol{i} + 12\boldsymbol{j})\ ms^{-1}$. Calculate its kinetic energy. $1690\ J$

Worked example	Your turn
A box of mass $3\ kg$ is pulled across a smooth horizontal surface by a horizontal force. The initial speed of the box is $u\ ms^{-1}$ and its final speed is $6\ ms^{-1}$. The work done by the force is $3.6\ J$. Calculate the value of u.	A box of mass $1.5\ kg$ is pulled across a smooth horizontal surface by a horizontal force. The initial speed of the box is $u\ ms^{-1}$ and its final speed is $3\ ms^{-1}$. The work done by the force is $1.8\ J$. Calculate the value of u. $u = 2.57$ (3 sf)

Worked example	Your turn
A car of mass 1000 kg starts from rest at some traffic lights. After travelling 200 m the van's speed is 6 ms^{-1}. A constant resistance of 250 N acts on the van. Calculate the driving force, which can be assumed to be constant.	A van of mass 2000 kg starts from rest at some traffic lights. After travelling 400 m the van's speed is 12 ms^{-1}. A constant resistance of 500 N acts on the van. Calculate the driving force, which can be assumed to be constant. 860 N

Worked example	Your turn
An object of mass $10\ kg$ is lowered vertically to the ground through a distance of $45\ m$. Find the loss in potential energy.	An object of mass $30\ kg$ is lowered vertically to the ground through a distance of $15\ m$. Find the loss in potential energy. $4410\ J$

Worked example	Your turn
A parcel of mass 6 kg is pulled 20 m up a plane inclined at an angle $\theta°$ to the horizontal, where tanθ = $\frac{5}{12}$. Assuming that the parcel moves up the line of greatest slope of the plane, (a) Calculate the potential energy gained by the parcel. (b) Find the speed of the parcel if the gain in gravitational potential energy was all transferred into kinetic energy.	A parcel of mass 3 kg is pulled 10 m up a plane inclined at an angle $\theta°$ to the horizontal, where tanθ = $\frac{3}{4}$. Assuming that the parcel moves up the line of greatest slope of the plane, (a) Calculate the potential energy gained by the parcel. (b) Find the speed of the parcel if the gain in gravitational potential energy was all transferred into kinetic energy. a) $176\ J$ (3 sf) b) $10.8\ ms^{-1}$ (3 sf)

Worked example	Your turn
An object P is modelled as a particle of mass $0.3\ kg$. P slides down a rough plane from a point S to a point T where ST = $6\ m$. The plane is inclined at an angle of $30°$ to the horizontal and ST is a line of greatest slope of the plane. The speed of P at S and T is $5\ ms^{-1}$ and $4.5\ ms^{-1}$ respectively. a) Calculate the total loss of energy of P in moving from S to T. b) Given that the work done against friction by P is equal to the total loss of energy of P in moving from S to T, calculate the coefficient of friction between P and the plane.	An object P is modelled as a particle of mass $0.6\ kg$. P slides down a rough plane from a point S to a point T where ST = $12\ m$. The plane is inclined at an angle of $30°$ to the horizontal and ST is a line of greatest slope of the plane. The speed of P at S and T is $10\ ms^{-1}$ and $9\ ms^{-1}$ respectively. a) Calculate the total loss of energy of P in moving from S to T. b) Given that the work done against friction by P is equal to the total loss of energy of P in moving from S to T, calculate the coefficient of friction between P and the plane. a) $41.0\ J$ (3 sf) b) 0.671 (3 sf)

2.3) Conservation of mechanical energy and the work-energy principle

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Worked example	Your turn
A smooth plane is inclined at $60°$ to the horizontal. A particle of mass $1\ kg$ slides down a line of greatest slope of the plane. The particle starts from rest at point A and passes point B with a speed of $12\ ms^{-1}$. Find the distance AB.	A smooth plane is inclined at $30°$ to the horizontal. A particle of mass $0.5\ kg$ slides down a line of greatest slope of the plane. The particle starts from rest at point A and passes point B with a speed of $6\ ms^{-1}$. Find the distance AB. $3.67\ m$ (3 sf)

Worked example	Your turn
A particle of mass 4 kg is projected with speed 16 ms^{-1} up the line of greatest slope of a rough plane inclined at 30° to the horizontal. The coefficient of friction between the particle and the plane is 0.8. Calculate the distance the particle travels up the plane before coming to instantaneous rest.	A particle of mass 2 kg is projected with speed 8 ms^{-1} up the line of greatest slope of a rough plane inclined at 45° to the horizontal. The coefficient of friction between the particle and the plane is 0.4. Calculate the distance the particle travels up the plane before coming to instantaneous rest. 3.30 m (3 sf)

Worked example	Your turn
A skier is moving downhill and passes point A on a ski run at 12 ms^{-1}. After descending 100 m vertically the run begins to ascend. When the skier has ascended 50 m to the point B her speed is 8 ms^{-1}. The skier and her skis have a combined mass of 55 kg. The total distance she travels from A to B is 2800 m. The non-gravitational resistances to motion are constant and have a total magnitude of 24 N. Calculate the work done by the skier.	A skier is moving downhill and passes point A on a ski run at 6 ms^{-1}. After descending 50 m vertically the run begins to ascend. When the skier has ascended 25 m to the point B her speed is 4 ms^{-1}. The skier and her skis have a combined mass of 55 kg. The total distance she travels from A to B is 1400 m. The non-gravitational resistances to motion are constant and have a total magnitude of 12 N. Calculate the work done by the skier. $2780\ J$ (3 sf)

Worked example	Your turn
Two particles, A and B, of mass m and $4m$ respectively, are attached to the ends of a light inextensible string. The particle A lies on a rough plane inclined at an angle α to the horizontal, where $\tan\alpha = \frac{5}{12}$. The string passes over a small light smooth pulley P fixed at the top of the plane. The particle B hangs freely below P. The particles are released from rest with the string taut and the section of the strig from A to P parallel to a line of greatest slope of the plane. The coefficient of friction between A and the plane is $\frac{3}{8}$. When each particle has moved a distance h, B has not reached the ground and A has not reached P. a) Find an expression for the potential energy lost by the system when each particle has moved a distance h b) When each particle has moved a distance h, they are moving with speed v. Using the work-energy principle, find an expression for v^2 in the form kgh where k is a number.	Two particles, A and B, of mass m and $2m$ respectively, are attached to the ends of a light inextensible string. The particle A lies on a rough plane inclined at an angle α to the horizontal, where $\tan\alpha = \frac{3}{4}$. The string passes over a small light smooth pulley P fixed at the top of the plane. The particle B hangs freely below P. The particles are released from rest with the string taut and the section of the strig from A to P parallel to a line of greatest slope of the plane. The coefficient of friction between A and the plane is $\frac{5}{8}$. When each particle has moved a distance h, B has not reached the ground and A has not reached P. a) Find an expression for the potential energy lost by the system when each particle has moved a distance h b) When each particle has moved a distance h, they are moving with speed v. Using the work-energy principle, find an expression for v^2 in the form kgh where k is a number. a) $\frac{7mgh}{5}$ b) $v^2 = \frac{3}{5}gh$

2.4) Power

Worked example	Your turn
A van of mass 2500 kg is travelling along a horizontal road. The van's engine is working at 48 kW. The constant resistance to motion has a magnitude of 1200 N. Calculate : a) the acceleration of the van when it is travelling at 12 ms^{-1} b) the maximum speed of the van.	A van of mass 1250 kg is travelling along a horizontal road. The van's engine is working at 24 kW. The constant resistance to motion has a magnitude of 600 N. Calculate : a) the acceleration of the van when it is travelling at 6 ms^{-1} b) the maximum speed of the van. a) 2.72 ms^{-2} (3 sf) b) 40 ms^{-1}

Worked example	Your turn
A van of mass 2200 kg is travelling at a constant speed of 30 ms^{-1} along a straight road inclined at 14° to the horizontal. The engine is working a rate of 48 kW. a) Calculate the magnitude of the non-gravitational resistance to motion. The rate of working of the engine is now increased to 56 kW. Assuming the resistances to motion are unchanged, b) Calculate the initial acceleration of the van.	A car of mass 1100 kg is travelling at a constant speed of 15 ms^{-1} along a straight road inclined at 7° to the horizontal. The engine is working a rate of 24 kW. a) Calculate the magnitude of the non-gravitational resistance to motion. The rate of working of the engine is now increased to 28 kW. Assuming the resistances to motion are unchanged, b) Calculate the initial acceleration of the car. a) 286 N (3 sf) b) 0.242 ms^{-2} (3 sf)

Worked example	Your turn
A car of mass $1300\ kg$ is travelling in a straight line. At the instant when the speed of the van is $v\ ms^{-1}$, the total resistances to motion are modelled as a variable force of magnitude $(400 + 2.5v^2)\ N$. The car has a cruise control feature which adjusts the power generated by the engine to maintain a constant speed of $9\ ms^{-1}$. Find the power generated by the engine when the car is travelling on a horizontal road.	A van of mass $2600\ kg$ is travelling in a straight line. At the instant when the speed of the van is $v\ ms^{-1}$, the total resistances to motion are modelled as a variable force of magnitude $(800 + 5v^2)\ N$. The van has a cruise control feature which adjusts the power generated by the engine to maintain a constant speed of $18\ ms^{-1}$. Find the power generated by the engine when the van is travelling on a horizontal road. $43600\ W$ (3 sf)

Worked example	Your turn
A car of mass $1300\ kg$ is travelling in a straight line. At the instant when the speed of the van is $v\ ms^{-1}$, the total resistances to motion are modelled as a variable force of magnitude $(400 + 2.5v^2)\ N$. The car has a cruise control feature which adjusts the power generated by the engine to maintain a constant speed of $9\ ms^{-1}$. Find the power generated by the engine when the car is travelling up a road that is inclined at $2°$ to the horizontal.	A van of mass $2600\ kg$ is travelling in a straight line. At the instant when the speed of the van is $v\ ms^{-1}$, the total resistances to motion are modelled as a variable force of magnitude $(800 + 5v^2)\ N$. The van has a cruise control feature which adjusts the power generated by the engine to maintain a constant speed of $18\ ms^{-1}$. Find the power generated by the engine when the van is travelling up a road that is inclined at $4°$ to the horizontal. $75600\ W$ (3 sf)

Worked example	Your turn
A child and his bicycle have a combined mass of $32\ kg$. He cycles up a straight stretch of road inclined at an angle α to the horizontal, where $\sin\alpha = \frac{1}{7}$. He cycles at a constant speed of $2.5\ ms^{-1}$. When he is cycling at this speed, the resistance to motion from non-gravitational forces has magnitude $10\ N$. Find the rate at which the cyclist is working.	A girl and her bicycle have a combined mass of $64\ kg$. She cycles up a straight stretch of road inclined at an angle α to the horizontal, where $\sin\alpha = \frac{1}{14}$. She cycles at a constant speed of $5\ ms^{-1}$. When she is cycling at this speed, the resistance to motion from non-gravitational forces has magnitude $20\ N$. Find the rate at which the cyclist is working. $324\ W$