## 2) Measures of location and spread

2.1) Measures of central tendency
2.2) Other measures of location
2.3) Measures of spread
2.4) Variance and standard deviation
2.5) Coding
2.1) Measures of central tendency

## Your turn

## Calculate the mean:

$2,3,7,9,1,8,6,3,0,1$

Calculate the mean:

$$
3,2,5,9,5,1,7,0
$$

Find the median:
$2,3,7,9,1,8,6,3,0,1$

Find the median:

$$
3,2,5,9,5,1,7,0
$$

Find the median:
$2,3,7,9,1,8,6,3,0,1,9$

Find the median:
$3,2,5,9,5,1,7,0,2$

## Your turn

Find the mode:
$2,3,7,9,1,8,6,3,0,1$

Find the mode:

$$
3,2,5,9,5,1,7,0
$$

Worked example

## Your turn

Calculate the mean height:

| Height, $\boldsymbol{x}$ (cm) |
| :---: |
| 0.9 |
| 1.8 |
| 2.7 |
| 3.6 |
| 4.5 |

Worked example
Calculate the mean score:

| Score, $\boldsymbol{x}$ | Frequency |
| :---: | :---: |
| 0 | 3 |
| 1 | 2 |
| 2 | 1 |
| 3 | 1 |
| 4 | 4 |

## Your turn

Calculate the mean score:

| Score, $\boldsymbol{x}$ | Frequency |
| :---: | :---: |
| 0 | 6 |
| 1 | 4 |
| 2 | 2 |
| 3 | 2 |
| 4 | 8 |
| $\bar{x}=2.09(3 \mathrm{sf})$ |  |

## Your turn

Find the median score:

| Score | Frequency |
| :---: | :---: |
| 0 | 3 |
| 1 | 2 |
| 2 | 1 |
| 3 | 1 |
| 4 | 4 |

Find the median score:

| Score | Frequency |
| :---: | :---: |
| 0 | 6 |
| 1 | 4 |
| 2 | 2 |
| 3 | 2 |
| 4 | 11 |
| 3 |  |

## Your turn

Find the median score:

| Score | Frequency |
| :---: | :---: |
| 0 | 3 |
| 1 | 2 |
| 2 | 1 |
| 3 | 1 |
| 4 | 5 |

Find the median score:

| Score | Frequency |
| :---: | :---: |
| 0 | 5 |
| 1 | 4 |
| 2 | 2 |
| 3 | 2 |
| 4 | 11 |
| 3 |  |

Find the mode of the scores:

| Score | Frequency |
| :---: | :---: |
| 0 | 3 |
| 1 | 2 |
| 2 | 1 |
| 3 | 1 |
| 4 | 4 |

Find the mode of the scores:

| Score | Frequency |
| :---: | :---: |
| 0 | 6 |
| 1 | 4 |
| 2 | 2 |
| 3 | 2 |
| 4 | 3 |

0

Worked example

## Your turn

Calculate an estimate for the mean score:

| Score, $\boldsymbol{x}$ | Frequency |
| :---: | :---: |
| $0 \leq x<1$ | 3 |
| $1 \leq x<2$ | 2 |
| $2 \leq x<4$ | 1 |
| $4 \leq x<9.5$ | 1 |
| $9.5 \leq x<10$ | 4 |

Calculate an estimate for the mean score:

| Score, $\boldsymbol{x}$ | Frequency |
| :---: | :---: |
| $0<x \leq 1$ | 6 |
| $1<x \leq 3$ | 4 |
| $3<x \leq 6$ | 2 |
| $6<x \leq 6.5$ | 2 |
| $6.5<x \leq 10$ | 8 |
| $=4.48(3 \mathrm{sf})$ |  |

Worked example

## Your turn

Calculate an estimate for the mean score:

| Score, $\boldsymbol{x}$ | Frequency |
| :---: | :---: |
| $0 \leq x<2$ | 3 |
| $2 \leq x<4$ | 2 |
| $4 \leq x<6$ | 1 |
| $6 \leq x<8$ | 1 |
| $8 \leq x<10$ | 4 |

Calculate an estimate for the mean score:

| Score, $\boldsymbol{x}$ | Frequency |
| :---: | :---: |
| $0 \leq x<2$ | 3 |
| $2 \leq x<4$ | 0 |
| $4 \leq x<6$ | 1 |
| $6 \leq x<8$ | 1 |
| $8 \leq x<10$ | 4 |
| $=5.67(3 \mathrm{sf})$ |  |

Worked example

## Your turn

Write down the interval containing the median

| Score, $\boldsymbol{x}$ | Frequency |
| :---: | :---: |
| $0 \leq x<1$ | 8 |
| $1 \leq x<2$ | 2 |
| $2 \leq x<4$ | 1 |
| $4 \leq x<9.5$ | 1 |
| $9.5 \leq x<10$ | 4 |

Write down the interval containing the median

| Score, $\boldsymbol{x}$ | Frequency |
| :---: | :---: |
| $0<x \leq 1$ | 6 |
| $1<x \leq 3$ | 4 |
| $3<x \leq 6$ | 2 |
| $6<x \leq 6.5$ | 2 |
| $6.5<x \leq 10$ | 8 |
| $3<x \leq 6$ |  |

Worked example
Your turn
Write down the interval containing the median

| Score, $\boldsymbol{x}$ | Frequency |
| :---: | :---: |
| $0 \leq x<1$ | 3 |
| $1 \leq x<2$ | 2 |
| $2 \leq x<4$ | 1 |
| $4 \leq x<9.5$ | 1 |
| $9.5 \leq x<10$ | 4 | median


| Score, $\boldsymbol{x}$ | Frequency |
| :---: | :---: |
| $0<x \leq 1$ | 11 |
| $1<x \leq 3$ | 4 |
| $3<x \leq 6$ | 2 |
| $6<x \leq 6.5$ | 2 |
| $6.5<x \leq 10$ | 8 |
| $1<x \leq 3$ |  |

Worked example
Write down the modal class

| Score, $\boldsymbol{x}$ | Frequency |
| :---: | :---: |
| $0 \leq x<1$ | 3 |
| $1 \leq x<2$ | 2 |
| $2 \leq x<4$ | 1 |
| $4 \leq x<9.5$ | 1 |
| $9.5 \leq x<10$ | 4 |

## Your turn

Write down the modal class

| Score, $\boldsymbol{x}$ | Frequency |
| :---: | :---: |
| $0<x \leq 1$ | 11 |
| $1<x \leq 3$ | 4 |
| $3<x \leq 6$ | 2 |
| $6<x \leq 6.5$ | 2 |
| $6.5<x \leq 10$ | 8 |

$$
0<x \leq 1
$$

Worked example
Times, $x$, have been rounded to the nearest minute. Find an estimate for the mean time:

| Time, $\boldsymbol{x}$ | Frequency |
| :---: | :---: |
| $0-2$ | 5 |
| $3-5$ | 2 |
| $6-10$ | 3 |

Times, $x$, have been rounded to the nearest minute. Find an estimate for the mean time:

| Time, $\boldsymbol{x}$ | Frequency |
| :---: | :---: |
| $0-3$ | 7 |
| $4-8$ | 11 |
| $9-10$ | 2 |

$$
\bar{x}=4.8125
$$

## Your turn

Write down the class containing the median

| Time, $\boldsymbol{x}$ | Frequency |
| :---: | :---: |
| $0-2$ | 5 |
| $3-5$ | 2 |
| $6-10$ | 3 |

Write down the class containing the median

| Time, $\boldsymbol{x}$ | Frequency |
| :---: | :---: |
| $0-3$ | 7 |
| $4-8$ | 11 |
| $9-10$ | 2 |

$$
4-8
$$

## Your turn

Write down the modal class

| Time, $\boldsymbol{x}$ | Frequency |
| :---: | :---: |
| $0-2$ | 5 |
| $3-5$ | 2 |
| $6-10$ | 3 |

Write down the modal class

| Time, $\boldsymbol{x}$ | Frequency |
| :---: | :---: |
| $0-3$ | 7 |
| $4-8$ | 11 |
| $9-10$ | 2 |

$$
4-8
$$

## Your turn

The table shows the length of time for different people to complete a task:

| Person | A | B | C | D | E | F |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Time (minutes) | 1 | 3 | 8 | 2 | 1 | 9 |

Calculate the mean.

The table shows the length of time for different people to complete a task:

| Person | A | B | C | D | E | F |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Time (minutes) | 7 | 3 | 8 | 2 | 1 | 3 |

Calculate the mean.
4 minutes

## Your turn

The table shows the length of time for different people to complete a task:

| Person | A | B | C | D | E | F |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Time (minutes) | 1 | 3 | 8 | 2 | 1 | 9 |

Find the median:

The table shows the length of time for different people to complete a task:

| Person | A | B | C | D | E | F |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Time (minutes) | 7 | 3 | 8 | 2 | 1 | 3 |

Find the median:
3 minutes

The table shows the length of time for different people to complete a task:

| Person | A | B | C | D | E | F |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Time (minutes) | 1 | 3 | 8 | 2 | 1 | 9 |

Write down the mode.

The table shows the length of time for different people to complete a task:

| Person | A | B | C | D | E | F |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Time (minutes) | 7 | 3 | 8 | 2 | 1 | 3 |

Write down the mode.
3 minutes

## Worked example

## Your turn

The mean maths score of 10 pupils in class $A$ is 26 . The mean maths score of 40 pupils in class $B$ is 57 .
a) Find the overall mean of all the pupils' marks.
b) The teacher realises they mismarked one student's paper; he should have received 90 instead of 95. Explain the effect on the mean and median.

The mean maths score of 20 pupils in class $A$ is 62. The mean maths score of 30 pupils in class $B$ is 75 .
a) Find the overall mean of all the pupils' marks.
b) The teacher realises they mismarked one student's paper; he should have received 100 instead of 95. Explain the effect on the mean and median.
a) 69.8
b) Mean increased, median unaffected.

## 2.2) Other measures of location

Worked example

## Estimate the median:

| Score, $\boldsymbol{x}$ | Frequency |
| :---: | :---: |
| $0 \leq x<1$ | 3 |
| $1 \leq x<2$ | 2 |
| $2 \leq x<4$ | 1 |
| $4 \leq x<9.5$ | 1 |
| $9.5 \leq x<10$ | 4 |

## Your turn

## Estimate the median:

| Score, $\boldsymbol{x}$ | Frequency |
| :---: | :---: |
| $0<x \leq 1$ | 11 |
| $1<x \leq 3$ | 4 |
| $3<x \leq 6$ | 2 |
| $6<x \leq 6.5$ | 2 |
| $6.5<x \leq 10$ | 8 |
| 2.25 |  |

## Your turn

Times, $x$, have been rounded to the nearest minute. Estimate the median:

| Time, $\boldsymbol{x}$ | Frequency |
| :---: | :---: |
| $0-2$ | 5 |
| $3-5$ | 2 |
| $6-10$ | 3 |

Times, $x$, have been rounded to the nearest minute. Estimate the median:

| Time, $\boldsymbol{x}$ | Frequency |
| :---: | :---: |
| $0-3$ | 7 |
| $4-8$ | 11 |
| $9-10$ | 2 |

$$
4.86 \text { (3 sf) }
$$

Worked example
Estimate the median:

| Height (m) | Frequency |
| :---: | :---: |
| $0.55 \leq h<0.6$ | 33 |
| $0.6 \leq h<0.65$ | 54 |
| $0.65 \leq h<0.7$ | 15 |
| $0.7 \leq h<0.75$ | 30 |
| $0.75 \leq h<0.8$ | 18 |

## Your turn

Estimate the median:

| Height (m) | Frequency |
| :---: | :---: |
| $0.55 \leq h<0.6$ | 55 |
| $0.6 \leq h<0.65$ | 45 |
| $0.65 \leq h<0.7$ | 30 |
| $0.7 \leq h<0.75$ | 15 |
| $0.75 \leq h<0.8$ | 5 |

$0.62 m$ (2 dp)

Find the lower quartile:
$2,3,7,9,1,8,6,3,0,1,9$

Find the lower quartile:
$3,2,5,9,5,1,7,0,2$
2

## Your turn

Find the upper quartile:
$2,3,7,9,1,8,6,3,0,1,9$
Find the upper quartile:
$3,2,5,9,5,1,7,0,2$
5

## Your turn

Find the lower quartile:
$2,3,7,9,1,8,6,3,0,1,9,12$

Find the lower quartile:
$3,2,5,9,5,1,7,0$
1.5

Find the upper quartile:
$2,3,7,9,1,8,6,3,0,1,9,12$

Find the upper quartile:
$3,2,5,9,5,1,7,0$
6

Worked example
Estimate the lower quartile:

| Score, $\boldsymbol{x}$ | Frequency |
| :---: | :---: |
| $0 \leq x<1$ | 3 |
| $1 \leq x<2$ | 2 |
| $2 \leq x<4$ | 1 |
| $4 \leq x<9.5$ | 1 |
| $9.5 \leq x<10$ | 4 |

## Your turn

Estimate the lower quartile:

| Score, $\boldsymbol{x}$ | Frequency |
| :---: | :---: |
| $0<x \leq 1$ | 11 |
| $1<x \leq 3$ | 4 |
| $3<x \leq 6$ | 2 |
| $6<x \leq 6.5$ | 2 |
| $6.5<x \leq 10$ | 8 |
| $0.61(2 \mathrm{sf})$ |  |

Worked example
Estimate the upper quartile:

| Score, $\boldsymbol{x}$ | Frequency |
| :---: | :---: |
| $0 \leq x<1$ | 3 |
| $1 \leq x<2$ | 2 |
| $2 \leq x<4$ | 1 |
| $4 \leq x<9.5$ | 1 |
| $9.5 \leq x<10$ | 4 |

## Your turn

Estimate the upper quartile:

| Score, $\boldsymbol{x}$ | Frequency |
| :---: | :---: |
| $0<x \leq 1$ | 11 |
| $1<x \leq 3$ | 4 |
| $3<x \leq 6$ | 2 |
| $6<x \leq 6.5$ | 2 |
| $6.5<x \leq 10$ | 8 |

7.05 ( 2 sf )

## Your turn

Times, $x$, have been rounded to the nearest minute. Estimate the lower quartile:

| Time, $\boldsymbol{x}$ | Frequency |
| :---: | :---: |
| $0-2$ | 5 |
| $3-5$ | 2 |
| $6-10$ | 3 |

Times, $x$, have been rounded to the nearest minute. Estimate the lower quartile:

| Time, $\boldsymbol{x}$ | Frequency |
| :---: | :---: |
| $0-3$ | 7 |
| $4-8$ | 11 |
| $9-10$ | 2 |

2.5

## Your turn

Times, $x$, have been rounded to the nearest minute. Estimate the upper quartile:

| Time, $\boldsymbol{x}$ | Frequency |
| :---: | :---: |
| $0-2$ | 5 |
| $3-5$ | 2 |
| $6-10$ | 3 |

Times, $x$, have been rounded to the nearest minute. Estimate the upper quartile:

| Time, $\boldsymbol{x}$ | Frequency |
| :---: | :---: |
| $0-3$ | 7 |
| $4-8$ | 11 |
| $9-10$ | 2 |

5.72 (3 sf)

Worked example
Estimate the $27^{\text {th }}$ percentile:

| Score, $\boldsymbol{x}$ | Frequency |
| :---: | :---: |
| $0 \leq x<1$ | 3 |
| $1 \leq x<2$ | 2 |
| $2 \leq x<4$ | 1 |
| $4 \leq x<9.5$ | 1 |
| $9.5 \leq x<10$ | 4 |

Your turn
Estimate the $72^{\text {nd }}$ percentile:

| Score, $\boldsymbol{x}$ | Frequency |
| :---: | :---: |
| $0<x \leq 1$ | 11 |
| $1<x \leq 3$ | 4 |
| $3<x \leq 6$ | 2 |
| $6<x \leq 6.5$ | 2 |
| $6.5<x \leq 10$ | 8 |

6.6925

Worked example

## Your turn

Times, $x$, have been rounded to the nearest minute. Estimate the $63^{\text {rd }}$ percentile:

| Time, $\boldsymbol{x}$ | Frequency |
| :---: | :---: |
| $0-2$ | 5 |
| $3-5$ | 2 |
| $6-10$ | 3 |


| Time, $\boldsymbol{x}$ | Frequency |
| :---: | :---: |
| $0-3$ | 7 |
| $4-8$ | 11 |
| $9-10$ | 2 |

3.59

## Your turn

Calculate the interquartile range:
$2,3,7,9,1,8,6,3,0,1,9$
Calculate the interquartile range:

$$
3,2,5,9,5,1,7,0,2
$$

## Your turn

Calculate the interquartile range:
$2,3,7,9,1,8,6,3,0,1,9,12$

Calculate the interquartile range:

$$
3,2,5,9,5,1,7,0
$$

4.5

Worked example
Estimate the interquartile range:

| Score, $\boldsymbol{x}$ | Frequency |
| :---: | :---: |
| $0 \leq x<1$ | 3 |
| $1 \leq x<2$ | 2 |
| $2 \leq x<4$ | 1 |
| $4 \leq x<9.5$ | 1 |
| $9.5 \leq x<10$ | 4 |

## Your turn

Estimate the interquartile range:

| Score, $\boldsymbol{x}$ | Frequency |
| :---: | :---: |
| $0<x \leq 1$ | 11 |
| $1<x \leq 3$ | 4 |
| $3<x \leq 6$ | 2 |
| $6<x \leq 6.5$ | 2 |
| $6.5<x \leq 10$ | 8 |

6.43 ( 3 sf )

Worked example

## Your turn

Times, $x$, have been rounded to the nearest minute. Estimate the interquartile range:

| Time, $\boldsymbol{x}$ | Frequency |
| :---: | :---: |
| $0-2$ | 5 |
| $3-5$ | 2 |
| $6-10$ | 3 |

Times, $x$, have been rounded to the nearest minute. Estimate the interquartile range:

| Time, $\boldsymbol{x}$ | Frequency |
| :---: | :---: |
| $0-3$ | 7 |
| $4-8$ | 11 |
| $9-10$ | 2 |

3.22 (3 sf)

## Your turn

Estimate the $20^{\text {th }}-80^{\text {th }}$ interpercentile range:

| Score, $\boldsymbol{x}$ | Frequency |
| :---: | :---: |
| $0 \leq x<1$ | 3 |
| $1 \leq x<2$ | 2 |
| $2 \leq x<4$ | 1 |
| $4 \leq x<9.5$ | 1 |
| $9.5 \leq x<10$ | 4 |

Estimate the $10^{\text {th }}-90^{\text {th }}$ interpercentile range:

| Score, $\boldsymbol{x}$ | Frequency |
| :---: | :---: |
| $0<x \leq 1$ | 11 |
| $1<x \leq 3$ | 4 |
| $3<x \leq 6$ | 2 |
| $6<x \leq 6.5$ | 2 |
| $6.5<x \leq 10$ | 8 |

$$
8.57 \text { (3 sf) }
$$

Worked example

## Your turn

Times, $x$, have been rounded to the nearest minute. Estimate the $5^{\text {th }}-95^{\text {th }}$ interpercentile range:

| Time, $\boldsymbol{x}$ | Frequency |
| :---: | :---: |
| $0-2$ | 5 |
| $3-5$ | 2 |
| $6-10$ | 3 |

Times, $x$, have been rounded to the nearest minute. Estimate the $15^{\text {th }}-85^{\text {th }}$ interpercentile range:

| Time, $\boldsymbol{x}$ | Frequency |
| :---: | :---: |
| $0-3$ | 7 |
| $4-8$ | 11 |
| $9-10$ | 2 |

6.55 ( 3 sf )
2.4) Variance and standard deviation Chapter CONTENTS

Calculate the variance and standard deviation:
$2,3,4,5,6$

Calculate the variance and standard deviation:

$$
2,3,4,5,7
$$

Variance $=\sigma^{2}=2.96$
Standard deviation $=\sigma=1.72(3 \mathrm{sf})$
Variance $=\sigma^{2}=2.96$
Standard deviation $=\sigma=1.72$ (3 sf)

Calculate the variance and standard deviation:
$2,3,4,5,6$

Calculate the variance and standard deviation:

$$
4,6,8,10,12
$$

Variance $=\sigma^{2}=8$
Standard deviation $=\sigma=2.83$ (3 sf)

## Your turn

Calculate the variance and standard deviation:
$2,4,6,8,10$
Calculate the variance and standard deviation:

$$
\begin{aligned}
& 1,2,3,4,5 \\
& \text { Variance }=\sigma^{2}=2 \\
& \text { Standard deviation }=\sigma=1.41(3 \mathrm{sf})
\end{aligned}
$$

Worked example
Calculate the variance and standard deviation:

| Score | Frequency |
| :---: | :---: |
| 0 | 3 |
| 1 | 2 |
| 2 | 1 |
| 3 | 1 |
| 4 | 4 |

Calculate the variance and standard deviation:

| Score | Frequency |
| :---: | :---: |
| 0 | 6 |
| 1 | 4 |
| 2 | 2 |
| 3 | 2 |
| 4 | 8 |

Variance $=\sigma^{2}=2.81(3 \mathrm{sf})$
Standard deviation $=\sigma=1.68(3 \mathrm{sf})$

## Your turn

Estimate the variance and standard deviation:

| Score, $\boldsymbol{x}$ | Frequency |
| :---: | :---: |
| $0 \leq x<1$ | 8 |
| $1 \leq x<2$ | 2 |
| $2 \leq x<4$ | 1 |
| $4 \leq x<9.5$ | 1 |
| $9.5 \leq x<10$ | 4 |

Estimate the variance and standard deviation:

| Score, $\boldsymbol{x}$ | Frequency |
| :---: | :---: |
| $0<x \leq 1$ | 6 |
| $1<x \leq 3$ | 4 |
| $3<x \leq 6$ | 2 |
| $6<x \leq 6.5$ | 2 |
| $6.5<x \leq 10$ | 8 |

Variance $=\sigma_{x}^{2} \approx 10.9$ (3 sf)
Standard deviation $=\sigma_{x} \approx 3.30$ (3 sf)

## Worked example

## Your turn

Times, $x$, have been rounded to the nearest minute. Estimate the variance and standard deviation:

| Time, $\boldsymbol{x}$ | Frequency |
| :---: | :---: |
| $0-2$ | 5 |
| $3-5$ | 2 |
| $6-10$ | 3 |

Times, $x$, have been rounded to the nearest minute. Estimate the variance and standard deviation:

| Time, $\boldsymbol{x}$ | Frequency |
| :---: | :---: |
| $0-3$ | 7 |
| $4-8$ | 11 |
| $9-10$ | 2 |

Variance $=\sigma_{x}^{2} \approx 5.81$ (3 sf)
Standard deviation $=\sigma_{x} \approx 2.41$ ( 3 sf )

## Your turn

Work out how many people had a score more than one standard deviation below the mean

Work out how many people had a score more than one standard deviation above the mean

| Score | Frequency |
| :---: | :---: |
| 0 | 3 |
| 1 | 2 |
| 2 | 1 |
| 3 | 1 |
| 4 | 4 |
| 5 | 9 |
| 6 | 5 |


| Score | Frequency |
| :---: | :---: |
| 0 | 6 |
| 1 | 4 |
| 2 | 2 |
| 3 | 2 |
| 4 | 8 |
| 5 | 18 |
| 6 | 10 |

10

The scores, $x$, were recorded for 20 people. The summary data is:

$$
S_{x x}=235
$$

Calculate the standard deviation

The scores, $x$, were recorded for 40 people. The summary data is:

$$
S_{x x}=532
$$

Calculate the standard deviation

$$
\sigma_{x}=3.65(3 \mathrm{sf})
$$

The scores, $x$, were recorded for 20 people. The summary data is:

$$
\sum x=34, \sum x^{2}=567
$$

Calculate the mean and standard deviation.

The scores, $x$, were recorded for 40 people. The summary data is:

$$
\sum x=76, \sum x^{2}=543
$$

Calculate the mean and standard deviation.
Mean $=\bar{x}=1.9$
Standard deviation $=\sigma_{x}=3.16$ (3 sf)

## Your turn

The scores, $x$, were recorded for 20 people. The summary data is:

$$
\sum x=34, \sum x^{2}=567
$$

The highest score was 8.5.
The lowest score was 0.2 .
Estimate the number of scores which were greater than one standard deviation above the mean.

The scores, $x$, were recorded for 40 people. The summary data is:

$$
\sum x=76, \sum x^{2}=543
$$

The highest score was 5.8.
The lowest score was 0.3.
Estimate the number of scores which were greater than one standard deviation above the mean.

## 2.5) Coding

## Your turn

Prove that if all values of $x$ are multiplied by 5 , the variance increases by a scale factor of 25

Prove that if all values of $x$ are multiplied by 3 , the variance increases by a scale factor of
9

$$
\begin{aligned}
\sigma^{2} & =\frac{\sum(3 x)^{2}}{n}-\left(\frac{\sum(3 x)}{n}\right)^{2} \\
& =\frac{\Sigma 9 x^{2}}{n}-\left(\frac{3 \Sigma x}{n}\right)^{2} \\
& =\frac{9 \cdot \Sigma x^{2}}{n}-9\left(\frac{\sum x}{n}\right)^{2} \\
& =9\left(\frac{\Sigma x^{2}}{n}-\left(\frac{\Sigma x}{n}\right)^{2}\right)
\end{aligned}
$$

Worked example
Scores, $x$ :
2090, 2080, 2070, 2060, 2050
a) Use the coding $y=x-2000$ to code this data
b) Calculate the mean and standard deviation of the coded data
c) Use your answer to b) to calculate the mean and standard deviation of the original data

Scores, $x$ :
1010, 1020, 1030, 1040, 1050
a) Use the coding $y=x-1000$ to code this data
b) Calculate the mean and standard deviation of the coded data
c) Use your answer to b) to calculate the mean and standard deviation of the original data
a) $y: 10,20,30,40,50$
b) $\bar{y}=30, \sigma_{y}=14.1(3 \mathrm{sf})$
c) $\bar{x}=1030, \sigma_{x}=14.1(3 \mathrm{sf})$

## Your turn

Scores, $x$ :
2090, 2080, 2070, 2060, 2050
a) Use the coding $y=2 x$ to code this data
b) Calculate the mean and standard deviation of the coded data
c) Use your answer to b) to calculate the mean and standard deviation of the original data

Scores, $x$ :
1010, 1020, 1030, 1040, 1050
a) Use the coding $y=3 x$ to code this data
b) Calculate the mean and standard deviation of the coded data
c) Use your answer to b) to calculate the mean and standard deviation of the original data
a) $y: 3030,3060,3090,3120,3150$
b) $\bar{y}=3090, \sigma_{y}=42.4(3 \mathrm{sf})$
c) $\bar{x}=1030, \sigma_{x}=14.1(3 \mathrm{sf})$

## Your turn

Scores, $x$ :
2090, 2080, 2070, 2060, 2050
a) Use the coding $y=\frac{x}{5}$ to code this data
b) Calculate the mean and standard deviation of the coded data
c) Use your answer to b) to calculate the mean and standard deviation of the original data

Scores, $x$ :
1010, 1020, 1030, 1040, 1050
a) Use the coding $y=\frac{x}{10}$ to code this data
b) Calculate the mean and standard deviation of the coded data
c) Use your answer to b) to calculate the mean and standard deviation of the original data
a) $y: 101,102,103,104,105$
b) $\bar{y}=103, \sigma_{y}=1.41(3 \mathrm{sf})$
c) $\bar{x}=1030, \sigma_{x}=14.1(3 \mathrm{sf})$

## Your turn

Scores, $x$ :
2090, 2080, 2070, 2060, 2050
a) Use the coding $y=\frac{x-2000}{10}$ to code this data
b) Calculate the mean and standard deviation of the coded data
c) Use your answer to b) to calculate the mean and standard deviation of the original data

Scores, $x$ :

$$
1010,1020,1030,1040,1050
$$

a) Use the coding $y=\frac{x-1000}{10}$ to code this data
b) Calculate the mean and standard deviation of the coded data
c) Use your answer to b) to calculate the mean and standard deviation of the original data
a) $y$ : $1,2,3,4,5$
b) $\bar{y}=3, \sigma_{y}=1.41(3 \mathrm{sf})$
c) $\bar{x}=1030, \sigma_{x}=14.1(3 \mathrm{sf})$

## Your turn

Scores, $x$ :
2090, 2080, 2070, 2060, 2050
a) Use the coding $y=\frac{x}{10}-200$ to code this data
b) Calculate the mean and standard deviation of the coded data
c) Use your answer to b) to calculate the mean and standard deviation of the original data

Scores, $x$ :
1010, 1020, 1030, 1040, 1050
a) Use the coding $y=\frac{x}{10}-100$ to code this data
b) Calculate the mean and standard deviation of the coded data
c) Use your answer to b) to calculate the mean and standard deviation of the original data
a) $y: 1,2,3,4,5$
b) $\bar{y}=3, \sigma_{y}=1.41(3 \mathrm{sf})$
c) $\bar{x}=1030, \sigma_{x}=14.1(3 \mathrm{sf})$

## Your turn

Scores, $x$, of 20 people were recorded. The data was coded using $y=\frac{x-10}{5}$ and the following summations were obtained:

$$
\Sigma y=23, \Sigma y^{2}=147.6
$$

Calculate the standard deviation of the actual scores.

Scores, $x$, of 40 people were recorded. The data was coded using $y=\frac{x-5}{10}$ and the following summations were obtained:

$$
\Sigma y=32, \Sigma y^{2}=764.1
$$

Calculate the standard deviation of the actual scores.

$$
\sigma_{x}=42.97(2 \mathrm{dp})
$$

## Your turn

Scores, $x$, of 20 people were recorded. The data was coded using $y=5 x-10$ and the following summations were obtained:

$$
\Sigma y=23, \Sigma y^{2}=147.6
$$

Calculate the standard deviation of the actual scores.

Scores, $x$, of 40 people were recorded.
The data was coded using $y=10 x-5$ and the following summations were obtained:

$$
\Sigma y=32, \Sigma y^{2}=764.1
$$

Calculate the standard deviation of the actual scores.

$$
\sigma_{x}=0.4297(4 \mathrm{dp})
$$

## Worked example

## Your turn

A teacher standardises scores, $x$, of his class by adding 10 to each score and then reducing the score by $8 \%$.
The following summary statistics are calculated for the standardised scores, $y$ :

$$
n=30, \bar{y}=23.4, S_{y y}=5.6
$$

Calculate the mean and standard deviation of the original scores

A teacher standardises scores, $x$, of his class by adding 8 to each score and then reducing the score by $10 \%$.
The following summary statistics are calculated for the standardised scores, $y$ :

$$
n=25, \bar{y}=43.2, S_{y y}=6.5
$$

Calculate the mean and standard deviation of the original scores
Mean $=\bar{x}=40$
Standard deviation $=\sigma_{x}=0.567(3 \mathrm{sf})$

## Your turn

A person was recording times, $x$, in a race. They then realised the stopwatch was slow and all times were actually 3 minutes more than the recorded times.
Explain the effect on:
a) The mean
b) The standard deviation
c) The median
d) The range
e) The lower quartile
f) The interquartile range

A person was recording times, $x$, in a race.
They then realised the stopwatch was slow and all times were actually 5 minutes less than the recorded times.
Explain the effect on:
a) The mean
b) The standard deviation
c) The median
d) The range
e) The lower quartile
f) The interquartile range
a) Decreases by 5
b) No effect
c) Decreases by 5
d) No effect
e) Decreases by 5
f) No effect

