2) Conditional probability

2.1) Set notation

2.2) Conditional probability

2.3) Conditional probabilities in Venn diagrams

2.4) Probability formulae

2.5) Tree diagrams

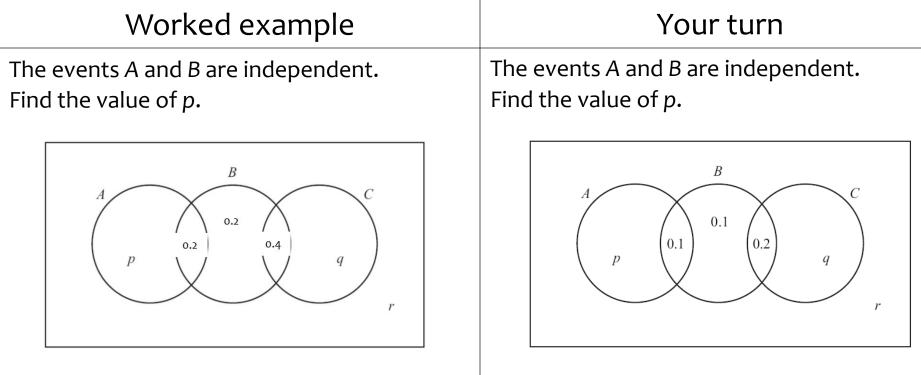
2.1) Set notation

Chapter CONTENTS

Worked example	Your turn
A card is selected at random from a pack of 52 playing cards. Let <i>R</i> be the event that the card is a royal (king, queen or jack). Let <i>S</i> be the event that the card is a spade. Find: a) $P(R \cap S)$ b) $P(R \cup S)$ c) $P(R')$ d) $P(R' \cap S)$	A card is selected at random from a pack of 52 playing cards. Let A be the event that the card is an ace. Let D be the event that the card is a diamond. Find: a) $P(A \cap D)$ b) $P(A \cup D)$ c) $P(A')$ d) $P(A' \cap D)$ a) $\frac{1}{52}$ b) $\frac{16}{52}$ c) $\frac{48}{52}$ d) $\frac{12}{52}$

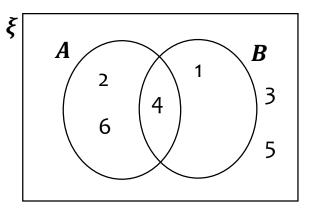
Worked example	Your turn
Given that: $P(A) = 0.5$	Given that: $P(A) = 0.3$
P(B) = 0.2 $P(A \cap B) = 0.1$ Explain why events A and B are independent	$P(B) = 0.4$ $P(A \cap B) = 0.25$ Explain why events A and B are not independent.
	If independent $P(A) \times P(B) = P(A \cap B)$ $0.3 \times 0.4 = 0.12 \neq 0.25$ $\therefore A$ and B are not independent.

Worked example	Your turn
Given that: $P(A) = 0.5$ $P(B) = 0.34$ $P(A \cap B) = 0.25$ $P(C) = 0.15$ A and C are mutually exclusive. Events B and C are independent. a) Draw a Venn diagram to illustrate the events A, B and C, showing the probabilities for each region. b) Find $P((C \cap B') \cup A)$	Given that: $P(A) = 0.3$ $P(B) = 0.4$ $P(A \cap B) = 0.25$ $P(C) = 0.2$ A and C are mutually exclusive. Events B and C are independent. a) Draw a Venn diagram to illustrate the events A, B and C, showing the probabilities for each region. b) Find P((A \cap B') \cup C) a) $A = \frac{B}{(0.05 + 0.07)} \frac{C}{(0.08 + 0.12)} \frac{C}{(0.05 + 0.07)} \frac{C}{(0.08 + 0.12)} \frac{C}{(0.$
	b) 0.25



p = 0.15

Worked example	Your turn
P(A) = x $P(B) = y$ Find: a) $P(A \cup B)$ b) $P(A' \cup B)$ Find a) $P(A \cup B)$ Find b) $P(A' \cup B)$ Find c) $P(A \cup B)$ F	$P(A \cap B)$ $P(A \cup B')$



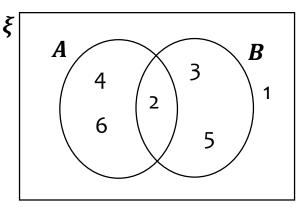
- ξ = the whole sample space (1 to 6)
- A = even number on a die thrown

B = square number on a die thrown

State what it means in this context, and the resulting set of outcomes:

A'

Your turn



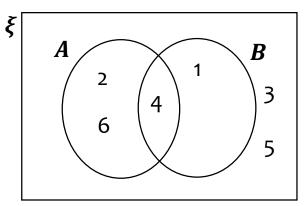
 ξ = the whole sample space (1 to 6)

- A = even number on a die thrown
- B = prime number on a die thrown

State what it means in this context, and the resulting set of outcomes:

A'

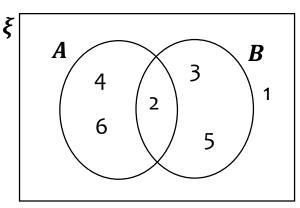
Not A (the complement of A) Not rolling an even number {1, 3, 5}



- ξ = the whole sample space (1 to 6)
- A = even number on a die thrown
- B = square number on a die thrown
- State what it means in this context, and the resulting set of outcomes:

B'

Your turn

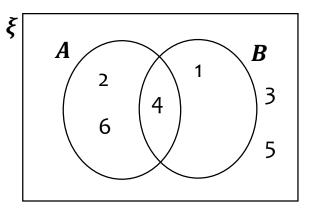


 ξ = the whole sample space (1 to 6)

- A = even number on a die thrown
- B = prime number on a die thrown

State what it means in this context, and the resulting set of outcomes:

B' Not B (the complement of B) Not rolling a prime number {1, 4, 6}



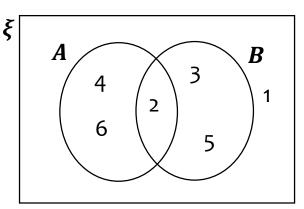
- ξ = the whole sample space (1 to 6)
- A = even number on a die thrown

B = square number on a die thrown

State what it means in this context, and the resulting set of outcomes:

 $A \cup B$

Your turn



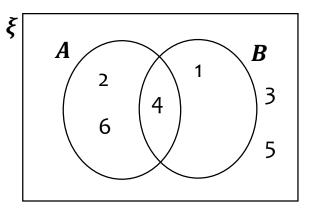
 ξ = the whole sample space (1 to 6)

- A = even number on a die thrown
- B = prime number on a die thrown

State what it means in this context, and the resulting set of outcomes:

 $A \cup B$

A or B (the union of A and B) Rolling an even number or a prime number {2, 3, 4, 5, 6}



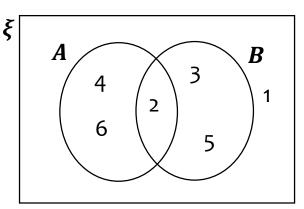
- ξ = the whole sample space (1 to 6)
- A = even number on a die thrown

B = square number on a die thrown

State what it means in this context, and the resulting set of outcomes:

 $A\cap B$

Your turn



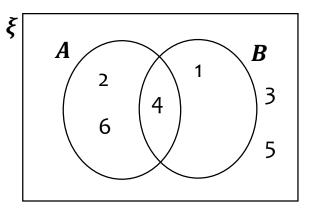
 ξ = the whole sample space (1 to 6)

- A = even number on a die thrown
- B = prime number on a die thrown

State what it means in this context, and the resulting set of outcomes:

 $A \cap B$

A and B (the intersection of A and B) Rolling a number which is even and prime {2}



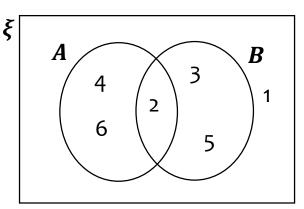
- ξ = the whole sample space (1 to 6)
- A = even number on a die thrown

B = square number on a die thrown

State what it means in this context, and the resulting set of outcomes:

 $A\cap B'$

Your turn



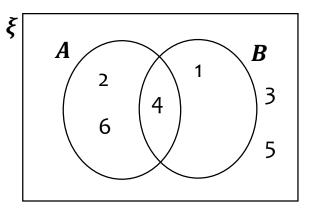
 ξ = the whole sample space (1 to 6)

- A = even number on a die thrown
- B = prime number on a die thrown

State what it means in this context, and the resulting set of outcomes:

 $A \cap B'$

A and not B Rolling a number which is even and not prime {4,6}



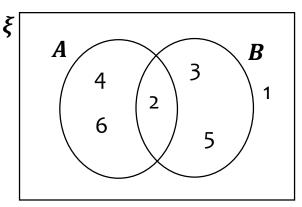
- ξ = the whole sample space (1 to 6)
- A = even number on a die thrown

B = square number on a die thrown

State what it means in this context, and the resulting set of outcomes:

 $A' \cap B$

Your turn



 ξ = the whole sample space (1 to 6)

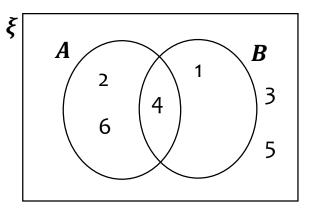
A = even number on a die thrown

B = prime number on a die thrown

State what it means in this context, and the resulting set of outcomes:

 $A' \cap B$

B and not A Rolling a number which is prime and not even {3, 5}



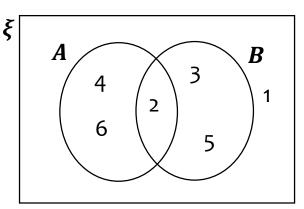
- ξ = the whole sample space (1 to 6)
- A = even number on a die thrown

B = square number on a die thrown

State what it means in this context, and the resulting set of outcomes:

 $(A \cup B)'$

Your turn



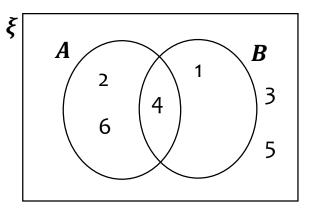
 ξ = the whole sample space (1 to 6)

- A = even number on a die thrown
- B = prime number on a die thrown

State what it means in this context, and the resulting set of outcomes:

 $(A \cup B)'$

Not (A or B) Rolling a number which is not (even or prime) {1}



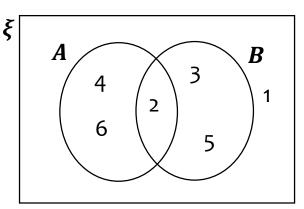
- ξ = the whole sample space (1 to 6)
- A = even number on a die thrown

B = square number on a die thrown

State what it means in this context, and the resulting set of outcomes:

 $(A \cap B)'$

Your turn



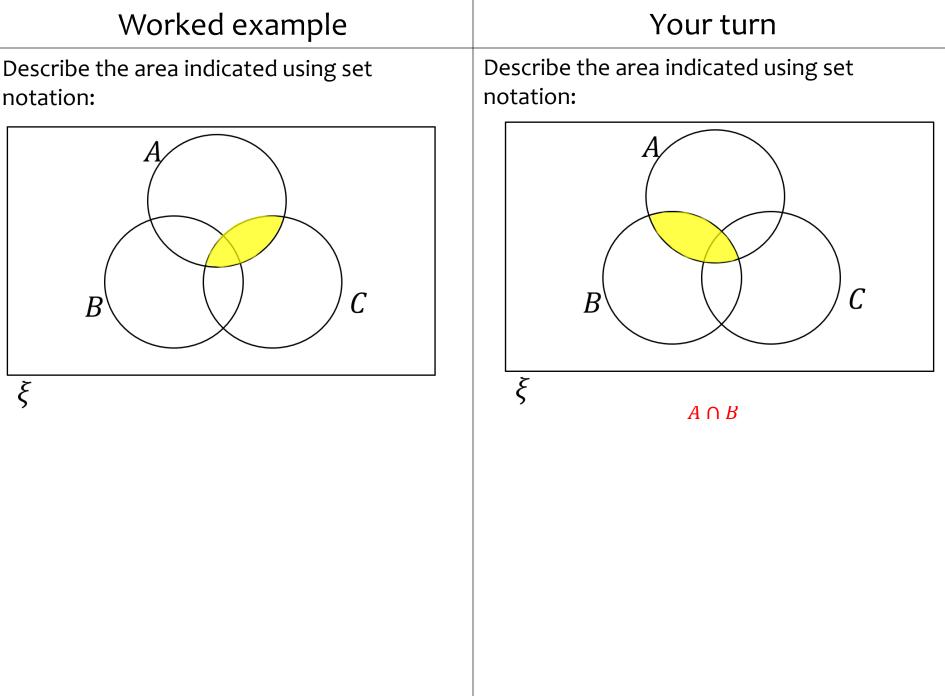
 ξ = the whole sample space (1 to 6)

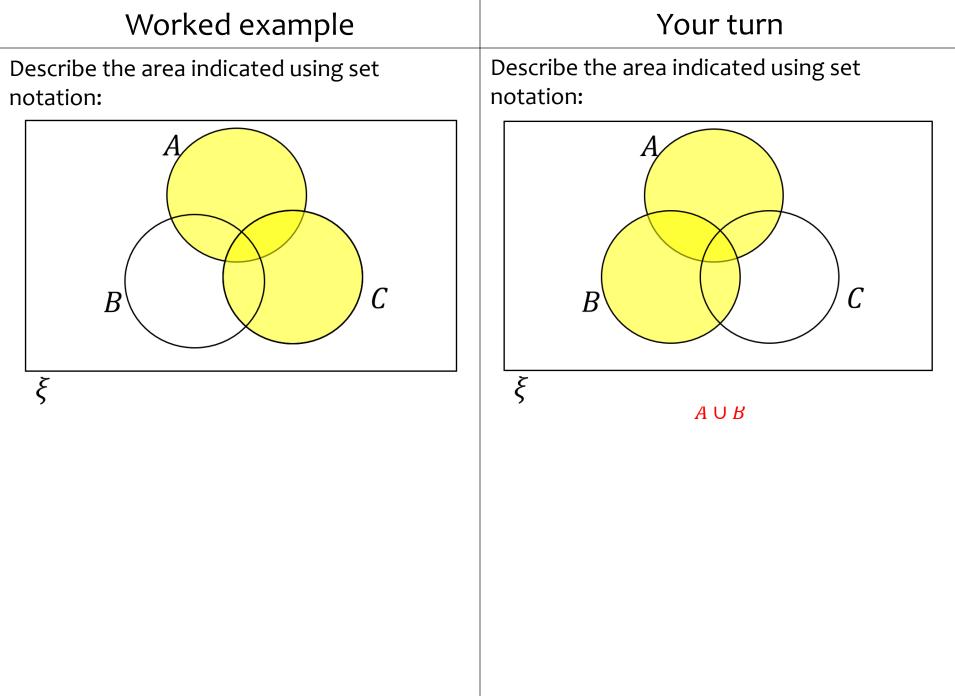
- A = even number on a die thrown
- B = prime number on a die thrown

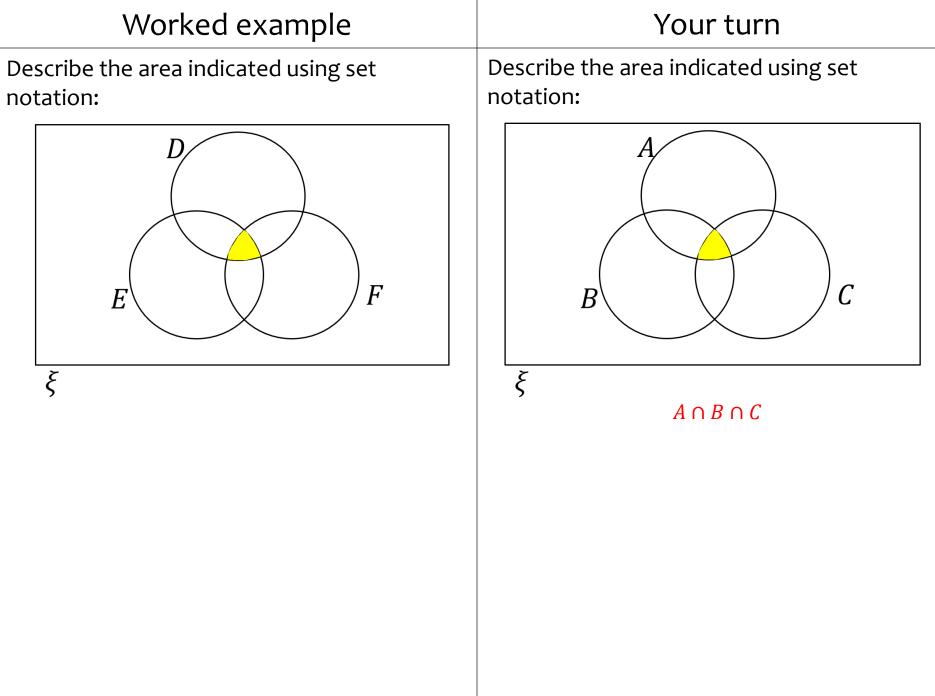
State what it means in this context, and the resulting set of outcomes:

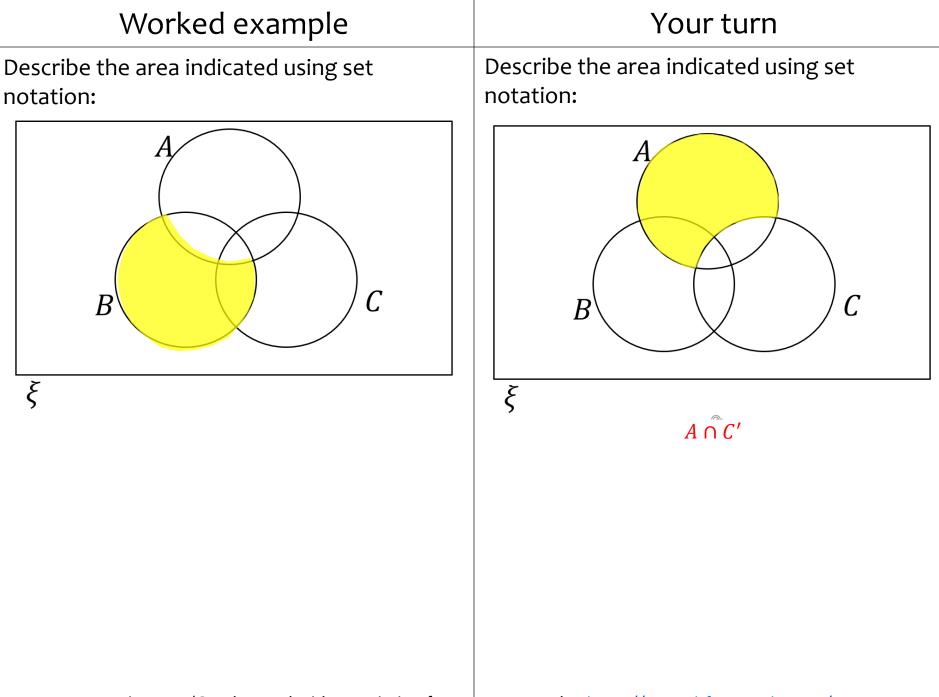
 $(A \cap B)'$

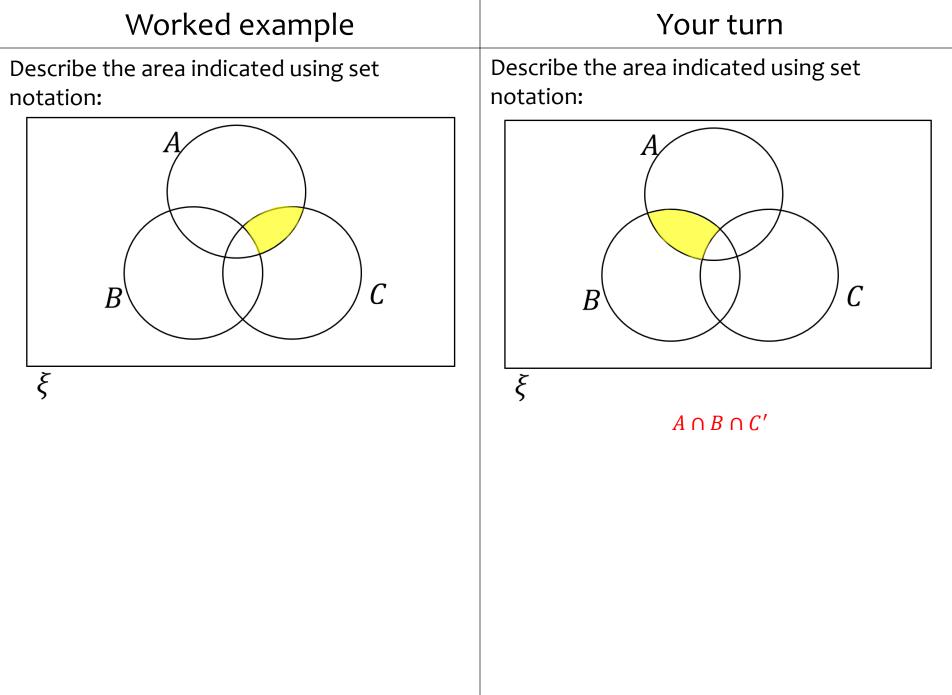
Not (A and B) Rolling a number which is not (even and prime) {1}

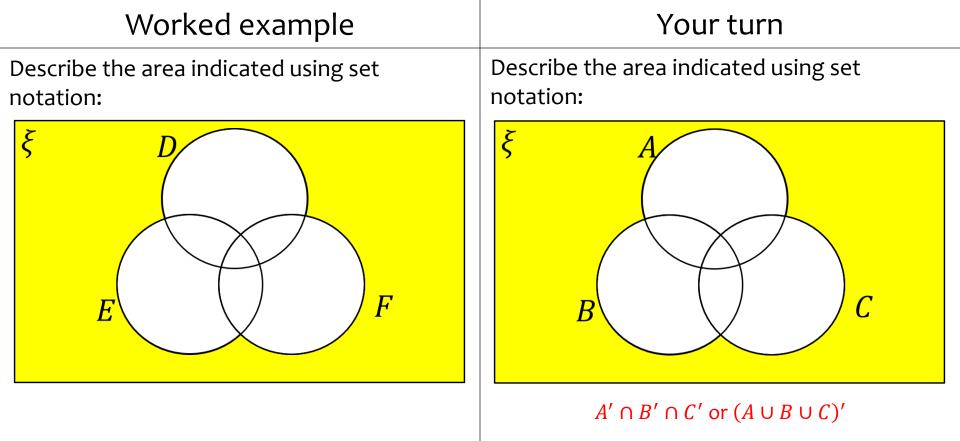


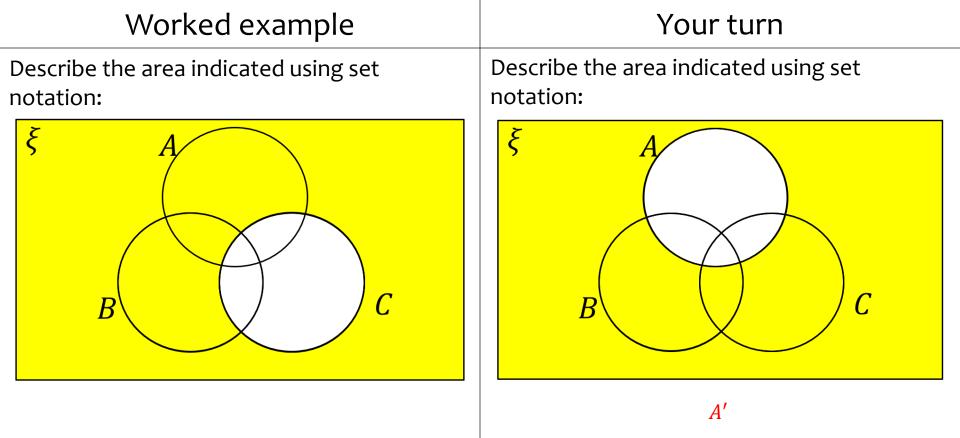


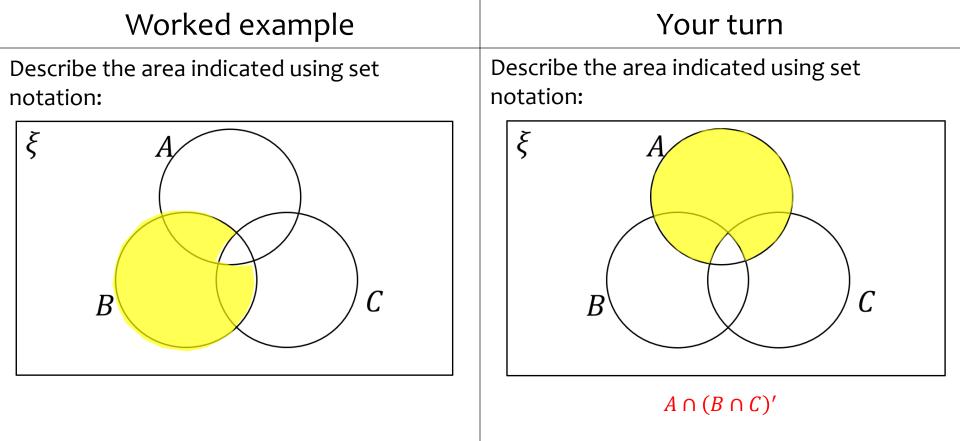


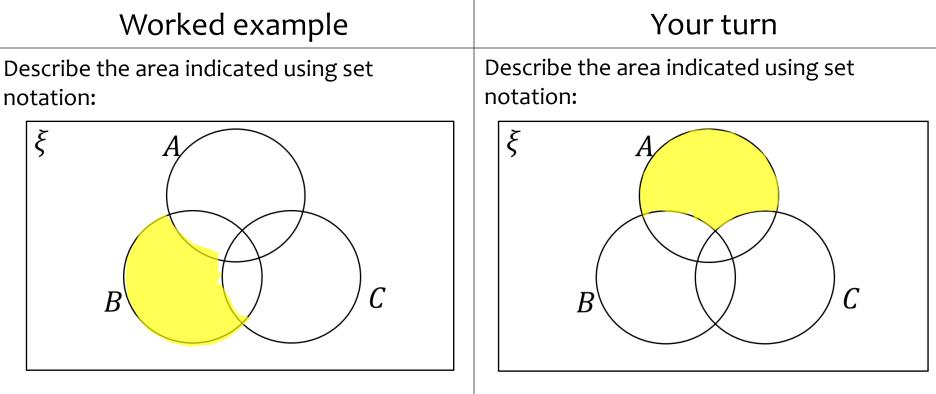






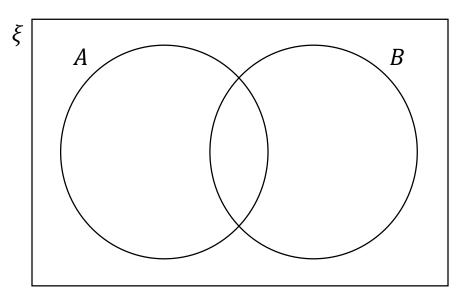




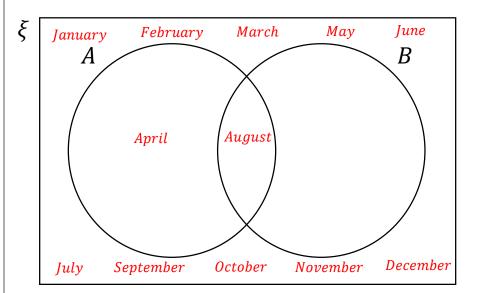


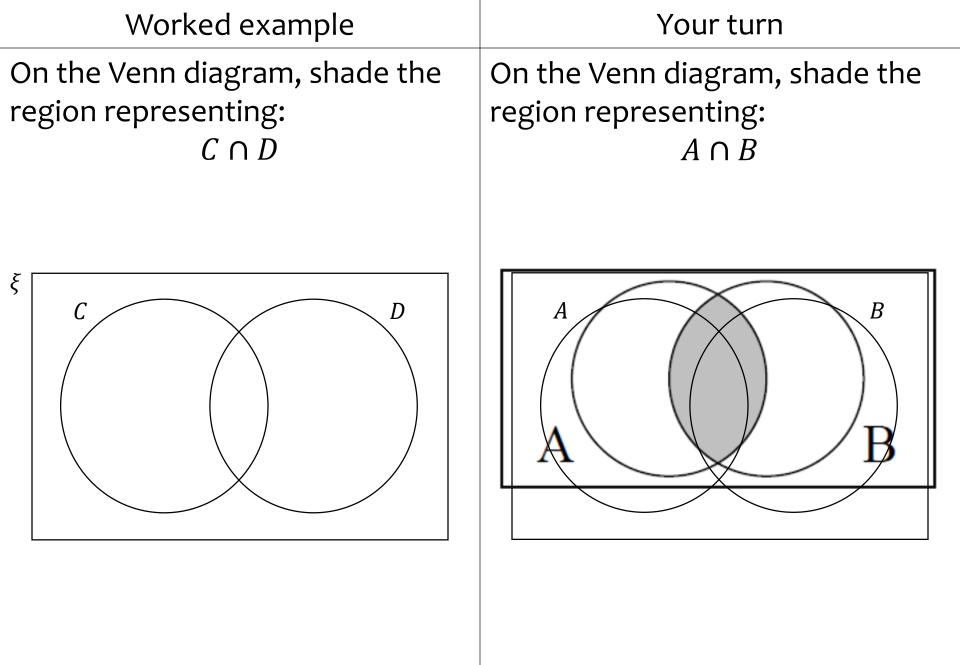
 $A \cap B' \cap C'$

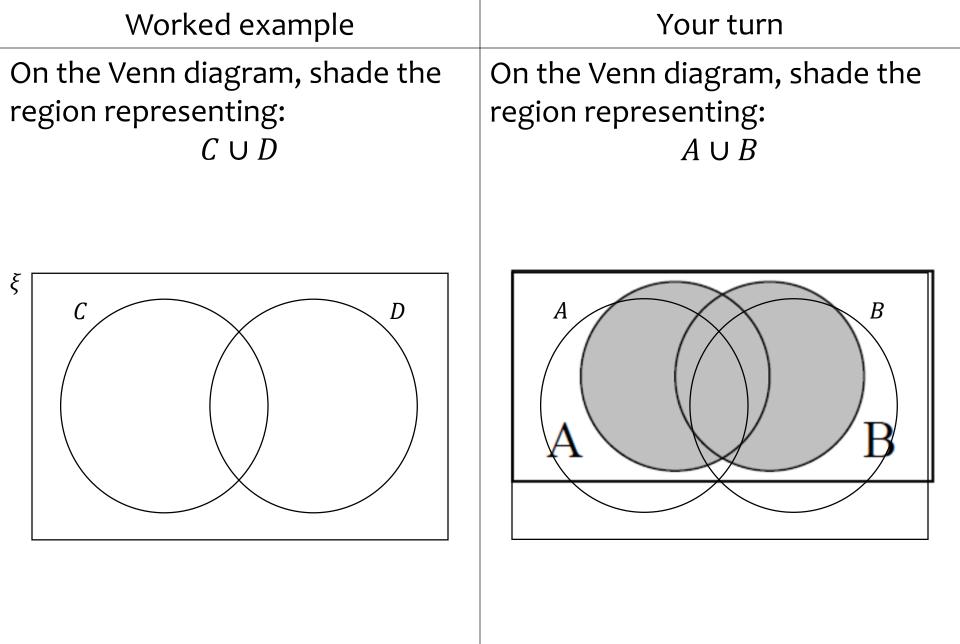
Y
$\xi = \{Months c$
$A = \{Months\}$
$B = \{Months\}$
Draw a Venn
represent this

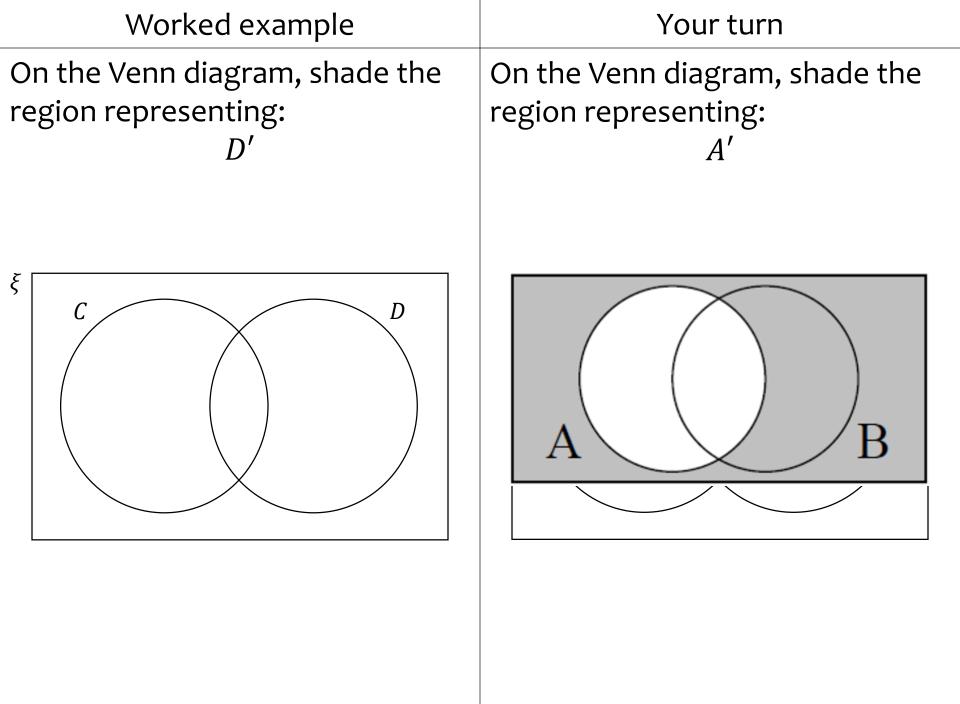


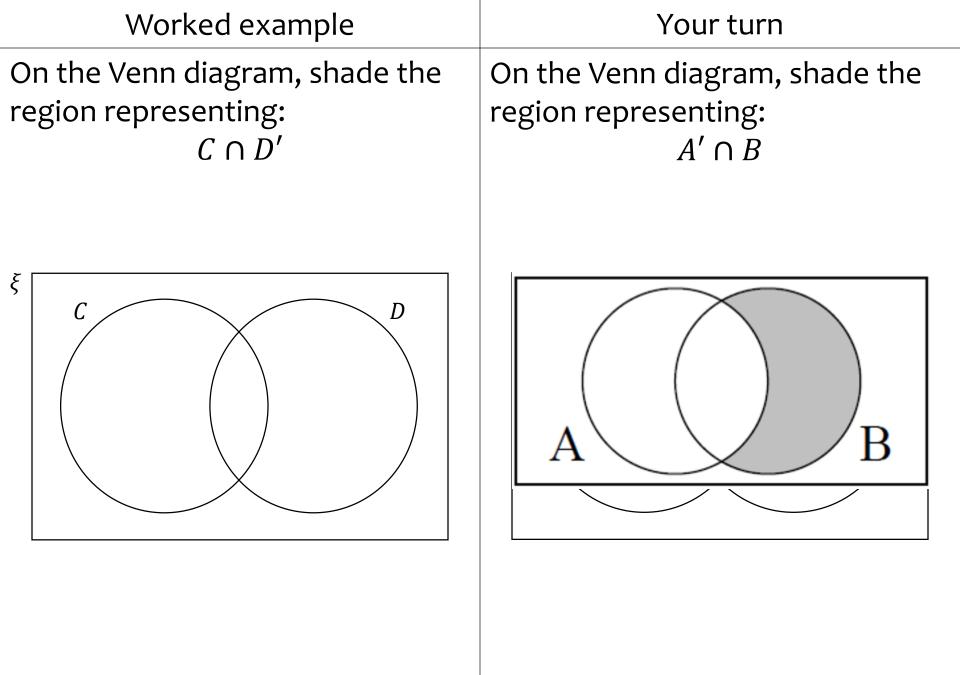
Your turn $\xi = \{Months \ of \ the \ year\}$ $A = \{Months \ starting \ with \ A\}$ $B = \{Months \ with \ six \ letters\}$ Draw a Venn diagram to represent this information.

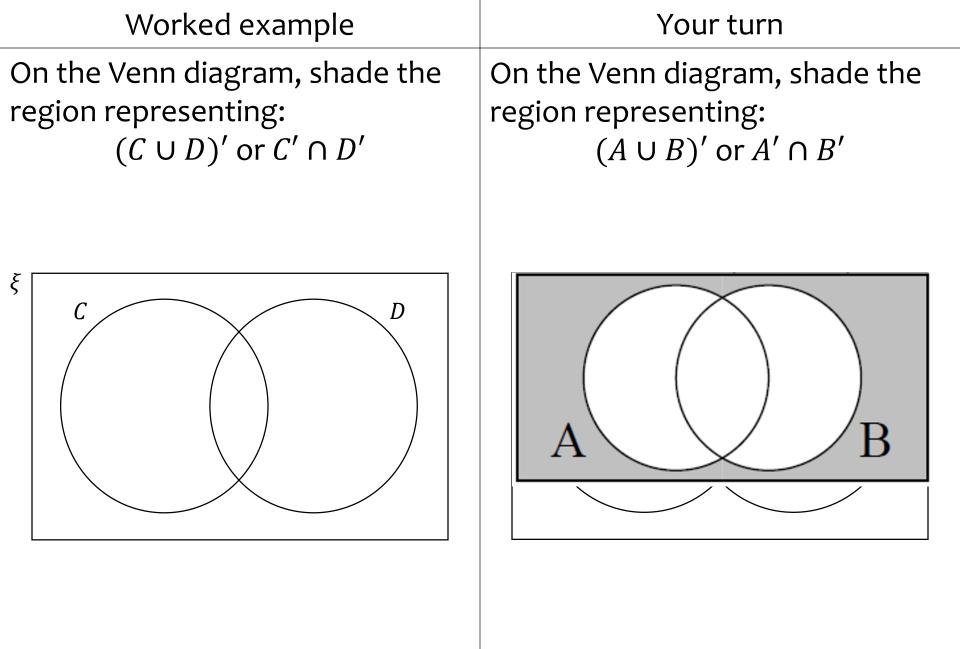


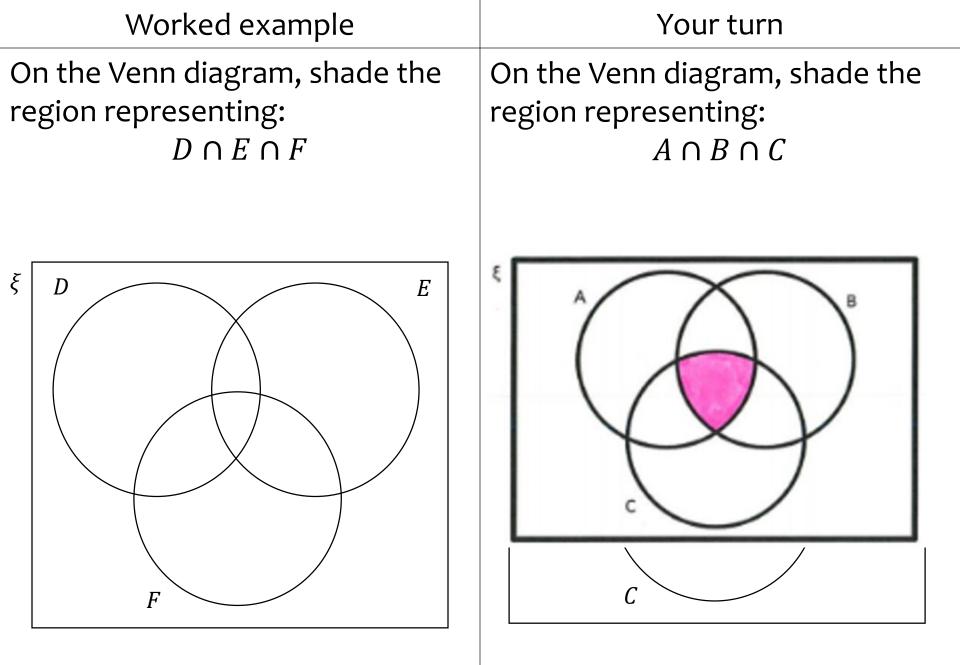


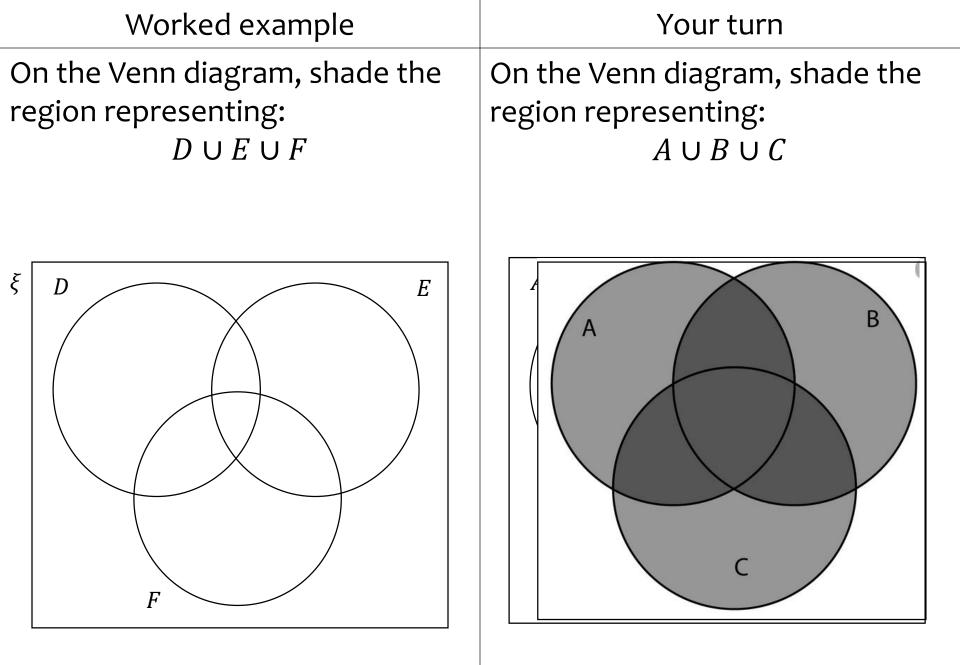


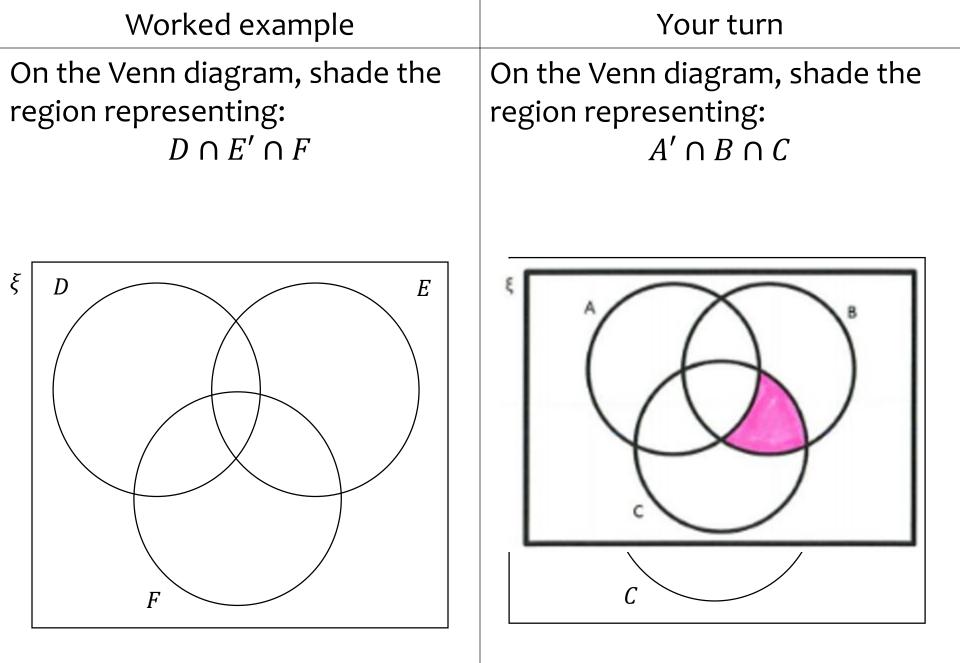


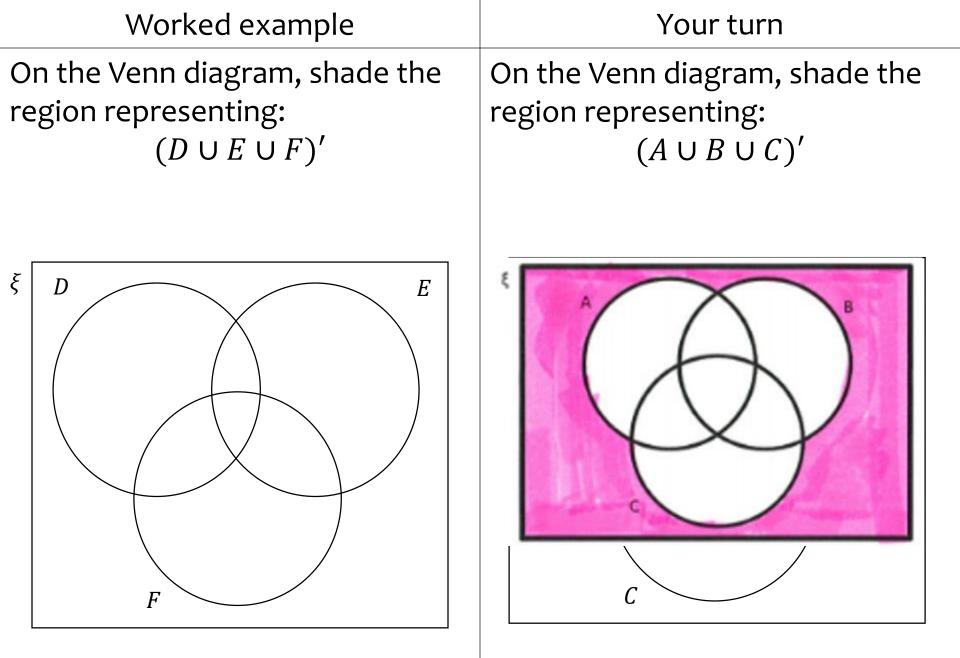


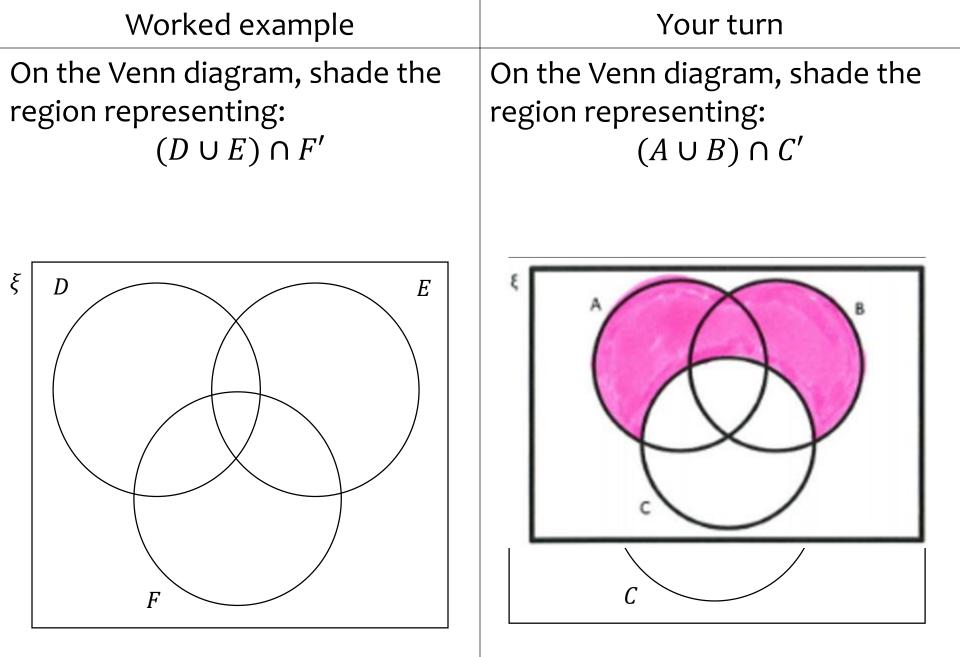


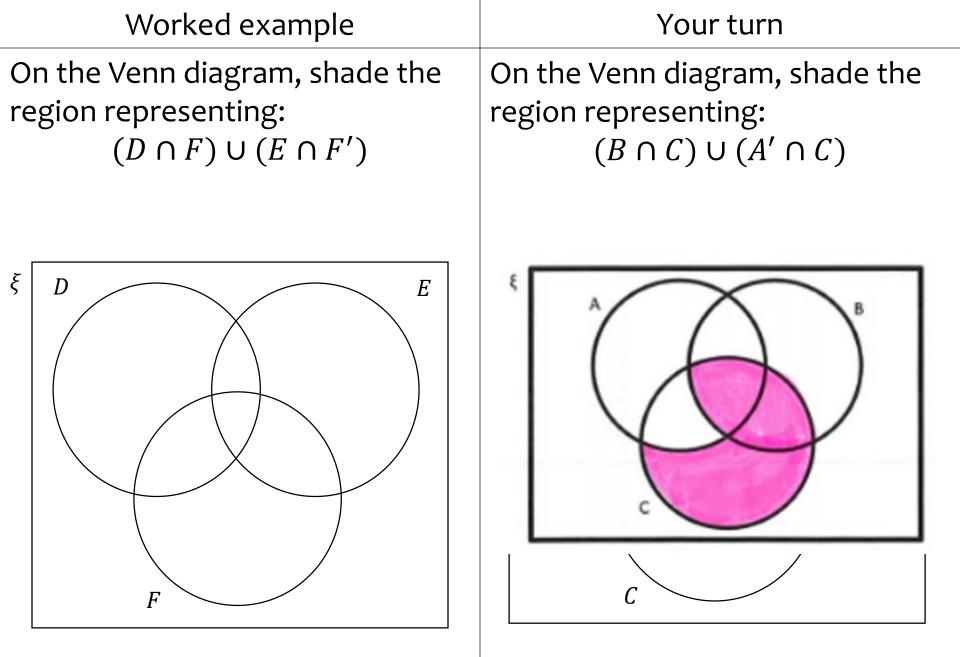


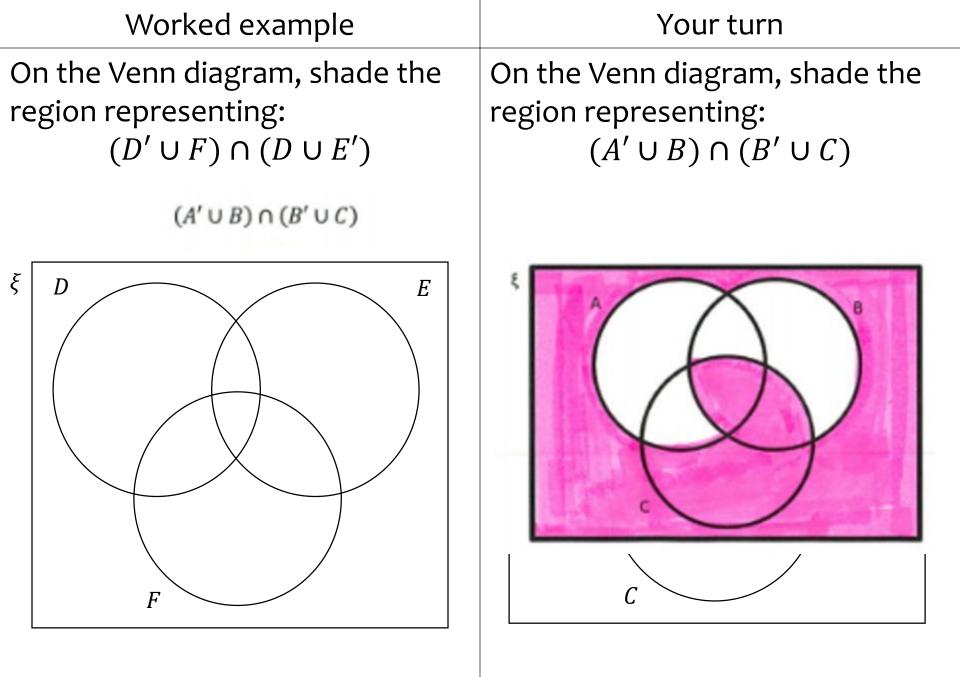












Worked example	Your turn
$\xi = Positive integers between 1$ and 10 inclusive $A = \{Multiples of 2\}$ $\xi = Positive 1$ and 20 $A = \{Multiples of 2\}$	B 12 17 18 19

Worked example	Your turn
 In a group of 28 scientists: 20 have degrees in Physics. 18 have degrees in Chemistry. Some have degrees in both 4 scientists have degrees which are neither Physics nor Chemistry. 	 In a group of 30 mathematicians: 15 have studied Calculus. 22 have studied Topology. Some have studied both. 3 mathematicians have not yet studied either Calculus or topology
Find the number of scientists who have degrees in both Physics and Chemistry.	Find the number of mathematicians who have studied both Calculus and Topology. 10

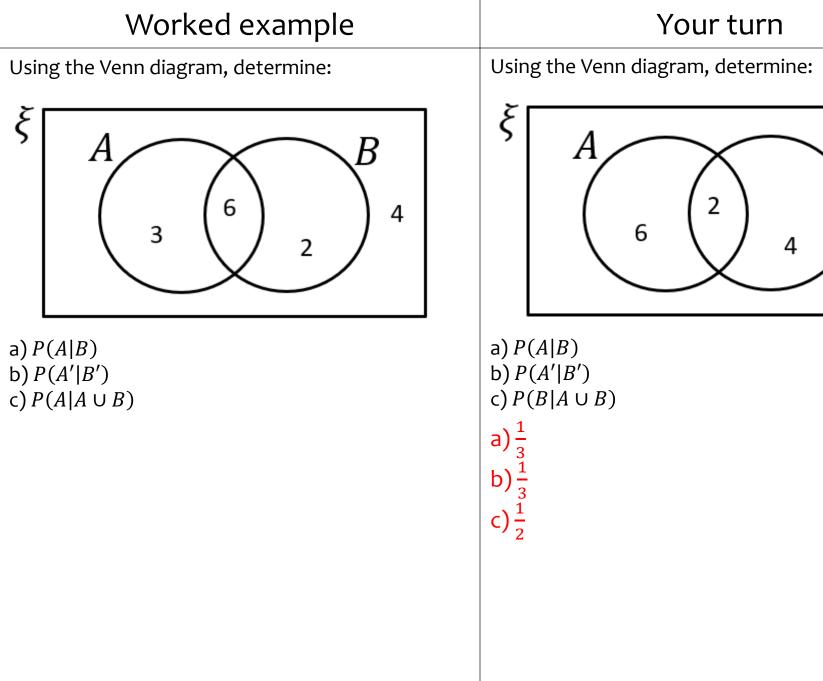
2.2) Conditional probability

Chapter CONTENTS

Worked example	Your turn				
 A group is made up of 62 men and 48 women. 32 of the men and 46 of the women are righthanded. a) Draw a two-way table to show this information. b) One person is chosen at random. Find: i) P(right-handed) ii) P(right-handed woman) iii) P(man right-handed) 	informat b) One pers i) P(left- ii) P(left- iii) P(wor	wo-way tal	24 of the ble to sh en at rar nan)	men are	e left-
	a)		L	L'	
		Μ	24	18	
		W	36	32	
	b) i) $\frac{60}{110} = \frac{6}{11}$ ii) $\frac{24}{42} = \frac{4}{7}$ iii) $\frac{36}{60} = \frac{3}{5}$				

Worked example			Your turn						
The following two-way table shows what foreign language students in Year 9 study. <i>G</i> is the event that the student is a girl. <i>S</i> is the event they chose Spanish as their language.		The following two-way table shows what foreign language students in Year 9 study. <i>B</i> is the event that the student is a boy. <i>F</i> is the event they chose French as their language.							
		G	G '				B	B'	
	S	18	34			F	14	38	
	<i>S</i> ′	16	32			F'	26	22	
Determine: a) $P(S' G)$ b) $P(G' S)$					Determine: a) $P(F B')$ b) $P(B F')$ a) $\frac{38}{60}$ b) $\frac{26}{48}$				

2.3) Conditional probabilities in Venn diagrams Chapter CONTENTS

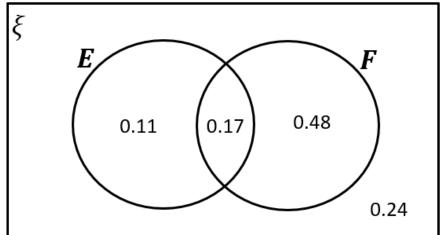


Worked example	Your turn		
Given that $P(X) = 0.7$ and $P(X \cap Y) = 0.2$, determine:	Given that $P(A) = 0.5$ and $P(A \cap B) = 0.3$, determine:		
P(Y X)	P(B A)		
	0.6		

Your turn
Given that $P(Y) = 0.6$ and $P(X \cap Y) = 0.4$, determine:
P(X' Y)
$\frac{1}{3}$

Worked example	Your turn
Given that $P(X) = 0.4$, $P(Y) = 0.4$ and $P(X \cap Y) = 0.3$, determine: P(Y X')	Given that $P(A) = 0.5$, $P(B) = 0.5$ and $P(A \cap B) = 0.4$, determine: P(B A')
	0.2

Worked example	Your turn
Given that	Given that
P(E) = 0.24 $P(E \cup F) = 0.79$ $P(E \cap F') = 0.12$	P(E) = 0.28 $P(E \cup F) = 0.76$ $P(E \cap F') = 0.11$
Draw a Venn diagram to illustrate the probabilities of each region.	Draw a Venn diagram to illustrate the probabilities of each region.
	7



Worked example	Your turn
Given that $P(A \cap P') = 0.2$	Given that $P(A \cap P') = 0.4$
$P(A \cap B') = 0.3$ $P(A \cup B) = 0.65$	$P(A \cap B') = 0.4$ $P(A \cup B) = 0.75$
Determine: a) $P(B)$ b) $P(A' \cap B')$	Determine: a) $P(B)$ b) $P(A' \cap B')$
	a) 0.35 b) 0.25

Worked example	Your turn
Given that	Given that
P(A') = 0.6,	P(A') = 0.7,
P(B') = 0.15	P(B') = 0.2
$P(A \cap B') = 0.05$	$P(A \cap B') = 0.1$
Determine:	Determine:
a) $P(A \cup B')$	a) $P(A \cup B')$
b) $P(B A')$	b) $P(B A')$
	a) 0.4
	$D_{\frac{7}{7}}$
	b) $\frac{6}{7}$

Worked example	Your turn
The events A and B are independent. $P(B C) = \frac{10}{11}$, a) Find the values of p, q and r b) Find $P(A \cup C B)$	The events A and B are independent. $P(B C) = \frac{5}{11}$, a) Find the values of p, q and r b) Find $P(A \cup C B)$
$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	$\begin{array}{ c c c c c c c c c c c c c c c c c c c$
	a) $p = 0.15, q = 0.24, r = 0.21$ b) 0.75

2.4) Probability formulae

Chapter CONTENTS

Worked example	Your turn
Two events A and B are independent. $P(A) = \frac{1}{5}$ $P(B) = \frac{1}{6}$ Find: a) $P(A \cap B)$ b) $P(B A)$	Two events A and B are independent. $P(A) = \frac{1}{3}$ $P(B) = \frac{1}{4}$ Find: a) $P(A \cap B)$ b) $P(A B)$
c) $P(A \cup B)$	c) $P(A \cup B)$ a) $\frac{1}{12}$ b) $\frac{1}{3}$ c) $\frac{1}{2}$

Worked example	Your turn
A and B are two events such that P(A) = 0.3 $P(B) = 0.4$ $P(A B) = 0.2$ Find: a) $P(A \cap B)$ b) $P(B A)$ c) $P(A \cup B)$	C and D are two events such that P(C) = 0.2 P(D) = 0.6 P(C D) = 0.3 Find: a) $P(C \cap D)$ b) $P(D C)$ c) $P(C \cup D)$ a) 0.18 b) 0.9 c) 0.62

Worked example	Your turn
<i>C</i> and <i>D</i> are two independent events such that $P(C) = \frac{1}{3}$ $P(C \cup D) = \frac{3}{5}$ Find: a) <i>P</i> (<i>D</i>) b) <i>P</i> (<i>C'</i> \cap <i>D</i>) c) <i>P</i> (<i>D'</i> <i>C</i>)	A and B are two independent events such that $P(A) = \frac{1}{4}$ $P(A \cup B) = \frac{2}{3}$ Find: a) P(B) b) P(A' \cap B) c) P(B' A)
	a) $\frac{5}{9}$ b) $\frac{5}{12}$ c) $\frac{4}{9}$

Worked example	Your turn
A and C are mutually exclusive.A and B are independent.A and B are independent. $P(A) = 0.4$ $P(C) = 0.3$ $P(C) = 0.3$ $P(A \cup B) = 0.6$ Find:a)a) $P(A B)$ b)b) $P(A \cup C)$ b)c) $P(B)$ c)	There are three events: A, B and C . A and B are mutually exclusive. A and C are independent. P(A) = 0.2 P(B) = 0.4 $P(A \cup C) = 0.7$ Find: a) $P(A C)$ b) $P(A \cup B)$ c) $P(C)$ a) 0.2 b) 0.6 c) 0.625

Worked example	Your turn
Write out the law for conditional probability:	Write out the law for conditional probability:
"Given John runs to school, find the probability that he's not late"	"Given Bob walks to school, find the probability that he's not late"
	$P(L' W) = \frac{P(L' \cap W)}{P(W)} = \cdots$
"Given an even number is rolled on a die, find the probability that the number is prime"	

Worked example	Your turn
Write out the relevant probability laws:	Write out the law for independent events:
• C and D are independent events.	• A and B are independent events. $P(A \cap B) = P(A)P(B)$ $P(A B) = P(A)$
• C and D are mutually exclusive events.	• C and D are mutually exclusive events. $P(C \cap D) = 0$ $P(C \cup D) = P(C) + P(D)$

2.5) Tree diagrams

Chapter CONTENTS

Worked example	Your turn
A bag contains 7 green beads and 3 yellow beads. A bead is taken from the bag at random, the colour is recorded and it is not replaced. A second bead is then taken from the bag and its colour recorded. Given that both balls are the same colour, find the probability that they are both green.	A bag contains 6 green beads and 4 yellow beads. A bead is taken from the bag at random, the colour is recorded and it is not replaced. A second bead is then taken from the bag and its colour recorded. Given that both balls are the same colour, find the probability that they are both yellow.
	$\frac{2}{7}$

Worked example	Your turn
 There are two bags. Bag A contains 5 red balls and 5 blue balls Bag B contains 3 red balls and 6 blue balls. One ball is taken from bag A and placed in bag B. Then one ball is taken from bag B. Find the probability that: a) A blue ball is taken from bag B. b) Given that a blue ball is taken from bag B, the ball taken from bag A was also blue. 	 There are two bags. Bag A contains 5 red balls and 5 blue balls Bag B contains 3 red balls and 6 blue balls. One ball is taken from bag A and placed in bag B. Then one ball is taken from bag B. Find the probability that: a) A red ball is taken from bag B. b) Given that a red ball is taken from bag B, the ball taken from bag A was also red. a) ⁷/₂₀ b) ⁴/₇

Worked example	Your turn
that a person travels to work by bus, train orthemotorbike is $\frac{2}{5}$, $\frac{1}{4}$ and $\frac{7}{20}$ respectively.orThe probability of being late when usingThethese methods of travel is $\frac{1}{3}$, $\frac{2}{7}$ and $\frac{3}{8}$ therespectively.respectively.Given that the person is late, find theGiven	On a randomly chosen day the probability hat a person travels to school by car, bicycle or on foot is $\frac{1}{2}$, $\frac{1}{6}$ and $\frac{1}{3}$ respectively. The probability of being late when using hese methods of travel is $\frac{1}{5}$, $\frac{2}{5}$ and $\frac{1}{10}$ respectively. Given that the person is late, find the probability that they did not travel on foot. $\frac{5}{6}$

Worked example	Your turn
 A bag contains 9 blue balls and 3 red balls. A ball is selected at random from the bag and its colour is recorded. The ball is not replaced. A second ball is selected at random and its colour is recorded. Find the probability that: a) The second ball selected is blue b) Both balls selected are blue, given that the second ball selected is blue. 	 A bag contains 9 blue balls and 3 red balls. A ball is selected at random from the bag and its colour is recorded. The ball is not replaced. A second ball is selected at random and its colour is recorded. Find the probability that: a) The second ball selected is red b) Both balls selected are red, given that the second ball selected is red. a) 1/4 b) 2/11

Worked example	Your turn
red counters. In bag B there are 7 white counters and 3 red counters. A person takes at random one counter from A and one counter from B. Find the probability that the	In bag A there are 5 white and 2 red counters. In bag B there are 3 white counters and 7 red counters. A person takes at random one counter from A and one counter from B. Find the probability that the counters are the same colour $\frac{29}{70}$

Worked example	Your turn
In bag A there are 2 white and 5 red counters. In bag B there are 7 white counters and 3 red counters. A person takes at random one counter from A and one counter from B. Find the probability that the counters are different colours	In bag A there are 5 white and 2 red counters. In bag B there are 3 white counters and 7 red counters. A person takes at random one counter from A and one counter from B. Find the probability that the counters are different colours $\frac{41}{70}$

Worked example	Your turn
A person plays a game of tennis and	A person plays a game of tennis and
then a game of golf.	then a game of golf.
They can only win or lose each game.	They can only win or lose each game.
The probability of winning tennis is 0.3	The probability of winning tennis is 0.6
The probability of winning golf is 0.7	The probability of winning golf is 0.35
The results of each game are	The results of each game are
independent of each other.	independent of each other.
Calculate the probability that the person	Calculate the probability that the person
wins at least one game	wins at least one game
	27

 $\frac{37}{50} = 0.74$

Worked example					Your turn				
The table shows 100 students, who each study one language. Two students are chosen at random.					The table shows 50 students, who each study one language. Two students are chosen at random.				
		French	German				Japanese	Spanish	
	Female	26	30			Female	13	15	

Male

Calculate the probability that the two chosen students study the same language.

10

34

Male

Calculate the probability that the two chosen students study the same language.

64

245

5

17

Worked example	Your turn				
There are two bags with numbered discs as shown. Bag 1 Bag 2 3 (2) 3 (1)	There are two bags with numbered discs as shown. Bag 1 Bag 2 (2 1) $(3 2)$				
A person chooses a disc at random from bag 1. If it is labelled 2, he puts the disc in bag 2. If it is labelled 1, he does not put the disc in bag 2. He then chooses a disc at random from bag 2. He then adds the numbers of the two discs he selected to give his score. Find the probability that his score is 5.	A person chooses a disc at random from bag 1. If it is labelled 1, he puts the disc in bag 2. If it is labelled 2, he does not put the disc in bag 2. He then chooses a disc at random from bag 2. He then adds the numbers of the two discs he selected to give his score. Find the probability that his score is 4. $\frac{23}{80}$				
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