2) Quadratics

2.1) Solving quadratic equations

Chapter CONTENTS

Worked example	Your turn
Solve: $(2x-3)^2 = 4$	Solve: $(3x-5)^2 = 9$ $x = \frac{2}{3}, x = \frac{8}{3}$

Worked example	Your turn
Solve: $x - 8\sqrt{x} + 15 = 0$	Solve: $x - 6\sqrt{x} + 8 = 0$ $x = 4, x = 16$

Worked example	Your turn
Solve: $3x + 2\sqrt{x} - 8 = 0$	Solve: $2x + \sqrt{x} - 1 = 0$ $x = \frac{1}{4}$

Worked example	Your turn
Solve: $\sqrt{x+6} = x-6$	Solve: $\sqrt{x+3} = x-3$ $x = 6$ only

Worked example	Your turn
Solve: $(2x-3)^2 = 4$	Solve: $(3x-5)^2 = 9$ $x = \frac{2}{3}, x = \frac{8}{3}$

Worked example	Your turn
Solve: $2x^2 - 1 = 17$	Solve: $3x^2 + 49 = 7x^2 + 13$
$45 + 2x^2 = 5x^2 - 3$	$x = \pm 3$
$1 - x^2 = -6x^2 + 6$	

Worked example	Your turn
Solve:	Solve:
$2x^2 = 18$	$3x^2 = 48$
	$x = \pm 4$
$(2x)^2 = 4$	$(3x)^2 = 36$
	$x = \pm 2$

Worked example	Your turn
Solve: $2x^2 = \frac{32}{25}$	Solve: $5x^2 = \frac{45}{121}$ $x = \pm\frac{3}{11}$
$3x^2 = \frac{27}{49}$	

Worked example	Your turn
Solve: $4^x - 12(2^x) + 32 = 0$	Solve: $9^x - 10(3^x) + 9 = 0$
	$x = 2, x = 0$
$16^x - 5(4^x) + 4 = 0$	

Worked example	Your turn
Solve: $x^4 - 5x^2 + 4 = 0$	Solve: $x^4 - 17x^2 + 16 = 0$
$x^4 - 13x^2 + 36 = 0$	$x = \pm 4, x = \pm 1$

Worked example	Your turn
Solve: $x^6 - 35x^3 + 215 = 0$	Solve: $x^6 - 9x^3 + 8 = 0$ $x = 2, x = 1$

Worked example	Your turn
Solve $6x^{\frac{2}{3}} + 5x^{\frac{1}{3}} - 4 = 0$	Solve $3y^{\frac{2}{3}} + 2y^{\frac{1}{3}} - 1 = 0$
	$y = \frac{1}{27}, y = -1$

Worked example	Your turn
Solve: $x + \frac{2}{x} = 3$	Solve: $x + \frac{4}{x} = 5$
	$x = 4, x = 1$
$x - \frac{3}{x} = 5$	

Worked example	Your turn
Solve: $\frac{3}{x^2}+\frac{2}{x}=1$	Solve: $\frac{2}{x^2}-\frac{5}{x}=3$
$\frac{2}{x^2}-\frac{7}{x}=3$	$x=\frac{1}{3}, x=-2$

Worked example	Your turn
Solve: $x^3 + 2x^2 - 8x = 0$	Solve: $x^3 - 3x^2 - 10x = 0$ $x = 0, x = 5, x = -2$

Worked example	Your turn
Solve by factorising: $x^2 + 6x + 9 = 0$ $x^2 + 8x + 16 = 0$ $x^2 + 10x + 25 = 0$ $x^2 + 2x + 1 = 0$	Solve by factorising: $x^2 + 12x + 36 = 0$ $x = -6$

Worked example	Your turn
Solve by factorising:	Solve by factorising:
$x^2 - 6x + 9 = 0$	$x^2 - 12x + 36 = 0$
	$x = 6$
$x^2 - 8x + 16 = 0$	
$x^2 - 10x + 25 = 0$	
$x^2 - 2x + 1 = 0$	

Worked example	Your turn
Solve by factorising:	Solve by factorising:
$x^2 + 17x + 16 = 0$	$x^2 + 37x + 36 = 0$ $x = -36, x = -1$
$x^2 + 10x + 16 = 0$	$x^2 + 20x + 36 = 0$ $x = -18, x = -2$
$x^2 + 8x + 16 = 0$	$x^2 + 15x + 36 = 0$ $x = -12, x = -3$
$x^2 - 8x + 16 = 0$	$x^2 + 13x + 36 = 0$ $x = -9, x = -4$
	$x^2 + 12x + 36 = 0$ $x = -6$

Worked example	Your turn
Solve by factorising:	Solve by factorising:
$x^2 + 10x + 9 = 0$	$x^2 + 12x + 11 = 0$
	$x = -11, x = -1$
$x^2 + 10x + 16 = 0$	$x^2 + 12x + 27 = 0$
	$x = -9, x = -3$
$x^2 + 10x + 25 = 0$	$x^2 + 12x + 36 = 0$
	$x = -6$
$x^2 + 10x = 0$	$x^2 + 12x = 0$
	$x = 0, x = -12$

Worked example	Your turn
Solve by factorising:	Solve by factorising:
$3x^2 + 10x + 3 = 0$	$5x^2 + 8x + 3 = 0$
	$x = -\frac{3}{5}, x = -1$
$3x^2 + 10x + 8 = 0$	$5x^2 + 16x + 12 = 0$
	$x = -\frac{6}{5}, x = -2$
$3x^2 + 14x + 8 = 0$	$5x^2 + 32x + 12 = 0$
	$x = -\frac{2}{5}, x = -6$

Worked example	Your turn
Solve by factorising: $2x^2 + 8x + 6 = 0$ $3x^2 + 21x + 30 = 0$ $5x^2 + 5x - 30 = 0$	Solve by factorising: $3x^2 + 15x - 42 = 0$ $x = -7, x = 2$

Worked example	Your turn
Solve by factorising: $6 + 5x - x^2 = 0$ $3 - 2x - x^2 = 0$	Solve by factorising: $12 - x - x^2 = 0$ $x = 3, x = -4$

Worked example	Your turn
Solve: $6 + 5x - x^2 = 0$	Solve: $6 - 5x - x^2 = 0$
$-6 + 5x - x^2 = 0$	$x = -6, x = 1$

Worked example	Your turn
Solve with the quadratic formula: $2x^2 + x - 3 = 0$	Solve with the quadratic formula: $5x^2 + 13x - 6 = 0$
	$x = \frac{2}{5}, x = -3$
$3x^2 + x - 10 = 0$	

Worked example	Your turn
Solve with the quadratic formula: $2x^2 + x - 4 = 0$	Solve with the quadratic formula: $5x^2 + 13x - 7 = 0$
	$x = \frac{-13 + \sqrt{309}}{10}, x = \frac{-13 - \sqrt{309}}{10}$
$3x^2 + x - 11 = 0$	

Worked example	Your turn
Solve with the quadratic formula: $x^2 + x - 11 = 0$	Solve with the quadratic formula: $-x^2 + 13x - 7 = 0$
$-2x^2 + x + 3 = 0$	$x = \frac{13 + \sqrt{141}}{2}, x = \frac{13 - \sqrt{141}}{2}$

Worked example	Your turn
The solutions to a quadratic equation are $x = \frac{5 \pm \sqrt{25+24}}{6}$ What is the quadratic equation?	The solutions to a quadratic equation are $x = \frac{6 \pm \sqrt{36+8}}{2}$ What is the quadratic equation? $x^2 - 6x - 2 = 0$

Worked example	Your turn
How many real roots are there to: $x^2 + 6x + 8 = 0$	How many real roots are there to: $x^2 + 8x + 12 = 0$ Two: $x = -6, x = -2$
$x^2 + 6x + 9 = 0$	$x^2 + 8x + 16 = 0$ One: $x = -4$
$x^2 + 6x + 10 = 0$	$x^2 + 8x + 17 = 0$ No real roots

Worked example	Your turn
Solve: $x + \frac{2}{x} = 3$	Solve: $x + \frac{4}{x} = 5$
	$x = 4, x = 1$
$x - \frac{3}{x} = 5$	

Worked example	Your turn
Solve: $\frac{3}{x^2}+\frac{2}{x}=1$	Solve: $\frac{2}{x^2}-\frac{5}{x}=3$
$\frac{2}{x^2}-\frac{7}{x}=3$	$x=\frac{1}{3}, x=-2$

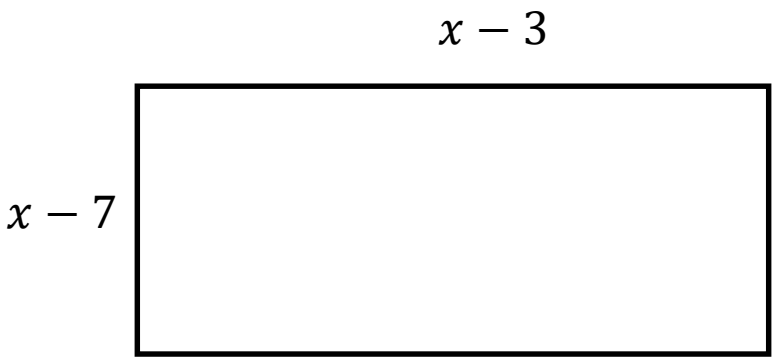
Worked example	Your turn
Solve by completing the square: $x^2 + 8x + 3 = 0$	Solve by completing the square: $x^2 + 6x + 4 = 0$
	$x = -3 + \sqrt{5}, x = -3 - \sqrt{5}$
$x^2 + 10x - 4 = 0$	

Worked example	Your turn
Solve by completing the square: $2x^2 - 8x + 3 = 0$	Solve by completing the square: $5x^2 - 6x - 2 = 0$ $x = \frac{3 + \sqrt{19}}{5}, x = \frac{3 - \sqrt{19}}{5}$
$3x^2 - 10x - 4 = 0$	

Worked example	Your turn
Solve using three methods: $x^2 + 6x + 8 = 0$ $x^2 + 6x + 8 = 0$ $x^2 + 6x + 8 = 0$	Solve using three methods: $x^2 + 6x + 5 = 0$ $x = -5, x = -1$

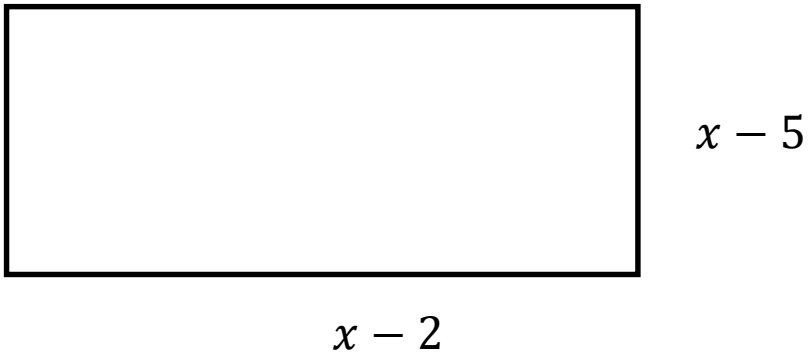
Worked example

The area of the rectangle is equal to 32 square units. Find x

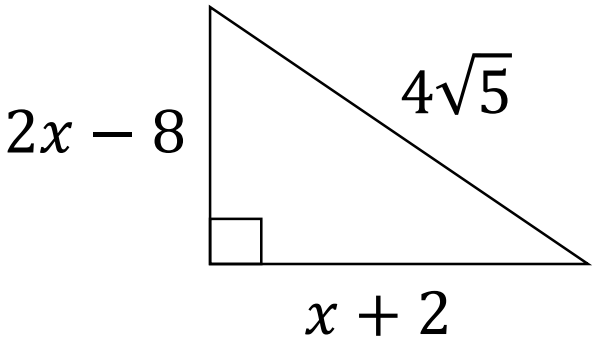
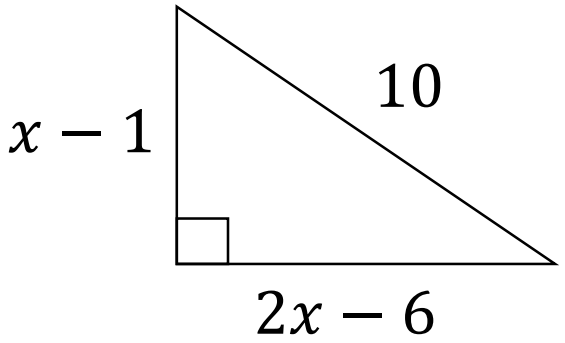


Your turn

The area of the rectangle is equal to 28 square units. Find x



$x = 9$

Worked example	Your turn
The area of the triangle is equal to 16 square units. Find x	The area of the triangle is equal to 24 square units. Find x
	
	$x = 7$

Worked example	Your turn
Two numbers have a difference of 3 and a product of 88. Find the two numbers.	Two numbers have a difference of 4 and a product of 12. Find the two numbers. $x = 6, y = 2$ $x = -2, y = -6$
Two numbers have a difference of 5 and a product of 14. Find the two numbers.	

2.2) Completing the square

Chapter CONTENTS

Worked example	Your turn
Complete the square for: $x^2 + 4x$ $x^2 - 6x$ $x^2 + 8x - 7$	Complete the square for: $x^2 - 10x + 3$ $(x - 5)^2 - 22$

Worked example	Your turn
Complete the square for: $x^2 + 6x + 5$	Complete the square for: $x^2 + 10x + 1$ $(x + 5)^2 - 24$
$x^2 + 8x + 3$	

Worked example	Your turn
Complete the square for: $x^2 - 6x + 5$	Complete the square for: $x^2 - 10x + 1$ $(x - 5)^2 - 24$
$x^2 - 8x - 3$	

Worked example	Your turn
Complete the square for: $x^2 + 9x + 5$ $x^2 + 7x + 3$	Complete the square for: $x^2 + 5x + 1$ $\left(x + \frac{5}{2}\right)^2 - \frac{21}{4}$

Worked example	Your turn
Complete the square for: $x^2 - 5x - 3$	Complete the square for: $x^2 - 3x - 2$
$x^2 - x + 2$	$\left(x - \frac{3}{2}\right)^2 - \frac{17}{4}$

Worked example	Your turn
Complete the square for: $2x^2 + 12x + 1$	Complete the square for: $5x^2 + 40x + 3$ $5(x + 4)^2 - 77$
$3x^2 + 12x + 2$	

Worked example	Your turn
Complete the square for: $2x^2 + 5x + 1$ $3x^2 + 7x + 2$	Complete the square for: $5x^2 + 9x + 3$ $5\left(x + \frac{9}{10}\right)^2 - \frac{21}{20}$

Worked example	Your turn
Complete the square for: $2x^2 - 5x + 3$	Complete the square for: $3x^2 - 7x + 2$ $3\left(x - \frac{7}{6}\right)^2 - \frac{25}{12}$
$5x^2 - 3x + 1$	

Worked example	Your turn
Express in the form $a(x+b)^2+c$: $2x^2-5x+3$	Express in the form $a(x+b)^2+c$: $3x^2-7x+2$ $3\left(x-\frac{7}{6}\right)^2-\frac{25}{12}$

Worked example	Your turn
Complete the square for: $3 + 5x - x^2$	Complete the square for: $5 - 7x - x^2$
	$-\left(x + \frac{7}{2}\right)^2 + \frac{69}{4}$
$2 - 3x - x^2$	

Worked example	Your turn
Complete the square for: $3 + 5x - 2x^2$	Complete the square for: $5 - 7x - 3x^2$
	$-3\left(x + \frac{7}{6}\right)^2 + \frac{109}{12}$
$2 - 3x - 5x^2$	

Worked example	Your turn
Solve by completing the square: $x^2 + 8x + 3 = 0$	Solve by completing the square: $x^2 + 6x + 4 = 0$
	$x = -3 + \sqrt{5}, x = -3 - \sqrt{5}$
$x^2 + 10x - 4 = 0$	

Worked example	Your turn
Solve by completing the square: $2x^2 - 8x + 3 = 0$	Solve by completing the square: $5x^2 - 6x - 2 = 0$ $x = \frac{3 + \sqrt{19}}{5}, x = \frac{3 - \sqrt{19}}{5}$
$3x^2 - 10x - 4 = 0$	

Worked example	Your turn
Solve using three methods: $x^2 + 6x + 8 = 0$ $x^2 + 6x + 8 = 0$ $x^2 + 6x + 8 = 0$	Solve using three methods: $x^2 + 6x + 5 = 0$ $x = -5, x = -1$

Worked example	Your turn
By completing the square, explain why the curve $y = 2x^2 - 8x + 9$ does not intersect the x-axis	By completing the square, explain why the curve $y = 2x^2 - 20x + 51$ does not intersect the x-axis $y = 2(x - 5)^2 + 1$ Turning point at $(5, 1)$

Worked example	Your turn
A sequence has the nth term $n^2 - 6n + 10$. By completing the square, show that every term is positive.	A sequence has the nth term $n^2 - 10n + 27$. By completing the square, show that every term is positive. $n^2 - 10n + 27 = (n-5)^2 + 2$ $k^2 \geq 0$ $(n-5)^2 \geq 0$ $(n-5)^2 + 2 \geq 2$

2.3) Functions

Chapter CONTENTS

Worked example	Your turn
If $f(x) = x^2 + 3$, evaluate: $f(4)$ $f(-2)$	If $h(x) = x^2 + 4$, evaluate: $h(5)$ 29
If $g(x) = x^3 - 5$, evaluate: $g(4)$ $g(-2)$	$h(-2)$ 8

Worked example

$$f(x) = 4x - 8, x \in \mathbb{R}$$
$$g(x) = x^2 - 4, x \in \mathbb{R}$$

Find:

a) $f(-2)$

b) $g(0)$

c) The value of x for which $f(x) = g(x)$

Your turn

$$f(x) = 2x - 10, x \in \mathbb{R}$$
$$g(x) = x^2 - 9, x \in \mathbb{R}$$

Find:

a) $f(5)$

b) $g(10)$

c) The value of x for which $f(x) = g(x)$

a) 0

b) 91

c) $x = 1$

Worked example

Determine the minimum/maximum value of the function and state the value of x for which this minimum occurs:

$$f(x) = x^2 + 8x + 17$$

$$g(x) = x^2 - 8x + 17$$

$$h(x) = 17 - x^2$$

$$i(x) = 17 - 8x - x^2$$

Your turn

Determine the minimum/maximum value of the function and state the value of x for which this minimum occurs:

$$f(x) = x^2 + 6x + 10$$

Minimum of 19 when $x = -3$

$$g(x) = x^2 - 6x + 10$$

Minimum of 1 when $x = 3$

$$h(x) = 10 - x^2$$

Maximum of 10 when $x = 0$

$$i(x) = 10 - 6x - x^2$$

Maximum of 19 when $x = -3$

Worked example

Find the roots of the function:

$$f(x) = x^4 + x^2 - 6$$

Your turn

Find the roots of the function:

$$f(x) = x^4 - x^2 - 6$$

$$x = \pm\sqrt{3}$$

Worked example

Find the roots of the function:

$$f(x) = x^6 - 7x^3 - 8$$

Your turn

Find the roots of the function:

$$f(x) = x^6 + 7x^3 - 8$$

$$x = -2, x = 1$$

Worked example

$$f(x) = 4x - 8, x \in \mathbb{R}$$
$$g(x) = x^2 - 4, x \in \mathbb{R}$$

Find:

a) $f(-2)$

b) $g(0)$

c) The value of x for which $f(x) = g(x)$

Your turn

$$f(x) = 2x - 10, x \in \mathbb{R}$$
$$g(x) = x^2 - 9, x \in \mathbb{R}$$

Find:

a) $f(5)$

b) $g(10)$

c) The value of x for which $f(x) = g(x)$

a) 0

b) 91

c) $x = 1$

Worked example

$$f(x) = 5x + 4$$

Find:

$f(x + 3)$

$f(x - 4)$

$f(5x)$

$f(\frac{1}{6}x)$

Your turn

$$g(x) = 3x + 2$$

Find:

$g(x + 2)$

$3x + 8$

$g(x - 2)$

$3x + 4$

$g(2x)$

$6x + 2$

$g(\frac{1}{2}x)$

$\frac{3}{2}x + 2$

Worked example	Your turn
$f(x) = 3x^2 - 2$	$g(x) = 5x^2 + 3$
$f(x+2)$	$g(x+4)$
	$5(x+4)^2 + 3$ $= 5x^2 + 40x + 83$
$f(x-2)$	$f(x-4)$
	$5(x-4)^2 + 3$ $= 5x^2 - 40x + 83$
$f(2x)$	$g(3x)$
	$5(3x)^2+3$ $= 45x^2 + 3$
$f(\frac{1}{2}x)$	$g(\frac{1}{3}x)$
	$5\left(\frac{1}{3}x\right)^2 + 3$ $= \frac{5}{9}x^2 + 3$

Worked example

$f(x) = 3x^2 - 5x - 2$

$f(x+2)$

$f(x-2)$

$f(2x)$

$f(\frac{1}{2}x)$

Your turn

$g(x) = 5x^2 - 2x + 3$

$g(x+4)$

$5(x+4)^2 - 2(x+4) + 3$
$= 5x^2 + 38x + 75$

$f(x-4)$

$5(x-4)^2 - 2(x-4) + 3$
$= 5x^2 - 42x + 91$

$g(3x)$

$5(3x)^2 - 2(3x) + 3$
$= 45x^2 - 6x + 3$

$g(\frac{1}{3}x)$

$5\left(\frac{1}{3}x\right)^2 - 2\left(\frac{1}{3}x\right) + 3$
$= \frac{5}{9}x^2 - \frac{2}{3}x + 3$

2.4) Quadratic graphs

Chapter CONTENTS

Worked example	Your turn
Write down the line of symmetry of: $y = x^2 + 4x - 5$	Write down the line of symmetry of: $y = x^2 + 8x - 17$
	$x = -4$
$y = x^2 - 6x + 10$	

Worked example

Write down the line of symmetry of:

$$y = 12x - 2x^2 - 5$$

$$y = 12x - 3x^2 + 5$$

Your turn

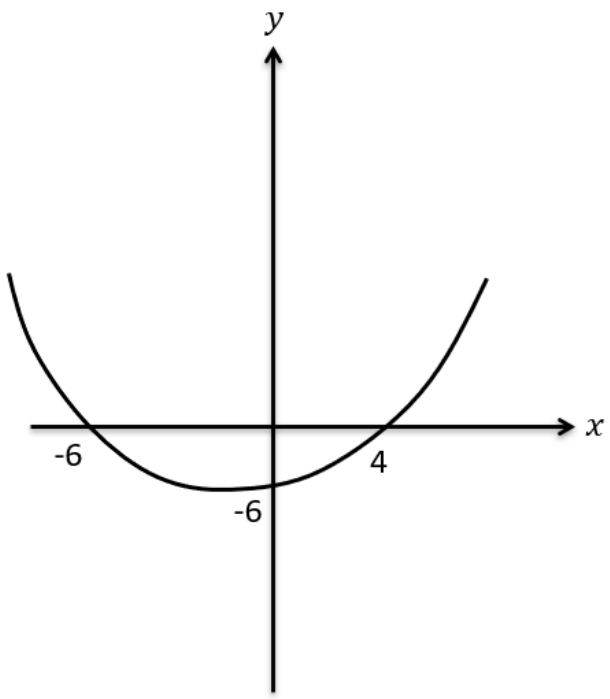
Write down the line of symmetry of:

$$y = 4x - 2x^2 - 3$$

$$x = 1$$

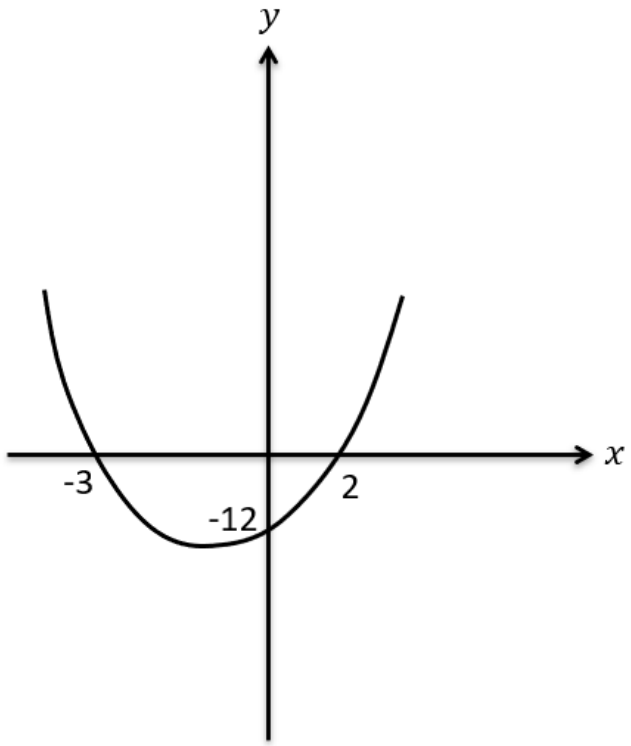
Worked example

Determine the equation of this quadratic graph, in the form $y = ax^2 + bx + c$.



Your turn

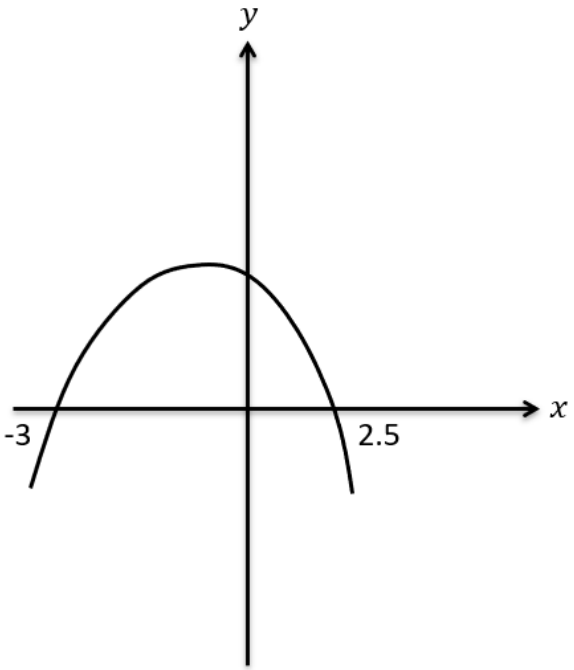
Determine the equation of this quadratic graph, in the form $y = ax^2 + bx + c$.



$y = 2x^2 + 2x - 12$

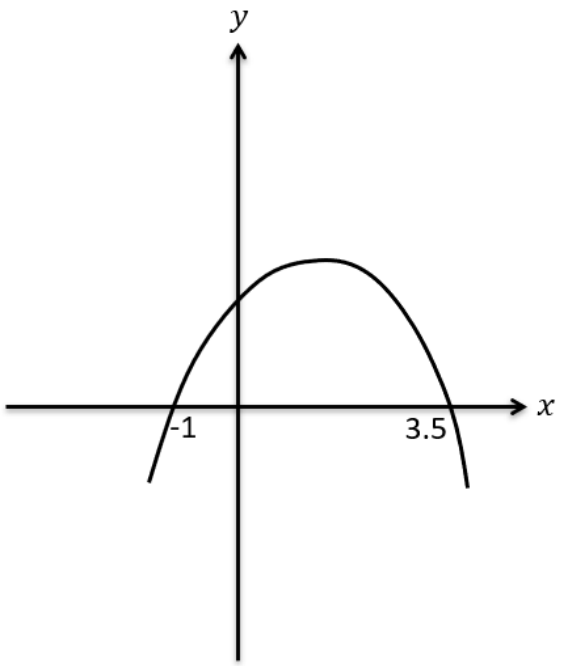
Worked example

Determine the equation of this quadratic graph, in the form $y = ax^2 + bx + c$, where a, b, c are integers.



Your turn

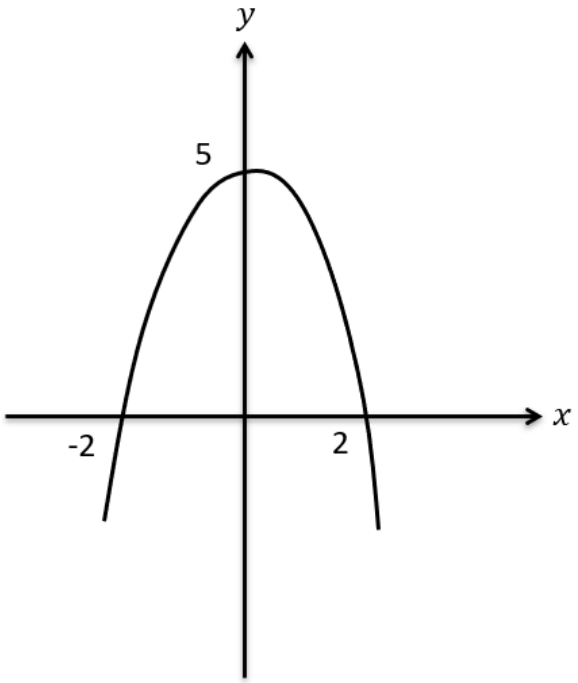
Determine the equation of this quadratic graph, in the form $y = ax^2 + bx + c$, where a, b, c are integers.



$y = -2x^2 + 5x + 7$

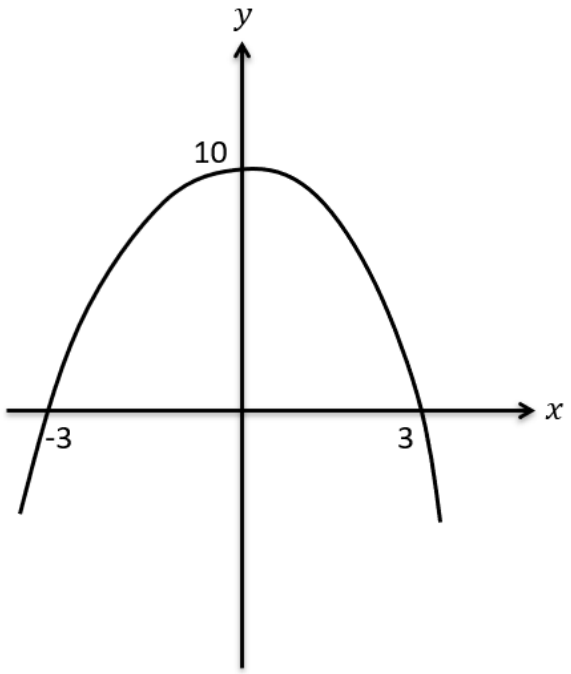
Worked example

Determine the equation of this quadratic graph, in the form $y = ax^2 + bx + c$, where a, b, c are integers.



Your turn

Determine the equation of this quadratic graph, in the form $y = ax^2 + bx + c$, where a, b, c are integers.



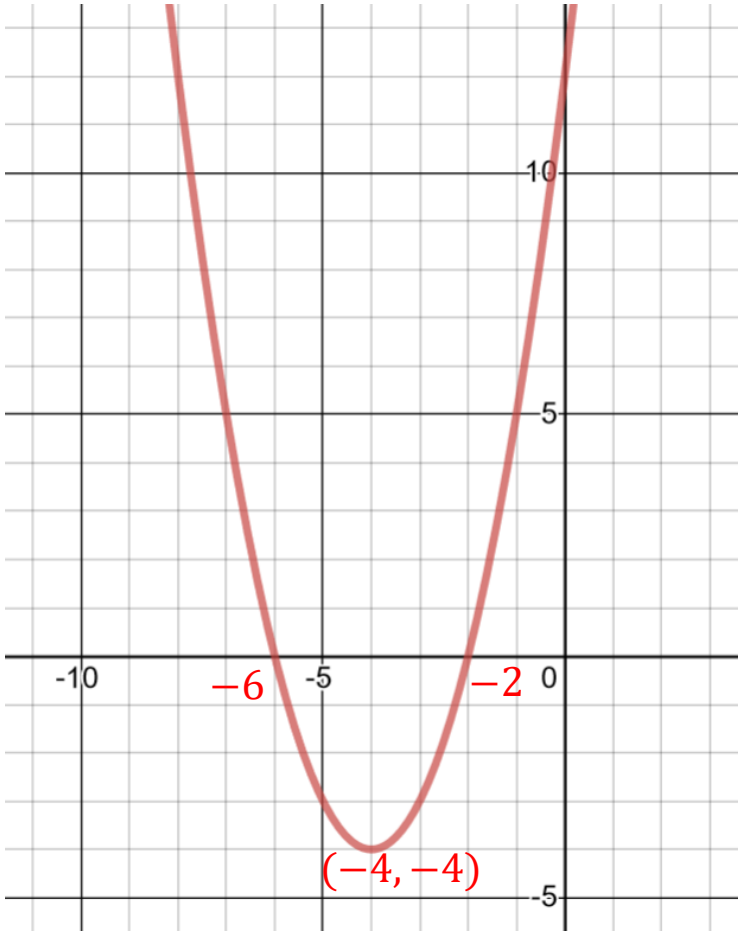
$$y = \frac{10}{9}(x + 3)(3 - x)$$

Worked example	Your turn
The graph of $y = ax^2 + bx + c$ has a minimum at $(3, -5)$ and passes through $(4, 0)$. Find the values of a, b and c	The graph of $y = ax^2 + bx + c$ has a minimum at $(7, -2)$ and passes through $(8, 0)$. Find the values of a, b and c $a = 2, b = -28, c = 96$

Worked example	Your turn
Find the coordinates of the turning point of: $y = x^2 + 6x - 5$ $y = x^2 - 8x + 3$	Find the coordinates of the turning point of: $y = x^2 + 8x - 2$ $(-4, -18)$

Worked example	Your turn
Find the coordinates of the turning point of: $y = 2x^2 + 6x - 5$	Find the coordinates of the turning point of: $y = 2x^2 + 10x - 3$ $(-\frac{5}{2}, -\frac{31}{2})$
$y = 2x^2 - 8x + 3$	

Worked example	Your turn
Sketch $y = x^2 + 6x + 8$, labelling the intercepts with the axes and the turning points.	Sketch $y = x^2 + 8x + 12$, labelling the intercepts with the axes and the turning points.



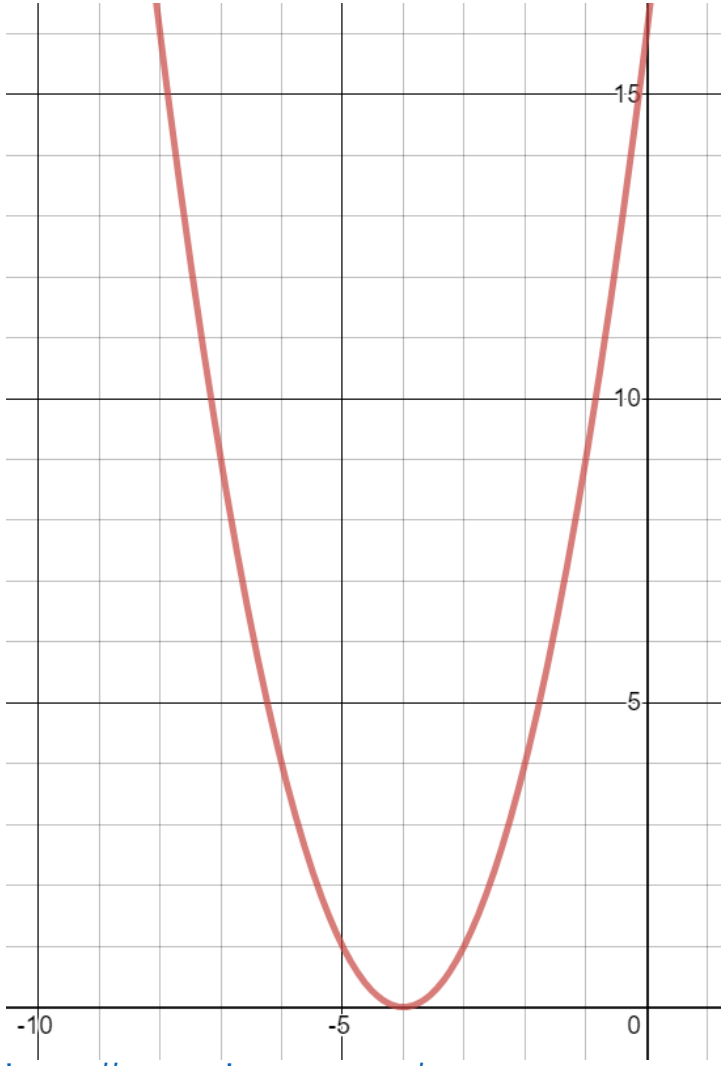
Graph used with permission from DESMOS: https://www.desmos.com/

Worked example

Sketch $y = x^2 + 6x + 9$, labelling the intercepts with the axes and the turning points.

Your turn

Sketch $y = x^2 + 8x + 16$, labelling the intercepts with the axes and the turning points.



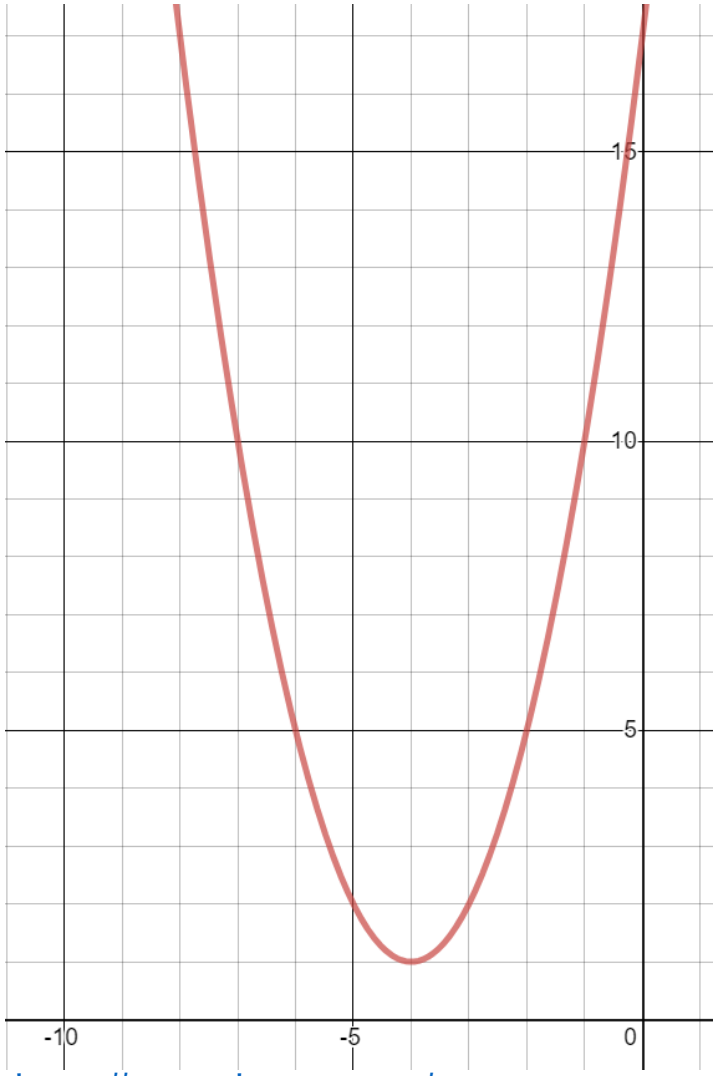
Graph used with permission from DESMOS: https://www.desmos.com/

Worked example

Sketch $y = x^2 + 6x + 10$, labelling the intercepts with the axes and the turning points.

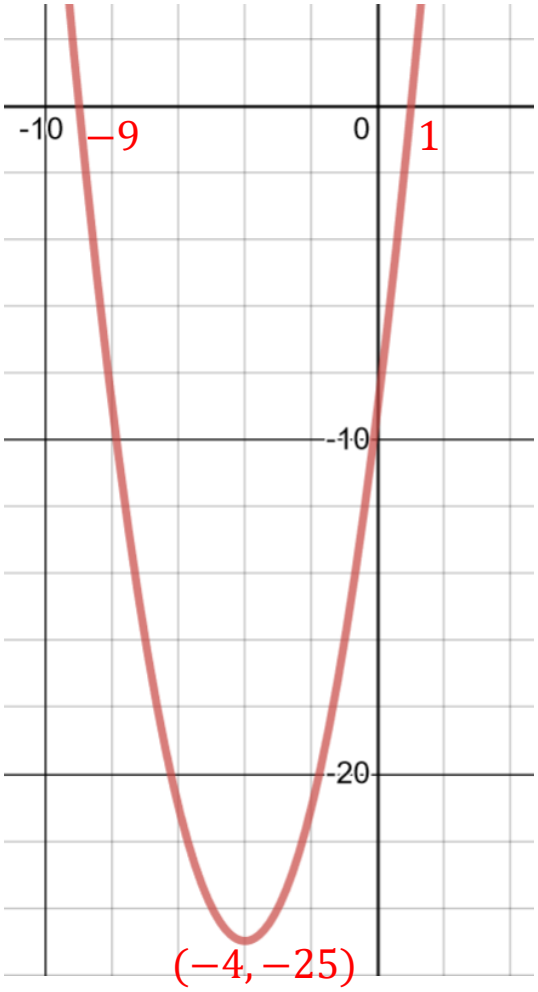
Your turn

Sketch $y = x^2 + 8x + 17$, labelling the intercepts with the axes and the turning points.



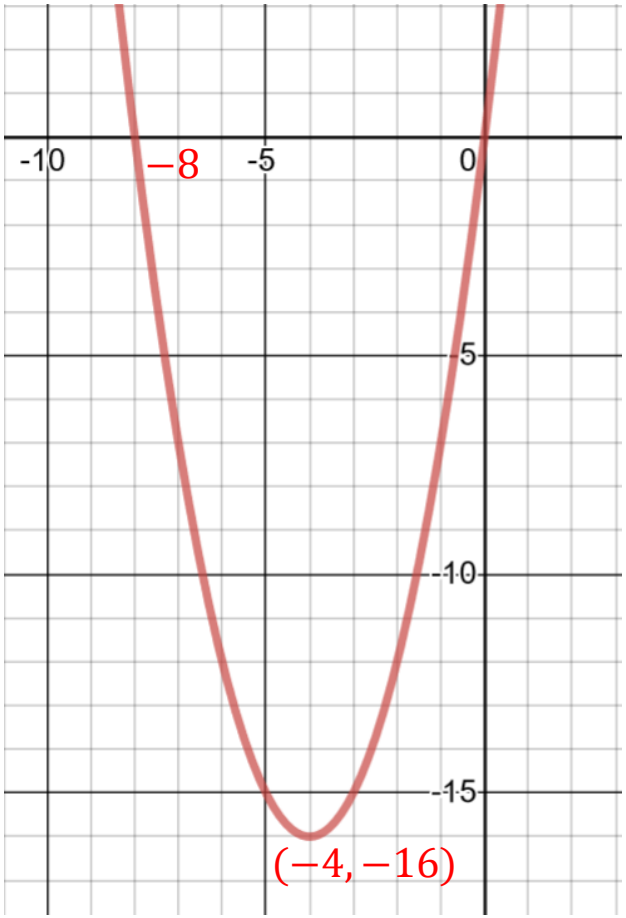
Graph used with permission from DESMOS: https://www.desmos.com/

Worked example	Your turn
Sketch $y = x^2 + 6x - 7$, labelling the intercepts with the axes and the turning points.	Sketch $y = x^2 + 8x - 9$, labelling the intercepts with the axes and the turning points.



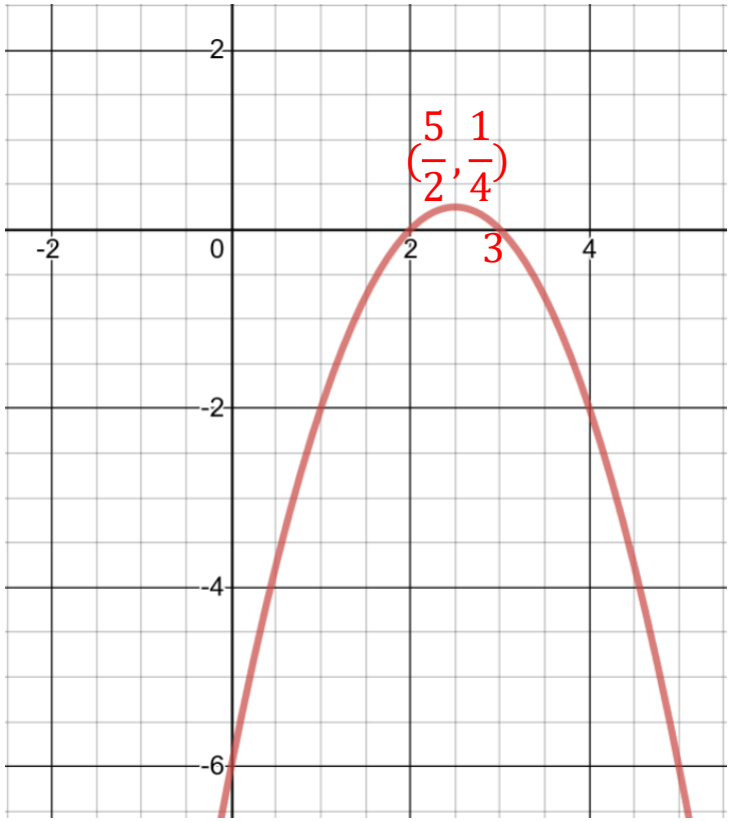
Graph used with permission from DESMOS: https://www.desmos.com/

Worked example	Your turn
Sketch $y = x^2 + 6x$, labelling the intercepts with the axes and the turning points.	Sketch $y = x^2 + 8x$, labelling the intercepts with the axes and the turning points.



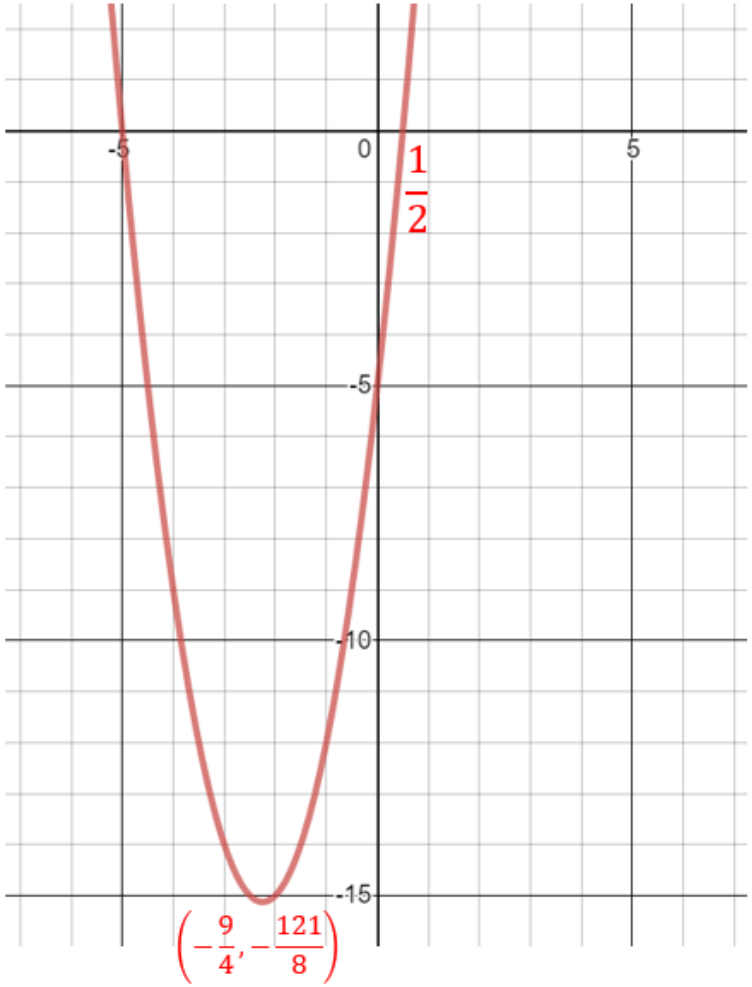
Graph used with permission from DESMOS: https://www.desmos.com/

Worked example	Your turn
Sketch $y = -x^2 + 3x - 2$, labelling the intercepts with the axes and the turning points.	Sketch $y = -x^2 + 5x - 6$, labelling the intercepts with the axes and the turning points.



Graph used with permission from DESMOS: https://www.desmos.com/

Worked example	Your turn
Sketch $y = 2x^2 + 5x - 3$, labelling the intercepts with the axes and the turning points.	Sketch $y = 2x^2 + 9x - 5$, labelling the intercepts with the axes and the turning points.



Graph used with permission from DESMOS: https://www.desmos.com/

2.5) The discriminant

Chapter CONTENTS

Worked example

How many distinct real solutions do these equations have?

$$x^2 + 6x + 8 = 0$$

$$x^2 + 6x + 9 = 0$$

$$x^2 + 6x + 10 = 0$$

Your turn

How many distinct real solutions do these equations have?

$$x^2 + 8x + 12 = 0$$

2

$$x^2 + 8x + 16 = 0$$

1 (equal roots)

$$x^2 + 8x + 17 = 0$$

0

Worked example	Your turn
Find the value of the discriminant: $x^2 + 5x + 6 = 0$	Find the value of the discriminant: $x^2 + 3x + 2 = 0$ 1
$x^2 - 5x + 6.25 = 0$	$x^2 - 3x + 2.25 = 0$ 0
$x^2 - 5x + 7 = 0$	$x^2 - 3x + 4 = 0$ -7

Worked example

Find the value of the discriminant:

$$6x^2 - 3x - 2 = 0$$

$$3x^2 - 2x - 6 = 0$$

Your turn

Find the value of the discriminant:

$$2x^2 - 6x - 3 = 0$$

60

Worked example	Your turn
Find the value of the discriminant: $4 + 3x - x^2$	Find the value of the discriminant: $9 - 5x - x^2$ 61
$4 - 3x - 2x^2$	$9 - 5x - 3x^2$ 133
$4 - x^2$	$9 - x^2$ 36

Worked example	Your turn
Find the range of values of k for which $f(x) = x^2 + kx + 25$ has equal roots	Find the range of values of k for which $f(x) = x^2 + kx + 9$ has equal roots $k = \pm 6$

Worked example	Your turn
Find the range of values of k for which $x^2 + 6x + k = 0$ has two distinct real solutions	Find the range of values of k for which $x^2 + 4x + k = 0$ has two distinct real solutions $k < 4$

Worked example

The equation $x^2 + 4px + (11p + 3) = 0$, where p is a positive constant, has equal roots.

a) Find the value of p

b) For this value of p solve the equation

Your turn

The equation $x^2 + 2px + (3p + 4) = 0$, where p is a positive constant, has equal roots.

a) Find the value of p

b) For this value of p solve the equation

a) $p = 4$

b) $x = -4$

Worked example

$$x^2 + 3kx + (6k + 12) = 0$$

where k is a negative constant.

Given that this equation has equal roots, determine the value of k.

Your turn

$$x^2 + 5kx + (10k + 5) = 0$$

where k is a negative constant.

Given that this equation has equal roots, determine the value of k.

$$k = -\frac{2}{5}$$

Worked example

Find the range of values of k for which $5x^2 - 3x + k = 0$ has no real solutions.

Your turn

Find the range of values of k for which $3x^2 - 5x + k = 0$ has no real solutions.

$$k > \frac{25}{12}$$

Worked example

Prove that the function

$$f(x) = 4x^2 + (k+8)x - k$$

has two distinct real roots for all values of k

Your turn

Prove that the function

$$f(x) = 3x^2 + (k+6)x + k$$

has two distinct real roots for all values of k

Proof

2.6) Modelling with quadratics

Chapter CONTENTS

Worked example

A spear is thrown over level ground from the top of a tower.
The height, in metres, of the spear above the ground after t seconds is modelled by the function: $h(t) = 1.65 + 24.5t - 4.9t^2, \quad t \geq 0$

a) Interpret the meaning of the constant term 12.25 in the model.
b) After how many seconds does the spear hit the ground?
c) Write $h(t)$ in the form $A - B(t - C)^2$, where A, B and C are constants to be found.
d) Using your answer to part c or otherwise, find the maximum height of the spear above the ground, and the time at which this maximum height is reached?

Your turn

A spear is thrown over level ground from the top of a tower.
The height, in metres, of the spear above the ground after t seconds is modelled by the function: $h(t) = 12.25 + 14.7t - 4.9t^2, \quad t \geq 0$

a) Interpret the meaning of the constant term 12.25 in the model.
b) After how many seconds does the spear hit the ground?
c) Write $h(t)$ in the form $A - B(t - C)^2$, where A, B and C are constants to be found.
d) Using your answer to part c or otherwise, find the maximum height of the spear above the ground, and the time at which this maximum height is reached?

a) The height of the tower is $12.25\ m$
b) 3.68 seconds (3 sf)
c) $h(t) = 23.275 - 4.9(t - 1.5)^2$
d) Maximum height $= 23.275\ m$ at $t = 1.5$ s