

## 1.4) Negative and fractional indices

## Worked example

Prove that:

$$x^{\frac{1}{2}} = \sqrt{x}$$

$$x^{\frac{1}{3}} = \sqrt[3]{x}$$

## Your turn

Prove that:

$$x^{\frac{1}{4}} = \sqrt[4]{x}$$

**Proof**

## Worked example

Evaluate:

$$64^{-\frac{2}{3}}$$

$$81^{-\frac{5}{4}}$$

## Your turn

Evaluate:

$$64^{-\frac{3}{2}}$$

$$\frac{1}{512}$$

## Worked example

Given that  $y = \frac{1}{125}x^3$  express the following in the form  $kx^n$  where  $k$  and  $n$  are constants

- a)  $y^{\frac{1}{3}}$
- b)  $5y^{-2}$

## Your turn

Given that  $y = \frac{1}{16}x^2$  express the following in the form  $kx^n$  where  $k$  and  $n$  are constants:

- a)  $y^{\frac{1}{2}}$
- b)  $4y^{-1}$
- a)  $\frac{x}{4}$
- b)  $64x^{-2}$

## Worked example

If  $125\sqrt{5} = 5^k$ , determine the value of  $k$ .

## Your turn

If  $9\sqrt{3} = 3^k$ , determine the value of  $k$ .

$$k = \frac{5}{2}$$

## Worked example

Given that

$$\left(3^{\frac{1}{4}}\right)^n = \frac{3^x}{81^y}$$

Express  $n$  in terms of  $x$  and  $y$ .

## Your turn

Given that

$$\left(2^{\frac{1}{2}}\right)^n = \frac{2^x}{8^y}$$

Express  $n$  in terms of  $x$  and  $y$ .

$$n = 2x - 6y$$

## Worked example

Given that  $y = 3^x$ , express the following in terms of  $y$ .

$$\frac{1}{27^{5x-2}}$$

Write your expression in its simplest form.

## Your turn

Given that  $y = 2^x$ , express the following in terms of  $y$ .

$$\frac{1}{4^{2x-3}}$$

Write your expression in its simplest form.

$$\frac{64}{y^4}$$

## Worked example

Simplify:

$$(16x^2y^6z^4)^{\frac{1}{2}}$$

$$(27x^4y^6z)^{\frac{1}{3}}$$

## Your turn

Simplify:

$$(9a^3b^6c^2d)^{\frac{1}{2}}$$

$$3a^{\frac{3}{2}}b^3cd^{\frac{1}{2}}$$



## Worked example

Convert to fraction form:

$$9^{-1}$$

$$5^{-2}$$

$$2^{-3}$$

$$x^{-5}$$

$$3y^{-4}$$

## Your turn

Convert to fraction form:

$$5b^{-2}$$

$$\frac{5}{b^2}$$

## Worked example

Convert to index form:

$$\frac{1}{2}$$

$$\frac{1}{100}$$

$$\frac{3}{x}$$

$$\frac{9}{x^2}$$

$$\frac{15}{\sqrt{x}}$$

## Your turn

Convert to index form:

$$\frac{7}{x^3}$$

$$7x^{-3}$$

$$\frac{12}{\sqrt[3]{x}}$$

$$12x^{-\frac{1}{3}}$$

## Worked example

## Your turn

Evaluate:

$$2^4$$

$$3^5$$

$$4^0$$

$$5^{-1}$$

$$6^{-2}$$

$$7^{-3}$$

$$9^{\frac{1}{2}}$$

$$8^{\frac{1}{3}}$$

$$16^{-\frac{1}{2}}$$

$$125^{-\frac{1}{3}}$$

$$25^{\frac{3}{2}}$$

$$64^{-\frac{2}{3}}$$

Evaluate:

$$36^{-\frac{1}{2}}$$

$$\frac{1}{6}$$

$$123456789^0$$

$$1$$

$$27^{-\frac{4}{3}}$$

$$\frac{1}{81}$$

## Worked example

Write  $\frac{1}{243}$  as  $3^n$

Write 0.04 as  $5^n$

## Your turn

Write 0.125 as  $2^n$

$$2^{-3}$$

## Worked example

Evaluate:

$$3^{-4}$$

$$2^{-5}$$

## Your turn

Evaluate:

$$5^{-3}$$

$$\frac{1}{125}$$

## Worked example

Write as a fraction:

$$x^{-5}$$

$$y^{-4}$$

## Your turn

Write as a fraction:

$$z^{-3}$$

$$\frac{1}{z^3}$$

## Worked example

Write as a fraction:

$$3x^{-5}$$

$$5y^{-4}$$

## Your turn

Write as a fraction:

$$2z^{-3}$$

$$\frac{2}{z^3}$$

## Worked example

Simplify the following:

$$(r^{-3})^4$$

$$(s^5)^{-2}$$

## Your turn

Simplify the following:

$$(t^{-4})^5$$

$$t^{-20}$$



## Worked example

Simplify the following:

$$(2r^{-3})^4$$

$$(3s^{-4})^{-2}$$

$$(5t^{-2})^3$$

## Your turn

Simplify the following:

$$(3u^{-4})^{-3}$$

$$\frac{u^{12}}{27}$$

## Worked example

Simplify the following:

$$(2r^{-3}s^4)^5$$

$$(3t^{-4}u^5)^{-2}$$

## Your turn

Simplify the following:

$$(5x^3y^{-2})^{-3}$$

$$\frac{y^6}{125x^9}$$

## Worked example

Simplify the following:

$$\frac{15x^7}{3x^{-4}}$$

$$\frac{56y^{-8}}{8y^{-7}}$$

## Your turn

Simplify the following:

$$\frac{42x^{-5}}{6x^{-3}}$$

$$7x^{-2} = \frac{7}{x^2}$$

## Worked example

Simplify the following:

$$\left(\frac{a^2 b^{-3}}{c^4}\right)^5$$

## Your turn

Simplify the following:

$$\left(\frac{a^{-4} b^3}{c^2}\right)^3$$
$$\frac{b^9}{a^{12} c^6}$$

## Worked example

Simplify the following:

$$\left(3a^{\frac{4}{5}}\right)^5$$

## Your turn

Simplify the following:

$$\left(4b^{\frac{2}{3}}\right)^3$$

$$64b^2$$

## Worked example

## Your turn

Evaluate:

$$64^{\frac{1}{2}}$$

$$64^{\frac{1}{3}}$$

$$64^{-\frac{1}{2}}$$

Simplify the following:

$$125^{-\frac{1}{3}}$$

$$\frac{1}{5}$$

## Worked example

## Your turn

Evaluate:

$$64^{\frac{3}{2}}$$

$$64^{\frac{2}{3}}$$

$$64^{-\frac{3}{2}}$$

Simplify the following:

$$125^{-\frac{2}{3}}$$

$$\frac{1}{125}$$

## Worked example

## Your turn

Evaluate:

$$81^{\frac{1}{2}}$$

$$81^{\frac{1}{4}}$$

$$81^{-\frac{1}{2}}$$

Simplify the following:

$$27^{-\frac{1}{3}}$$

$$\frac{1}{3}$$



## Worked example

## Your turn

Evaluate:

$$81^{\frac{3}{2}}$$

$$81^{\frac{3}{4}}$$

$$81^{-\frac{3}{2}}$$

Simplify the following:

$$27^{-\frac{4}{3}}$$

$$\frac{1}{81}$$

## Worked example

Evaluate:

$$\left(\frac{32}{243}\right)^{-\frac{3}{5}}$$

## Your turn

Simplify the following:

$$\left(\frac{64}{125}\right)^{-\frac{2}{3}}$$

$$\frac{25}{16}$$

## Worked example

Write in index form:

$$\sqrt[3]{25}$$

$$\sqrt{27}$$

## Your turn

Write in index form:

$$\sqrt[4]{32}$$

$$2^{\frac{5}{4}}$$

## Worked example

Write in index form:

$$\frac{1}{\sqrt[3]{25}}$$

$$\frac{1}{\sqrt{27}}$$

## Your turn

Write in index form:

$$\frac{1}{\sqrt[4]{32}}$$

$$2^{-\frac{5}{4}}$$

## Worked example

Simplify:

$$(64a^6)^{\frac{3}{2}}$$

$$(64a^6)^{\frac{2}{3}}$$

## Your turn

Simplify:

$$(27b^6)^{\frac{2}{3}}$$

$$9b^4$$

## Worked example

Evaluate:

$$\left(\sqrt{\frac{3}{2}}\right)^4$$

$$\left(\sqrt{\frac{5}{7}}\right)^4$$

## Your turn

Evaluate:

$$\left(\sqrt{\frac{3}{8}}\right)^4$$

$$\frac{9}{64}$$

## Worked example

Express 243 as a power of 9

Express 32 as a power of 4

## Your turn

Express 125 as a power of 25

$$25^{\frac{3}{2}}$$

## Worked example

Express in index form:

$$\sqrt[3]{x^4}$$

$$\sqrt{x^5}$$

## Your turn

Express in index form:

$$\sqrt[4]{x^7}$$

$$x^{\frac{7}{4}}$$



## Worked example

Express in index form:

$$\frac{1}{x}$$

$$\frac{1}{x^2}$$

$$\frac{1}{x^3}$$

## Your turn

Express in index form:

$$\frac{1}{x^4}$$

$$x^{-4}$$

## Worked example

Express in index form:

$$\frac{1}{\sqrt{x}}$$

$$\frac{1}{\sqrt[3]{x}}$$

$$\frac{1}{\sqrt[5]{x}}$$

## Your turn

Express in index form:

$$\frac{1}{\sqrt[7]{x}}$$

$$x^{-\frac{1}{7}}$$

## Worked example

Express in index form:

$$\frac{2}{x}$$

$$\frac{3}{x^2}$$

$$\frac{5}{x^3}$$

## Your turn

Express in index form:

$$\frac{7}{x^4}$$

$$7x^{-4}$$

## Worked example

Express in index form:

$$\frac{2}{\sqrt{x}}$$

$$\frac{5}{\sqrt[3]{x}}$$

$$\frac{7}{\sqrt[5]{x}}$$

## Your turn

Express in index form:

$$\frac{3}{\sqrt[7]{x}}$$

$$3x^{-\frac{1}{7}}$$

## Worked example

Express in index form:

$$\frac{2}{3\sqrt{x}}$$

$$\frac{5}{7\sqrt[3]{x}}$$

$$\frac{1}{4\sqrt[5]{x}}$$

## Your turn

Express in index form:

$$\frac{2}{5\sqrt[7]{x}}$$

$$\frac{2}{5}x^{-\frac{1}{7}}$$

## Worked example

Simplify fully:

$$\sqrt{a^{\frac{2}{3}} \times a^{\frac{4}{7}}}$$

$$\sqrt{b^{\frac{2}{5}} \times b^{\frac{6}{7}}}$$

## Your turn

Simplify fully:

$$\sqrt{a^{\frac{3}{4}} \times a^{\frac{3}{5}}}$$
$$a^{\frac{27}{40}}$$

## Worked example

Solve:

$$5^x = 125$$

$$2^x = 16$$

$$3^x = 243$$

## Your turn

Solve:

$$7^x = 343$$

$$x = 3$$

## Worked example

Solve:

$$5^{x-2} = 25$$

$$2^{2x+3} = 32$$

$$3^{1-3x} = 81$$

## Your turn

Solve:

$$2^{2-5x} = 16$$

$$x = -\frac{2}{5}$$



## Worked example

Solve:

$$5 = 25^x$$

$$3 = 27^x$$

$$2 = 16^{x-1}$$

## Your turn

Solve:

$$7 = 343^{x-2}$$

$$x = \frac{7}{3}$$

## Worked example

Solve:

$$5^x = \frac{1}{125}$$

$$3^x = \frac{1}{9}$$

$$2^x = \frac{1}{16}$$

## Your turn

Solve:

$$7^x = \frac{1}{343}$$

$$x = -3$$

## Worked example

Solve:

$$5^{-x} = \frac{1}{125}$$

$$3^{-x} = \frac{1}{9}$$

$$2^{-x} = \frac{1}{16}$$

## Your turn

Solve:

$$7^{-x} = \frac{1}{343}$$

$$x = 3$$

## Worked example

Solve:

$$5^{-x} = 125$$

$$3^{-x} = 9$$

$$2^{-x} = 16$$

## Your turn

Solve:

$$7^{-x} = 343$$

$$x = -3$$

## Worked example

Solve:

$$125^x = \frac{1}{5}$$

$$9^x = \frac{1}{3}$$

$$8^x = \frac{1}{2}$$

## Your turn

Solve:

$$343^x = \frac{1}{7}$$

$$x = -\frac{1}{3}$$

## Worked example

Solve:

$$2^{3x-2} = \frac{1}{64}$$

$$3^{2-5x} = \frac{1}{81}$$

## Your turn

Solve:

$$5^{3-2x} = \frac{1}{25}$$

$$x = \frac{5}{2}$$

## Worked example

Solve:

$$2^x = \sqrt[3]{16}$$

$$3^{x-2} = \sqrt{27}$$

## Your turn

Solve:

$$5^{x+1} = \sqrt[4]{25}$$

$$x = -\frac{1}{2}$$

## Worked example

Solve:

$$2^{3x-1} = \sqrt[4]{32}$$

$$3^{2-5x} = \sqrt{243}$$

## Your turn

Solve:

$$5^{3-2x} = \sqrt[3]{25}$$

$$x = \frac{7}{6}$$



## Worked example

Solve:

$$2^{3x-5} = \frac{16}{\sqrt[5]{64}}$$

## Your turn

Solve:

$$3^{2x-5} = \frac{27}{\sqrt{243}}$$

$$x = \frac{11}{4}$$

## Worked example

Solve:

$$16 \times 2^{3x-1} = \frac{1}{\sqrt{32}}$$

## Your turn

Solve:

$$27 \times 3^{5x-2} = \frac{1}{\sqrt[3]{9}}$$

$$x = -\frac{1}{3}$$

## Worked example

Solve:

$$(16^x)^3 = \frac{1}{8}$$

## Your turn

Solve:

$$(9^x)^5 = \frac{1}{27}$$

$$x = -\frac{3}{10}$$

## Worked example

Solve:

$$\sqrt{2} \times 8^{2x-5} = \frac{1}{16}$$

## Your turn

Solve:

$$\sqrt{3} \times 9^{4x-3} = \frac{1}{27}$$

$$x = \frac{5}{16}$$

## Worked example

Solve:

$$\frac{16^{4x-3}}{8^{5-2x}} = 4$$

## Your turn

Solve:

$$\frac{9^{3-5x}}{27^{4x-3}} = 81$$

$$x = \frac{1}{2}$$

## Worked example

Express  $y$  in terms of  $x$ , given:

$$2^x \times 2^y = 8$$

$$3^x \times 3^y = \sqrt{27}$$

## Your turn

Express  $y$  in terms of  $x$ , given:

$$5^x \times 5^y = 125$$

$$y = 3 - x$$

## Worked example

Express  $y$  in terms of  $x$ , given:

$$2^{3x} \times 8^{5y} = \frac{1}{4}$$

## Your turn

Express  $y$  in terms of  $x$ , given:

$$5^{3x} \times 25^{4y} = \frac{1}{\sqrt{125}}$$

$$y = -\frac{3}{8}x - \frac{3}{16}$$

## Worked example

Express  $y$  in terms of  $x$ , given:

$$\frac{2^{3x}}{2^{5y}} = 2\sqrt{2}$$

$$\frac{3^{4x}}{3^{2y}} = \frac{1}{27}$$

## Your turn

Express  $y$  in terms of  $x$ , given:

$$\frac{5^x}{5^{4y}} = 125\sqrt{5}$$

$$y = \frac{x}{4} - \frac{7}{8}$$



## Worked example

Express  $y$  in terms of  $x$ , given:

$$\frac{2^{3x}}{8^{5y}} = 2\sqrt{2}$$

$$\frac{3^{4x}}{9^{2y}} = \frac{1}{27}$$

## Your turn

Express  $y$  in terms of  $x$ , given:

$$\frac{5^x}{25^{4y}} = 125\sqrt{5}$$

$$y = \frac{x}{8} - \frac{7}{16}$$

## Worked example

Solve  $\left(\frac{1}{2}\right)^x = 8$

Solve  $25^{3-4x} = \frac{1}{125}$

## Your turn

Solve  $\left(\frac{1}{3}\right)^x = 9$

$x = -2$

Solve  $9^{4-3x} = \frac{1}{81}$

$x = 2$

## Worked example

Solve  $x^{\frac{1}{2}} = 5$

Solve  $x^{\frac{1}{3}} = 6$

## Your turn

Solve  $x^{\frac{1}{4}} = 2$

$x = 16$

## Worked example

Solve  $x^{-2} = 25$

Solve  $x^{-3} = 216$

## Your turn

Solve  $x^{-4} = 16$

$$x = \frac{1}{2}$$

## Worked example

Given that  $5^{-n} = 0.4$ , find the value of  $(5^3)^n$

## Your turn

Given that  $3^{-n} = 0.2$ , find the value of  $(3^4)^n$

625

## Worked example

Express  $\frac{(3+\sqrt[3]{x})^2}{x}$  as powers of  $x$ .

## Your turn

Express  $\frac{(2+\sqrt{x})^2}{x}$  as powers of  $x$ .

$$4x^{-1} + 4x^{-\frac{1}{2}} + 1$$

## Worked example

Express  $\frac{(x + \sqrt[3]{x})^2}{4\sqrt[3]{x}}$  as powers of  $x$ .

## Your turn

Express  $\frac{(x + \sqrt{x})^2}{2\sqrt{x}}$  as powers of  $x$ .

$$\frac{1}{2}x^{\frac{3}{2}} + x + \frac{1}{2}x^{\frac{1}{2}}$$