

1.1) Proof by contradiction

Worked example

Prove by deduction that $n^2 - 6n + 10$ is positive for all values of n

Your turn

Prove by deduction that $n^2 - 8n + 17$ is positive for all values of n

Proof

Worked example

Prove by exhaustion that $n^2 - 10 < 0$ for $-3 < n \leq 3$ where n is an integer

Your turn

Prove by exhaustion that $n^2 - 15 < 0$ for $-4 < n \leq 4$ where n is an integer

Proof

Worked example

Find a counter example to disprove the statement that $n^2 - n + 41$ is always a prime number.

Your turn

Find a counter example to disprove the statement that if p is an odd prime, then $p + 2$ is also a prime

$$p = 7, 13, 19, 23, \dots$$

Worked example

Prove by contradiction that there is no greatest even integer.

Your turn

Prove by contradiction that there is no greatest odd integer.

Proof

Worked example

Prove by contradiction that if n^2 is odd, then n must be odd

Your turn

Prove by contradiction that if n^2 is even, then n must be even

Proof

Worked example

Prove by contradiction that $\sqrt[3]{2}$ is an irrational number.

Your turn

Prove by contradiction that $\sqrt{2}$ is an irrational number.

Proof

Worked example

Prove by contradiction that there are no integer solutions to the equation $x^2 - y^2 = 6$

Your turn

Prove by contradiction that there are no integer solutions to the equation $x^2 - y^2 = 2$

Proof

Worked example

Prove by contradiction that there exist no integers for which $20a + 14b = 3$

Your turn

Prove by contradiction that there exist no integers for which $21a + 14b = 1$

Proof

Worked example

Prove by contradiction that there exist no rational solutions to the equation $x^2 - 3 = 0$

Your turn

Prove by contradiction that there exist no rational solutions to the equation $x^2 - 2 = 0$

Proof

Worked example

Prove that if $\frac{a}{b}$ is an irrational number then at least one of a and b is an irrational number

Your turn

Prove that if ab is an irrational number then at least one of a and b is an irrational number

Proof

Worked example

Prove that if $a - b$ is an irrational number then at least one of a and b is an irrational number

Your turn

Prove that if $a + b$ is an irrational number then at least one of a and b is an irrational number

Proof