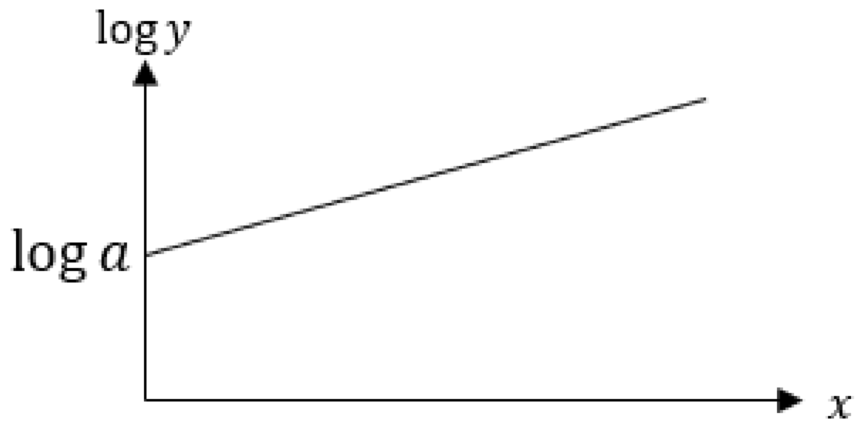
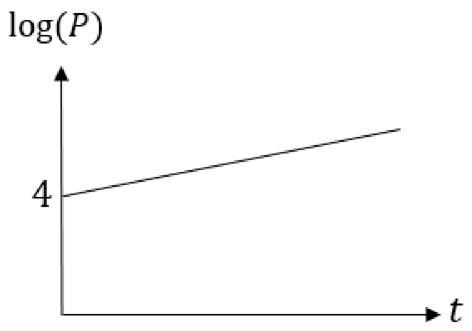
14.8) Logarithms and non-linear data

Worked example	Your turn
Use logarithms to convert the non-linear relationship into a linear form and sketch the resulting straight line. $$y = ax^n$$	Use logarithms to convert the non-linear relationship into a linear form and sketch the resulting straight line. $$y = ab^x$$ $$\log y = (\log b)\, x + \log a$$ 

Worked example

The graph represents the growth of a population of bacteria, P, over t hours.
The graph has a gradient of 0.3 and meets the vertical axis at $(0,4)$.
A scientist suggest that this growth can be modelled by the equation $P = ab^t$, where a and b are constants to be found.

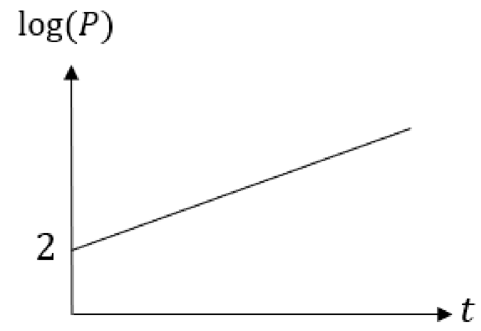
a. Write down an equation for the line.
b. Find the values of a and b, giving them to 3 sf where necessary.
c. Interpret the meaning of the constant a in this model.



Your turn

The graph represents the growth of a population of bacteria, P, over t hours.
The graph has a gradient of 0.6 and meets the vertical axis at $(0,2)$.
A scientist suggest that this growth can be modelled by the equation $P = ab^t$, where a and b are constants to be found.

a. Write down an equation for the line.
b. Find the values of a and b, giving them to 3 sf where necessary.
c. Interpret the meaning of the constant a in this model.



a) $\log P = 0.6t + 2$
b) $a = 100, b = 3.98$ (3 sf)
c) The initial size of the bacteria population was 100

Worked example

The table below gives the rank (by size) and population of a country's largest cities and districts (the capital city is number 1 but has been excluded as an outlier).

City	**A**	**B**	**C**	**D**	**E**
Rank, R	2	3	4	5	6
Population	2 000 000	1 400 000	1 200 000	1 000 000	900 000

The relationship between the rank and population can be modelled by the formula:

$P = aR^n$ where a and n are constants.

a) Draw a table giving values of $\log R$ and $\log P$ to 2dp.

b) Plot a graph of $\log R$ against $\log P$ using the values from your table and draw the line of best fit.

c) Use your graph to estimate the values of a and n to two significant figures.

Your turn

The table below gives the rank (by size) and population of the UK's largest cities and districts in the past (London is number 1 but has been excluded as an outlier).

City	**Birmingham**	**Leeds**	**Glasgow**	**Sheffield**	**Bradford**
Rank, R	2	3	4	5	6
Population	1 000 000	730 000	620 000	530 000	480 000

The relationship between the rank and population can be modelled by the formula:

$P = aR^n$ where a and n are constants.

a) Draw a table giving values of $\log R$ and $\log P$ to 2dp.

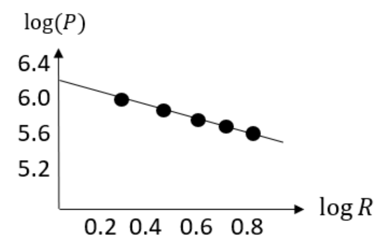
b) Plot a graph of $\log R$ against $\log P$ using the values from your table and draw the line of best fit.

c) Use your graph to estimate the values of a and n to two significant figures.

a)

$\log R$	0.30	0.48	0.60	0.70	0.78
$\log P$	6	5.86	5.79	5.72	5.68

b)



c) $a = 1600000, n = -0.67$ (2 sf)

Worked example

A population is increasing exponentially according to the model $P = ab^t$, where a, b are constants to be found.

The population is recorded as follows:

Years *t* after 2016	1.4	2.6	4.4
Population *P*	4706	7346	14324

a) Draw a table giving values of t and $\log P$ (to 3dp).

b) A line of best fit is drawn for the data in your new table, and it happens to go through the first data point above (where $t = 1.4$) and last (where $t = 4.4$). Determine the equation of this line of best fit.

c) Hence, determine the values of a and b in the model.

d) Estimate the population in 2020

Your turn

A population is increasing exponentially according to the model $P = ab^t$, where a, b are constants to be found.

The population is recorded as follows:

Years *t* after 2015	0.7	1.3	2.2
Population *P*	2353	3673	7162

a) Draw a table giving values of t and $\log P$ (to 3dp).

b) A line of best fit is drawn for the data in your new table, and it happens to go through the first data point above (where $t = 0.7$) and last (where $t = 2.2$). Determine the equation of this line of best fit

c) Hence, determine the values of a and b in the model.

d) Estimate the population in 2020

a)

t	0.7	1.3	2.2
$\log P$	3.372	3.565	3.855

b) $\log P = 0.322t + 3.147$

c) $a = 1403,\ b = 2.099$ (4 sf)

d) 57164