13) Integration

13.1) Integrating x^n

Worked example	Your turn
Find $f(x)$, given that $f'(x) =$ x^2	Find $f(x)$, given that $f'(x) =$ x^5
	$y = \dfrac{1}{6}x^6 + c$
x^3	
x^4	

Worked example	Your turn
Find y, given that $\frac{dy}{dx} = 3x^2$	Find y, given that $\frac{dy}{dx} = -3x^5$
	$y = -\frac{1}{2}x^6 + c$
$-2x^3$	
$5x^4$	

Worked example	Your turn
Find $f(x)$, given that $f'(x) = \sqrt{x}$	Find $f(x)$, given that $f'(x) = \sqrt[5]{x}$
	$y = \dfrac{5}{6}x^{\frac{6}{5}} + c$
$\sqrt[3]{x}$	
$\sqrt[4]{x}$	

Worked example	Your turn
Find y, given that $\dfrac{dy}{dx} =$ $\dfrac{1}{x^2}$	Find y, given that $\dfrac{dy}{dx} =$ $\dfrac{3}{x^4}$ $y = -\dfrac{1}{x^3} + c$
$\dfrac{2}{x^3}$	

Worked example	Your turn
Find $f(x)$, given that $f'(x) =$ $$\frac{3}{4x^2}$$	Find $f(x)$, given that $f'(x) =$ $$\frac{7}{8x^4}$$ $$y = -\frac{7}{24x^3} + c$$
$$\frac{6}{5x^3}$$	

Worked example	Your turn
Find y, given that $\frac{dy}{dx} =$	Find y, given that $\frac{dy}{dx} =$
$\frac{2}{3}\sqrt{x}$	$\frac{3}{5}\sqrt{x}$
	$y = \frac{2}{5}x^{\frac{3}{2}} + c$
	$y = \frac{2}{5}x\sqrt{x} + c$
$\frac{4}{7}\sqrt[3]{x}$	
$\frac{5}{6}\sqrt[4]{x}$	

Worked example	Your turn
Find $f(x)$, given that $f'(x) =$	Find $f(x)$, given that $f'(x) =$
$$\frac{2}{3\sqrt{x}}$$	$$\frac{3}{5\sqrt{x}}$$
	$$y = \frac{6}{5}x^{\frac{1}{2}} + c$$
	$$y = \frac{6}{5}\sqrt{x} + c$$
$$\frac{4}{7\sqrt[3]{x}}$$	
$$\frac{5}{6\sqrt[4]{x}}$$	

Worked example	Your turn
Find y, given that $\dfrac{dy}{dx} = \sqrt{36x^7}$	Find y, given that $\dfrac{dy}{dx} = \sqrt{16x^8}$
	$y = \dfrac{4}{5}x^5 + c$
$\sqrt{25}x^7$	$\sqrt{9}x^8$
	$y = \dfrac{1}{3}x^9 + c$

Worked example	Your turn
Find $f(x)$, given that $f'(x) =$ $$2x^{-\frac{7}{10}}$$	Find $f(x)$, given that $f'(x) =$ $$10x^{-\frac{2}{7}}$$ $$y = 14x^{\frac{5}{7}} + c$$

Worked example	Your turn
Find y, given that $\frac{dy}{dx} = 39x^{\frac{5}{8}}$	Find y, given that $\frac{dy}{dx} = 33x^{\frac{5}{6}}$ $$y = 18x^{\frac{11}{6}} + c$$

Worked example	Your turn
Find $f(x)$, given that $f'(x) = (3x - 2)^2$	Find $f(x)$, given that $f'(x) = (2x - 3)^2$
	$y = \dfrac{4}{3}x^3 - 6x^2 + 9x + c$

13.2) Indefinite integrals

Worked example	Your turn
Find:	Find:
$$\int 10x \ dx$$	$$\int 20x^3 \ dx$$
	$$5x^4 + c$$
$$\int 15x^2 \ dx$$	

Worked example	Your turn
Find: $$\int \left(x^{-\frac{5}{2}} - 3\right) dx$$	Find: $$\int \left(x^{-\frac{3}{2}} + 2\right) dx$$ $$-2x^{-\frac{1}{2}} + 2x + c$$

Worked example	Your turn
Find:	Find:
$$\int (2\theta^6 + 3)\, d\theta$$	$$\int (6t^2 - 1)\, dt$$ $$2t^3 - t + c$$

Worked example	Your turn
Find $\int (rx^4 - 5s)\, dx$ where r and s are constants.	Find $\int (px^3 + q)\, dx$ where p and q are constants.
	$$\frac{1}{4}px^4 + qx + c$$

Worked example	Your turn
Find: $$\int \left(\frac{3}{x^2} - 2\sqrt[3]{x}\right) dx$$	Find: $$\int \left(\frac{2}{x^3} - 3\sqrt{x}\right) dx$$ $$-\frac{1}{x^2} - 2\sqrt{x^3} + c$$

Worked example	Your turn
Find:	Find:
$$\int \left(x^2 \left(x^3 - \frac{4}{x^2} \right) \right) dx$$	$$\int \left(x \left(x^2 + \frac{2}{x} \right) \right) dx$$
	$\frac{1}{4}x^4 + 2x + c$

Worked example	Your turn
Find: $$\int \left((3x)^5 - \frac{\sqrt[3]{x} - 2}{x^4} \right) dx$$	Find: $$\int \left((2x)^2 + \frac{\sqrt{x} + 5}{x^2} \right) dx$$ $$\frac{4}{3}x^3 - \frac{2}{\sqrt{x}} - \frac{5}{x} + c$$

Worked example	Your turn
$\int \left(\dfrac{p}{2x^2} + pq \right) dx = \dfrac{2}{x} + 12 + c$	$\int \left(\dfrac{p}{3x^3} + pq^3 \right) dx = \dfrac{-4}{3x} - 108 + c$
Find the value of p and the value of q	Find the value of p and the value of q

$$p = 4, q = -3$$

13.3) Finding functions

Worked example	Your turn
The curve with equation $y = f(x)$ passes through $(3, 1)$. Given that $f'(x) = 4x^3$, find the equation of the curve.	The curve with equation $y = f(x)$ passes through $(1,3)$. Given that $f'(x) = 3x^2$, find the equation of the curve.
	$y = x^3 + 2$

Worked example	Your turn
The curve with equation $y = f(x)$ passes through $\left(8, \frac{6408}{11}\right)$. Given that $f'(x) = \frac{x^3+4}{\sqrt[3]{x}}$, find the equation of the curve.	The curve with equation $y = f(x)$ passes through $(4, 5)$. Given that $f'(x) = \frac{x^2-2}{\sqrt{x}}$, find the equation of the curve. $$y = \frac{2}{5}x^{\frac{5}{2}} - 4x^{\frac{1}{2}} + \frac{1}{5}$$

13.4) Definite integrals

Worked example	Your turn
Evaluate: $$\int_1^4 2x\ dx$$	Evaluate: $$\int_1^2 3x^2\ dx$$ 7

Worked example	Your turn
Evaluate: $$\int_2^4 5x^4 \, dx$$	Evaluate: $$\int_1^5 4x^3 \, dx$$ 624

Worked example	Your turn
Evaluate: $$\int_0^2 \left(x^{\frac{1}{4}} - 3\right)^2 dx$$	Evaluate: $$\int_0^1 \left(x^{\frac{1}{3}} - 1\right)^2 dx$$ $$\frac{1}{10}$$

Worked example	Your turn
Evaluate: $$\int_{-4}^{4} x^3 - 2\, dx$$	Evaluate: $$\int_{-3}^{3} x^2 + 1\, dx$$ 24

Worked example	Your turn
Given that P is a constant and $$\int_{3}^{7} (4Px + 7)\, dx = 108P^2$$ find the possible values of P	Given that P is a constant and $$\int_{1}^{5} (2Px + 7)\, dx = 4P^2$$ find the possible values of P $P = -1, P = 7$

Worked example	Your turn
Given that $\int_1^k \frac{1}{\sqrt[4]{x}} \, dx = \frac{28}{3}$, calculate the value of k	Given that $\int_1^k \frac{1}{\sqrt[3]{x}} \, dx = \frac{9}{2}$, calculate the value of k
	$k = 8$

13.5) Areas under curves

Worked example	Your turn
Find the area of the finite region bounded by the curve with equation $y = x^3$, the lines with equation $x = 1$ and $x = 4$ and the x-axis.	Find the area of the finite region bounded by the curve with equation $y = x^4$, the lines with equation $x = 3$ and $x = 5$ and the x-axis. $$\dfrac{2882}{5}$$

Worked example	Your turn
Find the area of the finite region between the curve with equation $y = 6 + x - x^2$ and the x-axis.	Find the area of the finite region between the curve with equation $y = 20 - x - x^2$ and the x-axis. $$\dfrac{243}{2}$$

Worked example	Your turn
Find the area of the finite region bounded by the curve with equation $y = x^2(x + 2)$ and the x-axis	Find the area of the finite region bounded by the curve with equation $y = x^2(3 - x)$ and the x-axis

$$\frac{27}{4}$$ |

13.6) Areas under the x-axis

Worked example	Your turn
Find the area of the finite region bounded by the curve with equation $y = x(x-5)$ and the x-axis	Find the area of the finite region bounded by the curve with equation $y = x(x-3)$ and the x-axis
	$$\frac{9}{2}$$

Worked example	Your turn
Find the total area bound between the curve $y = x(x-2)(x-4)$ and the x-axis.	Find the total area bound between the curve $y = x(x-1)(x-2)$ and the x-axis.

$$\frac{1}{2}$$ |

Worked example	Your turn
Find the total area bound between the curve $y = x^3 + 2x^2 - 15x$ and the x-axis.	Find the total area bound between the curve $y = x^3 + 2x^2 - 8x$ and the x-axis. $$\frac{148}{3}$$

13.7) Areas between curves and lines

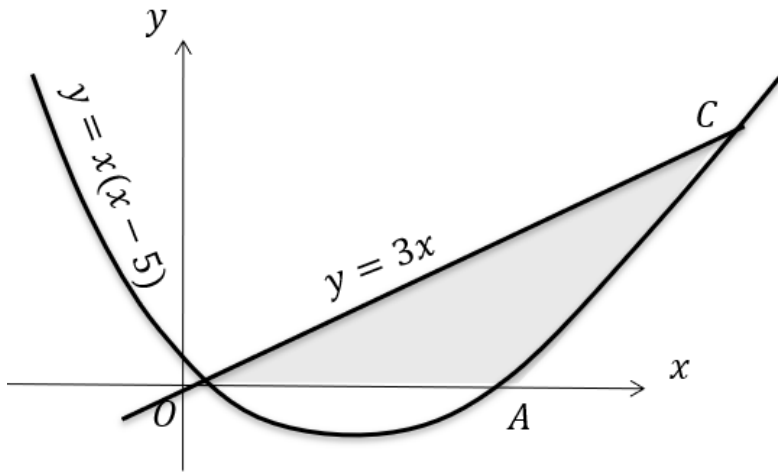| Worked example	Your turn
Determine the area bounded by the curve with equation $y = x(7 - x)$ and the line with equation $y = 2x$	Determine the area bounded by the curve with equation $y = x(4 - x)$ and the line with equation $y = x$ $$\frac{9}{2}$$

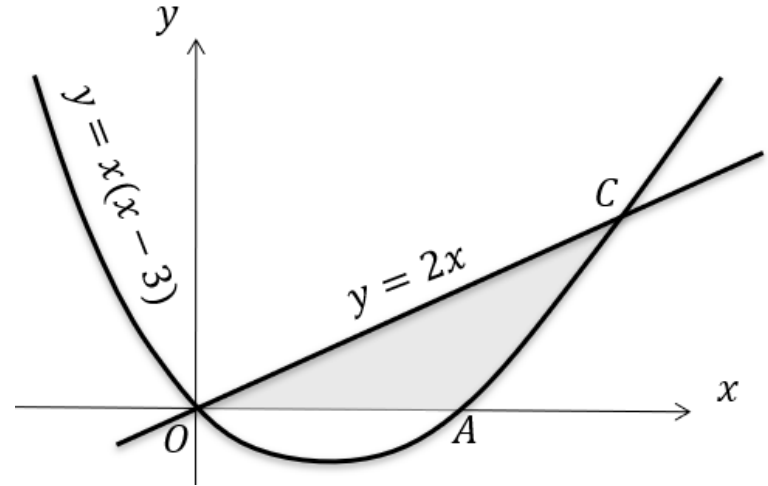| Worked example	Your turn
The diagram shows a sketch of the curve with equation $y = x(x - 5)$ and the line with equation $y = 3x$. Find the area of the shaded region OAC.	The diagram shows a sketch of the curve with equation $y = x(x - 3)$ and the line with equation $y = 2x$. Find the area of the shaded region OAC.

$$\frac{49}{3}$$

Worked example	Your turn
Determine the area bounded by the curve with equation $y = 5x - x^2 - 3$ and the line with equation $y = 5 - x$	Determine the area bounded by the curve with equation $y = 10x - x^2 - 8$ and the line with equation $y = 10 - x$ $$\frac{343}{6}$$