

## 12.11) Modelling with differentiation

## Worked example

Given that the area,  $A \text{ cm}^2$ , of an expanding circle is related to its radius,  $r \text{ cm}$ , by the formula  $A = \pi r^2$ , find the rate of change of area with respect to radius at the instant when the radius is  $10 \text{ cm}$ .

## Your turn

Given that the volume,  $V \text{ cm}^3$ , of an expanding sphere is related to its radius,  $r \text{ cm}$ , by the formula  $V = \frac{4}{3}\pi r^3$ , find the rate of change of volume with respect to radius at the instant when the radius is  $5 \text{ cm}$ .

**$314 \text{ cm}^3 \text{ per cm}$**

## Worked example

A cuboid is to be made with volume  $81 \text{ cm}^3$ .

The cuboid has a rectangular cross-section where the length of the rectangle is equal to twice its width,  $x \text{ cm}$ .

The volume of the cuboid is  $81 \text{ cm}^3$ .

- Show that the total length,  $L$ , of the twelve edges of the cuboid is given by  $L = 12x + \frac{162}{x^2}$
- Given that  $x$  can vary, use differentiation to find the maximum or minimum value of  $L$
- Justify that the value of  $L$  you have found is a minimum

## Your turn

A cuboid is to be made from  $54\text{m}^2$  of sheet metal.

The cuboid has a horizontal base and no top.

The height of the cuboid is  $x$  metres.

Two of the opposite vertical faces are squares.

- Show that the volume,  $V \text{ m}^3$ , of the tank is given by

$$V = 18x - \frac{2}{3}x^3.$$

- Given that  $x$  can vary, use differentiation to find the maximum or minimum value of  $V$ .
- Justify that the value of  $V$  you have found is a maximum

a) Shown

b)  $V = 36$

c)  $\frac{d^2V}{dx^2} = -4x ; x = 3, \frac{d^2V}{dx^2} = -12 < 0$