

12.9) Stationary points

Worked example

Find the least value of

$$f(x) = x^2 + 6x - 9$$

Your turn

Find the least value of

$$f(x) = x^2 - 4x + 9$$

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Worked example

Find the turning point of

$$y = \sqrt[4]{x} - 2x$$

Your turn

Find the turning point of

$$y = \sqrt{x} - x$$

$$\left(\frac{1}{4}, \frac{1}{4}\right)$$

Worked example

Find the coordinates of the turning/stationary point(s) of the curves by differentiation:

$$y = x^2 + 6x - 2$$

$$y = 2x^3 + 6x^2 - 4$$

Your turn

Find the coordinates of the turning/stationary point(s) of the curves by differentiation:

$$y = x^3 + 3x^2 - 4$$

$(-2, 0)$ and $(0, -4)$

Worked example

Find the coordinates of the turning/stationary point(s) of the curves by differentiation:

$$y = \frac{2}{3}x^3 - 3.5x^2 + 3x + 5$$

Your turn

Find the coordinates of the turning/stationary point(s) of the curves by differentiation:

$$y = x^3 + \frac{1}{2}x^2 - 2x + 4$$

$$\left(-1, \frac{11}{2}\right) \text{ and } \left(\frac{2}{3}, \frac{86}{27}\right)$$

Worked example

Find the stationary points on the curve $y = \frac{5}{3}x^3 - 80x$

Your turn

Find the stationary points on the curve $y = x^3 - 12x$

$(-2, 16)$ and $(2, -16)$

Worked example

Find the stationary point on the curve with equation

$y = x^4 - 108x$, and determine whether it is a local maximum, a local minimum or a point of inflection.

Your turn

Find the stationary point on the curve with equation

$y = x^4 - 32x$, and determine whether it is a local maximum, a local minimum or a point of inflection.

$(2, -48)$ Local minimum

Worked example

Find the coordinates of the stationary points on the curve with equation $y = 4x^3 + 30x^2 + 48x - 3$ and use the second derivative to determine their nature

Your turn

Find the coordinates of the stationary points on the curve with equation $y = 2x^3 - 15x^2 + 24x + 6$ and use the second derivative to determine their nature

(1, 17) Local maximum

(4, -10) Local minimum

Worked example

Sketch the graph of $y = \frac{1}{x} + \frac{256}{3}x^3$ labelling the stationary points.

Your turn

Sketch the graph of $y = \frac{1}{x} + 27x^3$ labelling the stationary points.

