12.6) Gradients, tangents and normal

Worked example	Your turn
Find the gradient of the curve: $y = 8\sqrt{x} + \frac{48}{x}$ at $(4, 28)$	Find the gradient of the curve: $y = 5\sqrt{x} - \frac{3}{x}$ at $(16, \frac{317}{16})$ $\frac{163}{256}$
$y = \frac{3}{x^2} - \frac{18}{\sqrt{x}}$ at $(9, -\frac{161}{27})$	

Worked example	Your turn
Find the coordinates of the point(s) where the gradient is 10: $y = x^3 + 6x^2 - 11x + 7$	Find the coordinates of the point(s) where the gradient is 2: $y = x^3 - 3x^2 - 7x + 8$ $(-1, 11)$ and $(3, -13)$

Worked example	Your turn
For the curve $y = f(x)$, $\frac{dy}{dx} = 723 + kx^5 + 2k$, where k is a constant. When $x = -3$, the gradient of the curve is 241. Find k.	For the curve $y = f(x)$, $\frac{dy}{dx} = \frac{3}{2} - kx^4 + k$, where k is a constant. When $x = -2$, the gradient of the curve is -6. Find k. $k = \frac{1}{2}$

Worked example	Your turn
Find the equation of the tangent to the curve $y = x^4$ when $x = 2$	Find the equation of the tangent to the curve $y = x^3$ when $x = 2$ $y - 8 = 12(x - 2)$ $y = 12x - 16$

Worked example

Find the equation of the normal to the curve $y = x^4$ when $x = 2$

Your turn

Find the equation of the normal to the curve $y = x^3$ when $x = 2$

$$y - 8 = -\frac{1}{12}(x - 2)$$

$$y = -\frac{1}{12}x + \frac{49}{6}$$

Worked example

Find the equation of the tangent to the curve with equation $y = x^3 - 5x^2 - 3x + 2$ at the point $(5, -13)$

Your turn

Find the equation of the tangent to the curve with equation $y = x^3 - 3x^2 + 2x - 1$ at the point $(3, 5)$

$y = 11x - 28$

Worked example

Find the equation of the normal to the curve with equation $y = 3 - 4\sqrt[3]{x}$ at the point where $x = 8$.

Give your answer in the form $ax + by + c = 0$

Your turn

Find the equation of the normal to the curve with equation $y = 8 - 3\sqrt{x}$ at the point where $x = 4$.

Give your answer in the form $ax + by + c = 0$

$$3y - 4x + 10 = 0$$

Worked example

The point P with x-coordinate $\frac{1}{4}$ lies on the curve with equation $y = 2x^2$.
The normal to the curve at P intersects the curve at points P and Q.
Find the coordinates of Q

Your turn

The point P with x-coordinate $\frac{1}{2}$ lies on the curve with equation $y = 4x^2$.
The normal to the curve at P intersects the curve at points P and Q.
Find the coordinates of Q

$$\left(-\frac{9}{16}, \frac{81}{64}\right)$$