12.3) Solving geometric problems

Worked example

A, B, C and D are the points $(3, -4, -9)$, $(1, -7, -3)$, $(1, 0, -15)$ and $(7, 9, -33)$ respectively.

a) Find $\overrightarrow{AB}$ and $\overrightarrow{DC}$, giving your answers in the form $p\boldsymbol{i} + q\boldsymbol{j} + r\boldsymbol{k}$.

b) Show that the lines AB and DC are parallel and that $\overrightarrow{DC} = 3\overrightarrow{AB}$.

c) Hence describe the quadrilateral $ABCD$.

Your turn

A, B, C and D are the points $(2, -5, -8)$, $(1, -7, -3)$, $(0, 15, -10)$ and $(2, 19, -20)$ respectively.

a) Find $\overrightarrow{AB}$ and $\overrightarrow{DC}$, giving your answers in the form $p\boldsymbol{i} + q\boldsymbol{j} + r\boldsymbol{k}$.

b) Show that the lines AB and DC are parallel and that $\overrightarrow{DC} = 2\overrightarrow{AB}$.

c) Hence describe the quadrilateral $ABCD$.

a)

$\overrightarrow{AB} = -\boldsymbol{i} - 2\boldsymbol{j} + 5\boldsymbol{k}$

$\overrightarrow{DC} = -2\boldsymbol{i} - 4\boldsymbol{j} + 10\boldsymbol{k}$

b)

$\overrightarrow{DC} = 2(-\boldsymbol{i} + 2\boldsymbol{j} + 5\boldsymbol{k}) = 2\overrightarrow{AB}$

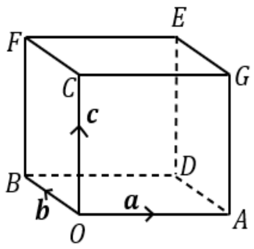
They are multiples ∴ parallel.

c)

AB and DC are parallel but different in length. Therefore $ABCD$ is a trapezium.

Worked example	Your turn
P, Q and R are the points $(9,3,-4), (-5,5,5)$ and $(0,2,-8)$ respectively. Find the coordinates of the point S so that $PQRS$ forms a parallelogram.	P, Q and R are the points $(4,-9,-3), (7,-7,-7)$ and $(8,-2,0)$ respectively. Find the coordinates of the point S so that $PQRS$ forms a parallelogram. $S(5,-4,4)$

Worked example	Your turn
Given that $(q-5)\boldsymbol{i}+2\boldsymbol{j}-120\boldsymbol{k}=p\boldsymbol{i}+q\boldsymbol{j}+4pqr\boldsymbol{k}$, find the values of p, q and r.	Given that $3\boldsymbol{i}+(p+2)\boldsymbol{j}+120\boldsymbol{k}=p\boldsymbol{i}-q\boldsymbol{j}+4pqr\boldsymbol{k}$, find the values of p, q and r. $p=3, q=-5, r=-2$

Worked example	Your turn
The diagram shows a cuboid whose vertices are O, A, B, C, D, E, F and G. Vectors a, b and c are the position vectors of the vertices A, B and C respectively. Prove that the diagonals OE and AF bisect each other. 	The diagram shows a cuboid whose vertices are O, A, B, C, D, E, F and G. Vectors a, b and c are the position vectors of the vertices A, B and C respectively. Prove that the diagonals OE and BG bisect each other. Proof