12.2) Vectors in 3D

Worked example

Consider the points $A(-1, -5, 2)$ and $B(-7, 3, 0)$.

a) Find the position vectors of A and B in ijk notation

b) Find the vector $\overrightarrow{AB}$ as a column vector.

Your turn

Consider the points $A(1, 5, -2)$ and $B(0, -3, 7)$.

a) Find the position vectors of A and B in ijk notation

b) Find the vector $\overrightarrow{AB}$ as a column vector.

a) $\overrightarrow{OA} = \boldsymbol{i} + 5\boldsymbol{j} - 2\boldsymbol{k}$

$\overrightarrow{OB} = -\mathbf{3}\boldsymbol{j} + 7\boldsymbol{k}$

b) $\overrightarrow{AB} = \begin{pmatrix} -1 \\ -8 \\ 9 \end{pmatrix}$

Worked example

The vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ are given by:

$$\boldsymbol{a} = \begin{pmatrix} 3 \\ -2 \\ -5 \end{pmatrix} \text{ and } \boldsymbol{b} = \begin{pmatrix} 2 \\ 0 \\ -4 \end{pmatrix}$$

a) Find:

i) $\boldsymbol{a} + 3\boldsymbol{b}$

ii) $4\boldsymbol{a} - 5\boldsymbol{b}$

b) State whether these vectors are parallel to $-4\boldsymbol{i} + 16\boldsymbol{j}$

Your turn

The vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ are given by:

$$\boldsymbol{a} = \begin{pmatrix} 2 \\ -3 \\ 5 \end{pmatrix} \text{ and } \boldsymbol{b} = \begin{pmatrix} 4 \\ -2 \\ 0 \end{pmatrix}$$

a) Find:

i) $4\boldsymbol{a} + \boldsymbol{b}$

ii) $2\boldsymbol{a} - 3\boldsymbol{b}$

b) State whether these vectors are parallel to $4\boldsymbol{i} - 5\boldsymbol{k}$

a)

i) $\begin{pmatrix} 12 \\ -14 \\ 20 \end{pmatrix}$

ii) $\begin{pmatrix} -8 \\ 0 \\ 10 \end{pmatrix}$

b)

i) $\begin{pmatrix} 12 \\ -14 \\ 20 \end{pmatrix} = 3\begin{pmatrix} 4 \\ -\frac{14}{3} \\ \frac{20}{3} \end{pmatrix} \neq k\begin{pmatrix} 4 \\ 0 \\ -5 \end{pmatrix}$ ∴ Not parallel

ii) $\begin{pmatrix} -8 \\ 0 \\ 10 \end{pmatrix} = -2\begin{pmatrix} 4 \\ 0 \\ -5 \end{pmatrix} = k\begin{pmatrix} 4 \\ 0 \\ -5 \end{pmatrix}$ $(k = -2)$ ∴ Parallel

Worked example

Find the magnitude of the vector $\begin{pmatrix} 6 \\ 8 \\ 24 \end{pmatrix}$

Your turn

Find the magnitude of the vector $\begin{pmatrix} 3 \\ 4 \\ 12 \end{pmatrix}$

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Worked example

Find the magnitude of the vector

$$\boldsymbol{a} = 6\boldsymbol{i} - 8\boldsymbol{j} + 24\boldsymbol{k}$$

and hence find $\hat{\boldsymbol{a}}$, the unit vector in the direction of $\boldsymbol{a}$.

Find the magnitude of the vector

$$\boldsymbol{a} = \boldsymbol{i} - 4\boldsymbol{j} + 2\boldsymbol{k}$$

and hence find $\hat{\boldsymbol{a}}$, the unit vector in the direction of $\boldsymbol{a}$.

Your turn

Find the magnitude of the vector

$$\boldsymbol{a} = 2\boldsymbol{i} - \boldsymbol{j} + 4\boldsymbol{k}$$

and hence find $\hat{\boldsymbol{a}}$, the unit vector in the direction of $\boldsymbol{a}$.

$$\hat{\boldsymbol{a}} = \frac{1}{\sqrt{21}}(2\boldsymbol{i} - \boldsymbol{j} + 4\boldsymbol{k})$$

Worked example

Find the angles that the vector

$$\boldsymbol{a} = \boldsymbol{i} - 2\boldsymbol{j} + 3\boldsymbol{k}$$

makes with each of the positive coordinate axes. Give your answers to 1 decimal place.

Your turn

Find the angles that the vector

$$\boldsymbol{a} = 2\boldsymbol{i} - 3\boldsymbol{j} - \boldsymbol{k}$$

makes with each of the positive coordinate axes. Give your answers to 1 decimal place.

$$\theta_x = 57.7°$$
$$\theta_y = 143.3°$$
$$\theta_z = 105.5°$$

Worked example

The points A and B have position vectors $\boldsymbol{i} + 5\boldsymbol{j} + 3\boldsymbol{k}$ and $-2\boldsymbol{i} + 4\boldsymbol{j} + 8\boldsymbol{k}$ relative to a fixed origin, O.
Show that ΔOAB is isosceles.

Your turn

The points A and B have position vectors $4\boldsymbol{i} + 2\boldsymbol{j} + 7\boldsymbol{k}$ and $3\boldsymbol{i} + 4\boldsymbol{j} - \boldsymbol{k}$ relative to a fixed origin, O.
Show that ΔOAB is isosceles.

$$\left|\overrightarrow{AB}\right| = \sqrt{69}$$
$$\left|\overrightarrow{OA}\right| = \sqrt{69}$$
$$\left|\overrightarrow{OB}\right| = \sqrt{26}$$
$$\left|\overrightarrow{AB}\right| = \left|\overrightarrow{OA}\right| \neq \left|\overrightarrow{OB}\right|$$
$$\therefore\ OAB \text{ is isosceles.}$$

Worked example

$\boldsymbol{a} = 3\boldsymbol{i} + 2\boldsymbol{j} + \boldsymbol{k}$ and $\boldsymbol{b} = \boldsymbol{i} + 3\boldsymbol{j} + 5\boldsymbol{k}$

By considering the angles that $\boldsymbol{a}$ and $\boldsymbol{b}$ make with the x-axis, determine the area of OAB where $\overrightarrow{OA} = \boldsymbol{a}$ and $\overrightarrow{OB} = \boldsymbol{b}$.

Your turn

$\boldsymbol{a} = 2\boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k}$ and $\boldsymbol{b} = \boldsymbol{i} + 3\boldsymbol{j} + 2\boldsymbol{k}$

By considering the angles that $\boldsymbol{a}$ and $\boldsymbol{b}$ make with the x-axis, determine the area of OAB where $\overrightarrow{OA} = \boldsymbol{a}$ and $\overrightarrow{OB} = \boldsymbol{b}$.

2.90 (3 sf)

Worked example

A triangle PQR is such that
$\overrightarrow{PQ} = -2\boldsymbol{i} + 3\boldsymbol{j} - \boldsymbol{k}$ and $\overrightarrow{QR} = 4\boldsymbol{i} - 3\boldsymbol{j} - 2\boldsymbol{k}$
Find $< PQR$ to 1 decimal place

Your turn

A triangle PQR is such that
$\overrightarrow{PQ} = 2\boldsymbol{i} - \mathbf{3}\boldsymbol{j} + \boldsymbol{k}$ and $\overrightarrow{QR} = -4\boldsymbol{i} + 3\boldsymbol{j} + 2\boldsymbol{k}$
Find $< PQR$ to 1 decimal place

$41.9°$