12.2) Finding the derivative

Worked example	Your turn
The point A with coordinates $(8,64)$ lies on the curve with equation $y = x^2$. At point A the curve has gradient g. a) Show that $g = \lim_{h \to 0}(16 + h)$ b) Deduce the value of g.	The point A with coordinates $(4,16)$ lies on the curve with equation $y = x^2$. At point A the curve has gradient g. a) Show that $g = \lim_{h \to 0}(8 + h)$ b) Deduce the value of g. Shown

Worked example	Your turn
Prove from first principles that the derivative of $3x$ is 3	Prove from first principles that the derivative of $5x$ is 5

$$f(x) = 5x$$

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h\to 0} \frac{5(x+h) - 5(x)}{h} \\ &= \lim_{h\to 0} \frac{5x + 5h - 5x}{h} \\ &= \lim_{h\to 0} \frac{5h}{h} \\ &= \lim_{h\to 0} 5 \\ &= 5 \end{aligned}$$

Worked example	Your turn
Prove from first principles that the derivative of $3x^2$ is $6x$	Prove from first principles that the derivative of $5x^2$ is $10x$

$$f(x) = 5x^2$$

$$\begin{aligned} f'(x) &= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} \\ &= \lim_{h\to 0}\frac{5(x+h)^2-5(x)^2}{h} \\ &= \lim_{h\to 0}\frac{5x^2+10xh+5h^2-5x^2}{h} \\ &= \lim_{h\to 0}\frac{10xh+5h^2}{h} \\ &= \lim_{h\to 0}\frac{h(10x+5h)}{h} \\ &= \lim_{h\to 0}(10x+5h) \\ &= 10x \end{aligned}$$

[As $h \to 0, 5h \to 0$]

Worked example	Your turn
Prove from first principles that the derivative of x^4 is $4x^3$.	Prove from first principles that the derivative of x^3 is $3x^2$

$$f(x) = x^3$$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{(x+h)^3 - (x)^3}{h}$$

$$= \lim_{h\to 0} \frac{x^3 + 3x^2h + 3xh^2 + h^3 - x^3}{h}$$

$$= \lim_{h\to 0} \frac{3x^2h + 3xh^2 + h^3}{h}$$

$$= \lim_{h\to 0} \frac{h(3x^2 + 3xh^2 + h^3)}{h}$$

$$= \lim_{h\to 0} (3x^2 + 3xh + h^2)$$

$$= 3x^2$$

[As $h \to 0, 3xh \to 0$ and $h^2 \to 0$]