

11.7) Partial fractions

Worked example

Find:

$$\int \frac{x + 5}{(x - 1)(x + 2)} dx$$

Your turn

Find:

$$\int \frac{x - 5}{(x + 1)(x - 2)} dx$$

$$\ln \left| \frac{(x + 1)^2}{x - 2} \right| + c$$

Worked example

Find:

$$\int \frac{3x + 15 - 4x^2}{(2x + 1)(x - 2)^2} dx$$

Your turn

Find:

$$\int \frac{8x^2 - 19x + 1}{(2x + 1)(x - 2)^2} dx$$

$$\ln|(2x + 1)(x - 2)^3| + \frac{1}{x - 2} + c$$

Worked example

Evaluate:

$$\int_0^2 \frac{8x^2 + 34x + 20}{(2x + 1)(x + 1)(x + 3)}$$

Your turn

Evaluate:

$$\int_0^2 \frac{4 - 2x}{(2x + 1)(x + 1)(x + 3)}$$

$$\ln\left(\frac{125}{81}\right)$$

Worked example

Find:

$$\int \frac{4}{x^2 - 4} dx$$

Your turn

Find:

$$\int \frac{2}{x^2 - 1} dx$$
$$\ln \left| \frac{x - 1}{x + 1} \right| + c$$

Worked example

Find:

$$\int \frac{x^2}{x-1} dx$$

Your turn

Find:

$$\int \frac{x^2}{x+1} dx$$

$$\frac{1}{2}x^2 - x + \ln|x+1| + c$$

Worked example

Find:

$$\int \frac{x}{x-2} dx$$

$$\int \frac{x-2}{x} dx$$

Your turn

Find:

$$\int \frac{x}{x-1} dx$$

$$x + \ln|x-1| + c$$

$$\int \frac{x-1}{x} dx$$

$$x - \ln|x| + c$$

Worked example

Find:

$$\int \frac{x^3 - 2}{x - 1} dx$$

Your turn

Find:

$$\int \frac{x^3 + 2}{x + 1} dx$$

$$\frac{1}{3}x^3 - \frac{1}{2}x^2 + x + \ln|x + 1| + c$$

Worked example

Find:

$$\int \frac{4x^2 - 2x - 18}{4x^2 - 9} dx$$

Your turn

Find:

$$\int \frac{9x^2 - 3x + 2}{9x^2 - 4} dx$$

$$x + \frac{1}{3} \ln \left| \frac{3x - 2}{(3x + 2)^2} \right| + c$$