11.5) Integration by substitution

Worked example	Your turn
Find: $$\int x(2x-5)^9 \ dx$$ using the substitution $u = 2x - 5$	Find: $$\int x(5x-2)^8 \ dx$$ using the substitution $u = 5x - 2$ $$\frac{(5x-2)^{10}}{250} + \frac{2(5x-2)^9}{225} + c$$

Worked example	Your turn
Find: $$\int x\sqrt{5x+2}\ dx$$ using the substitution $u = 5x + 2$	Find: $$\int x\sqrt{2x+5}\ dx$$ using the substitution $u = 2x + 5$ $$\frac{(2x+5)^{\frac{5}{2}}}{10} - \frac{5(2x+5)^{\frac{3}{2}}}{6} + c$$

Worked example	Your turn
Find: $$\int x\sqrt{5x+2}\ dx$$ using the substitution $u^2 = 5x + 2$	Find: $$\int x\sqrt{2x+5}\ dx$$ using the substitution $u^2 = 2x + 5$ $$\frac{(2x+5)^{\frac{5}{2}}}{10} - \frac{5(2x+5)^{\frac{3}{2}}}{6} + c$$

Worked example	Your turn
Find: $$\int \cos x \sin x\,(2+\sin x)^4\ dx$$ using the substitution $u = \sin x + 2$	Find: $$\int \cos x \sin x\,(1+\sin x)^3\ dx$$ using the substitution $u = \sin x + 1$ $$\frac{1}{5}(\sin x+1)^5 - \frac{1}{4}(\sin x+1)^4 + c$$

Worked example

Find:

$$\int_0^{\frac{\pi}{2}} \cos x \sin x \,(2+\sin x)^4 \, dx$$

using the substitution $u = \sin x + 2$

Your turn

Find:

$$\int_0^{\frac{\pi}{2}} \cos x \sin x \,(1+\sin x)^3 \, dx$$

using the substitution $u = \sin x + 1$

$$\frac{49}{20}$$

Worked example	Your turn
Find: $$\int \frac{3\sin 2x}{2+\sin x}\,dx$$ using the substitution $u = 2 + \sin x$	Find: $$\int \frac{2\sin 2x}{1+\cos x}\,dx$$ using the substitution $u = 1 + \cos x$ $4\ln\|1+\cos x\| - 4\cos x + c$

Worked example

Calculate:

$$\int_0^{\frac{\pi}{2}} \sin x \sqrt{2+\cos x}\ dx$$

using the substitution $u = \cos x + 2$

Your turn

Calculate:

$$\int_0^{\frac{\pi}{2}} \cos x \sqrt{1+\sin x}\ dx$$

using the substitution $u = \sin x + 1$

$$\frac{2}{3}\left(2\sqrt{2}-1\right)$$

Worked example	Your turn
Use the substitution $u = \sqrt{x} - 1$ to evaluate: $$\int_{36}^{49} \frac{1}{\sqrt{x} - 1} \, dx$$	Use the substitution $u = 1 + \sqrt{x}$ to evaluate: $$\int_{16}^{25} \frac{1}{1 + \sqrt{x}} \, dx$$ $$2 + 2\ln\left\lvert\frac{5}{6}\right\rvert$$

Worked example	Your turn
A finite region is bounded by the curve with equation $y = x^3 \ln(x^2 + 3)$, the x-axis and the lines $x = 0$ and $x = \sqrt{5}$. Use the substitution $u = x^2 + 3$ to show that the area of R is $\frac{1}{2}\int_3^8 (u - 3) \ln u \, du$	A finite region is bounded by the curve with equation $y = x^3 \ln(x^2 + 2)$, the x-axis and the lines $x = 0$ and $x = \sqrt{2}$. Use the substitution $u = x^2 + 2$ to show that the area of R is $\frac{1}{2}\int_2^4 (u - 2) \ln u \, du$ Shown

Worked example	Your turn
Using integration by substitution, prove that: $$-\int \frac{1}{\sqrt{1-x^2}}\,dx = \arccos x + c$$	Using integration by substitution, prove that: $$\int \frac{1}{\sqrt{1-x^2}}\,dx = \arcsin x + c$$ Shown

Worked example

Use the substitution $u = \cos x$ to evaluate

$$\int_0^{\frac{\pi}{3}} \sin^3 x \cos^2 x \, dx$$

Your turn

Use the substitution $u = \sin x$ to evaluate

$$\int_0^{\frac{\pi}{6}} \sin^2 x \cos^3 x \, dx$$

$$\frac{17}{480}$$

Worked example	Your turn
Use the substitution $x = \cos u$ to evaluate $$\int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} x^2\sqrt{1-x^2}\,dx$$	Use the substitution $x = \sin u$ to evaluate $$\int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} x^2\sqrt{1-x^2}\,dx$$ $$\frac{2\pi+3\sqrt{3}}{96}$$