11.1) Functions of time

Worked example

A body moves in a straight line, such that its displacement, s metres, from a point O at time t seconds, is given by

$$s = 5t^3 - 2t, t > 0$$

Find:

a) s when $t = 3$

b) The time taken for the particle to return to O

Your turn

A body moves in a straight line, such that its displacement, s metres, from a point O at time t seconds, is given by

$$s = 2t^3 - 3t, t > 0$$

Find:

a) s when $t = 2$

b) The time taken for the particle to return to O

a) $10\ m$

b) $\sqrt{\frac{3}{2}}\ s = 1.2\ s$ (2 sf)

Worked example

A train travels along a straight track, leaving the start of the track at time $t = 0$. It then returns to the start of the track. The distance, s metres, from the start of the track at time t seconds is modelled by:

$$s = 8t^2 - 5t^3, \quad 0 \leq t \leq 1.6$$

Explain the restriction $0 \leq t \leq 1.6$

Your turn

A train travels along a straight track, leaving the start of the track at time $t = 0$. It then returns to the start of the track. The distance, s metres, from the start of the track at time t seconds is modelled by:

$$s = 4t^2 - t^3, 0 \leq t \leq 4$$

Explain the restriction $0 \leq t \leq 4$

s is the distance from the start of the track: $s \geq 0$

$$4t^2 - t^3 \geq 0$$

$$t^2(4 - t) \geq 0$$

$t^2 \geq 0$ for all t and $(4 - t) < 0$ for all $t > 4$.

So $t^2(4 - t)$ is only non-negative for $t \leq 4$

Motion begins at $t = 0$, hence $t \geq 0$

Hence $0 \leq t \leq 4$

Worked example

A body moves in a straight line such that its velocity, v ms^{-1}, at time t seconds is given by $v = 3t^2 - 24t + 36$. Find

(a) The initial velocity
(b) The values of t when the body is instantaneously at rest.
(c) The value of t when the velocity is 63 ms^{-1}.
(d) The greatest speed of the body in the interval $0 \leq t \leq 7$.

Your turn

A body moves in a straight line such that its velocity, v ms^{-1}, at time t seconds is given by $v = 2t^2 - 16t + 24$. Find

(a) The initial velocity
(b) The values of t when the body is instantaneously at rest.
(c) The value of t when the velocity is 64 ms^{-1}.
(d) The greatest speed of the body in the interval $0 \leq t \leq 5$.

a) $24\ ms^{-1}$
b) $t = 2, t = 6$
c) $t = 10$
d) $24\ ms^{-1}$