11) Vectors

11.1) Vectors

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Worked example	Your turn
$PQRS$ is a parallelogram. N is the point on SQ such that $SN : NQ = 3 : 4$ $\overrightarrow{PQ} = \boldsymbol{b}$ and $\overrightarrow{PS} = \boldsymbol{a}$ Express $\overrightarrow{NR}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$	$PQRS$ is a parallelogram. N is the point on SQ such that $SN : NQ = 3 : 2$ $\overrightarrow{PQ} = \boldsymbol{a}$ and $\overrightarrow{PS} = \boldsymbol{b}$ Express $\overrightarrow{NR}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$ $\frac{2}{5}\boldsymbol{a} + \frac{3}{5}\boldsymbol{b}$

Worked example	Your turn
OAB is a triangle. $\overrightarrow{OA} = \boldsymbol{b}$ and $\overrightarrow{OB} = \boldsymbol{a}$ P is the point on AB such that $AP:PB = 2:3$. Find $\overrightarrow{OP}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$	OAB is a triangle. $\overrightarrow{OA} = \boldsymbol{a}$ and $\overrightarrow{OB} = \boldsymbol{b}$ P is the point on AB such that $AP:PB = 3:1$. Find $\overrightarrow{OP}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$ $\frac{1}{4}\boldsymbol{a} + \frac{3}{4}\boldsymbol{b}$

Worked example	Your turn
Show that the vectors are parallel: $3\boldsymbol{a}+4\boldsymbol{b}$ and $15\boldsymbol{a}+20\boldsymbol{b}$	Show that the vectors are parallel: $6\boldsymbol{a}+8\boldsymbol{b}$ and $9\boldsymbol{a}+12\boldsymbol{b}$
	$9\boldsymbol{a}+12\boldsymbol{b}=\frac{3}{2}(6\boldsymbol{a}+8\boldsymbol{b})$
$3\boldsymbol{a}+4\boldsymbol{b}$ and $-0.75\boldsymbol{a}-\boldsymbol{b}$	

11.2) Representing vectors

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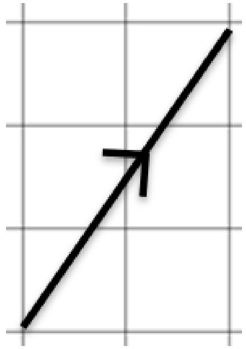
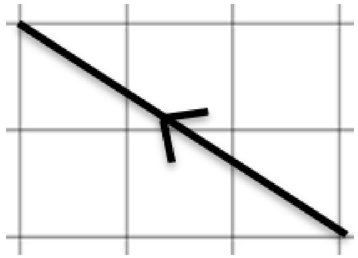
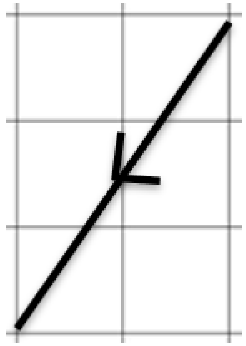
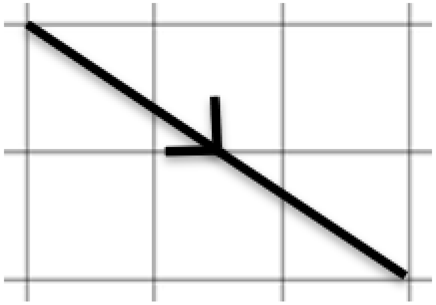
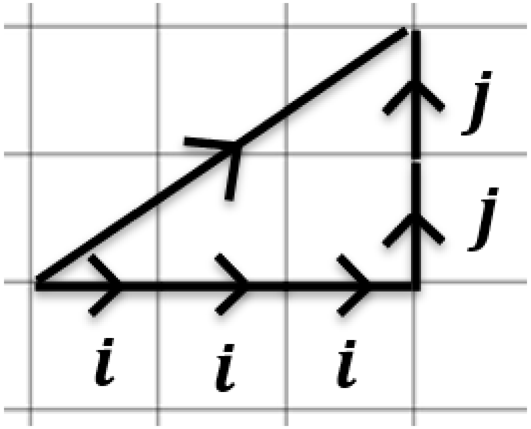
Worked example	Your turn
Represent in column vector form:   	Represent in column vector form:  $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$

Worked example	Your turn
Draw a diagram to represent the vector: $2\boldsymbol{i} + 3\boldsymbol{j}$	Draw a diagram to represent the vector: $3\boldsymbol{i} + 2\boldsymbol{j}$



$-3\boldsymbol{i} + 2\boldsymbol{j}$

$2\boldsymbol{i} - 3\boldsymbol{j}$

$-2\boldsymbol{i} - 3\boldsymbol{j}$

Worked example	Your turn
Given $\boldsymbol{a} = 8\boldsymbol{i} - 6\boldsymbol{j}$ and $\boldsymbol{b} = 9\boldsymbol{i} + 7\boldsymbol{j}$, find: • $4\boldsymbol{b} - 2\boldsymbol{a}$ • $-\boldsymbol{b} + \frac{1}{4}\boldsymbol{a}$	Given $\boldsymbol{a} = 5\boldsymbol{i} + 2\boldsymbol{j}$ and $\boldsymbol{b} = 3\boldsymbol{i} - 4\boldsymbol{j}$, find: • $2\boldsymbol{a} - \boldsymbol{b}$ • $-\boldsymbol{a} + \frac{1}{2}\boldsymbol{b}$ • $7\boldsymbol{i} + 8\boldsymbol{j}$ • $-\frac{7}{2}\boldsymbol{i} - 4\boldsymbol{j}$

11.3) Magnitude and direction

Worked example	Your turn
Find the magnitude of the vector: $3\boldsymbol{i} + 4\boldsymbol{j}$ $-5\boldsymbol{i} + 12\boldsymbol{j}$ $7\boldsymbol{i} - 24\boldsymbol{j}$	Find the magnitude of the vector: $-6\boldsymbol{i} - 8\boldsymbol{j}$ 10

Worked example	Your turn
Find a unit vector in the direction of: $\boldsymbol{a} = 8\boldsymbol{i} + 15\boldsymbol{j}$ $\boldsymbol{b} = -9\boldsymbol{i} + 12\boldsymbol{j}$	Find a unit vector in the direction of: $\boldsymbol{c} = 3\boldsymbol{i} - 4\boldsymbol{j}$ $\hat{c} = \frac{1}{5}(3\boldsymbol{i} - 4\boldsymbol{j})$ or $\begin{pmatrix} 0.6 \\ -0.8 \end{pmatrix}$

Worked example	Your turn
Given $\boldsymbol{a} = 8\boldsymbol{i} - 6\boldsymbol{j}$ and $\boldsymbol{b} = 9\boldsymbol{i} + 7\boldsymbol{j}$, find $\lvert 2\boldsymbol{b} - 3\boldsymbol{a} \rvert$	Given $\boldsymbol{a} = 5\boldsymbol{i} + 2\boldsymbol{j}$ and $\boldsymbol{b} = 3\boldsymbol{i} - 4\boldsymbol{j}$, find: $\lvert 4\boldsymbol{a} - 5\boldsymbol{b} \rvert$ $\sqrt{809}$

Worked example	Your turn
Find the angle between the vector $2\boldsymbol{i} + 3\boldsymbol{j}$ and the positive y-axis.	Find the angle between the vector $4\boldsymbol{i} + 5\boldsymbol{j}$ and the positive x-axis. 51.3° (3 sf)

Worked example	Your turn
Vector $\boldsymbol{a}$ has magnitude 5 and make an angle of 60° with $\boldsymbol{i}$. Find $\boldsymbol{a}$ in $\boldsymbol{i}, \boldsymbol{j}$ and column vector format.	Vector $\boldsymbol{b}$ has magnitude 10 and make an angle of 30° with $\boldsymbol{j}$. Find $\boldsymbol{b}$ in $\boldsymbol{i}, \boldsymbol{j}$ and column vector format. $\boldsymbol{b} = 5\boldsymbol{i} + 5\sqrt{3}\boldsymbol{j} = \begin{pmatrix} 5 \\ 5\sqrt{3} \end{pmatrix}$

Worked example	Your turn
A vector $\boldsymbol{a} = p\boldsymbol{i} + q\boldsymbol{j}$ has magnitude 68 and makes an angle θ with the positive x-axis where $\sin\theta = \frac{8}{17}$. Find all the possible vectors	A vector $\boldsymbol{a} = p\boldsymbol{i} + q\boldsymbol{j}$ has magnitude 26 and makes an angle θ with the positive x-axis where $\sin\theta = \frac{5}{13}$. Find all the possible vectors $p = 10, q = 24$ $p = 10, q = -24$ $p = -10, q = 24$ $p = -10, q = -24$

Worked example	Your turn
In triangle PQR, $\overrightarrow{PQ} = \boldsymbol{i} + 2\boldsymbol{j}$ and $\overrightarrow{PR} = 8\boldsymbol{i} - 15\boldsymbol{j}.$ Find the area of triangle PQR	In triangle PQR, $\overrightarrow{PQ} = 2\boldsymbol{i} + \boldsymbol{j}$ and $\overrightarrow{PR} = 9\boldsymbol{i} - 12\boldsymbol{j}.$ Find the area of triangle PQR 16.5

11.4) Position vectors

Worked example	Your turn
The points A and B have coordinates $(2,5)$ and $(6,13)$ respectively. Find, in terms of $\boldsymbol{i}$ and $\boldsymbol{j}$: a) The position vector of A b) The position vector of B c) The vector $\overrightarrow{AB}$	The points A and B have coordinates $(3,4)$ and $(11,2)$ respectively. Find, in terms of $\boldsymbol{i}$ and $\boldsymbol{j}$: a) The position vector of A b) The position vector of B c) The vector $\overrightarrow{AB}$ a) $\overrightarrow{OA} = 3\boldsymbol{i} + 4\boldsymbol{j}$ b) $\overrightarrow{OB} = 11\boldsymbol{i} + 2\boldsymbol{j}$ c) $\overrightarrow{AB} = 8\boldsymbol{i} - 2\boldsymbol{j}$

Worked example	Your turn
$\overrightarrow{OA} = 4\boldsymbol{i} + 3\boldsymbol{j}$ and $\overrightarrow{AB} = 2\boldsymbol{i} - 5\boldsymbol{j}$. Find: a) The position vector of B. b) The exact value of $\left\|\overrightarrow{OB}\right\|$ in simplified surd form.	$\overrightarrow{OA} = 5\boldsymbol{i} - 2\boldsymbol{j}$ and $\overrightarrow{AB} = 3\boldsymbol{i} + 4\boldsymbol{j}$. Find: a) The position vector of B. b) The exact value of $\left\|\overrightarrow{OB}\right\|$ in simplified surd form. a) $\overrightarrow{OB} = 8\boldsymbol{i} + 2\boldsymbol{j} = \binom{8}{2}$ b) $2\sqrt{17}$

11.5) Solving geometric problems

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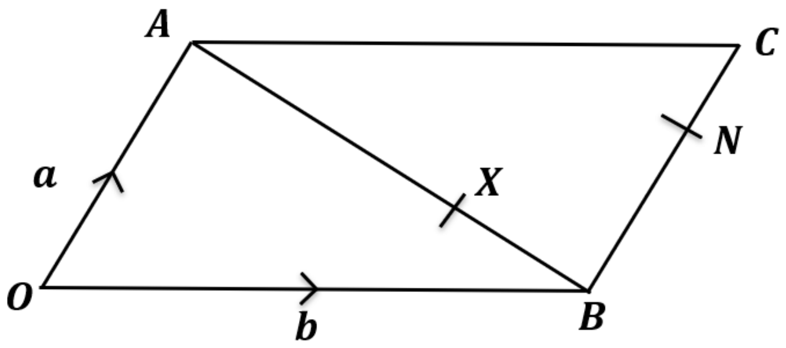
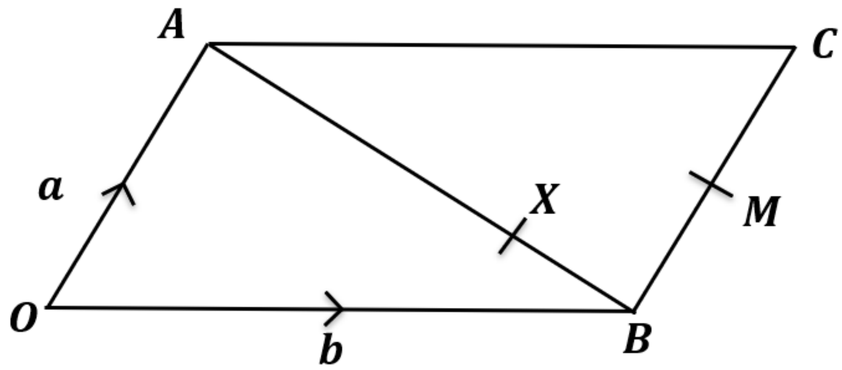
Worked example	Your turn
$OACB$ is a parallelogram. X is a point on AB such that $AX:XB = 2:1$. N is the point such that NC is half of BN. Show that $\overrightarrow{XN}$ is parallel to $\overrightarrow{OC}$.  	$OACB$ is a parallelogram. X is a point on AB such that $AX:XB = 3:1$. M is the midpoint of BC. Show that $\overrightarrow{XM}$ is parallel to $\overrightarrow{OC}$.  Shown

Worked example	Your turn
$\overrightarrow{AB} = 2\boldsymbol{i} - 5\boldsymbol{j}$ and $\overrightarrow{AC} = 3\boldsymbol{i} - 7\boldsymbol{j}$. Determine $\angle BAC$.	$\overrightarrow{AB} = 3\boldsymbol{i} - 2\boldsymbol{j}$ and $\overrightarrow{AC} = \boldsymbol{i} - 5\boldsymbol{j}$. Determine $\angle BAC$. $45°$

11.6) Modelling with vectors

Worked example	Your turn
A girl walks 6 km due east from a fixed point O to A, and then 4 km due south from A to B. Find: a) the total distance travelled b) the position vector of B relative to O c) $\|\overrightarrow{OB}\|$ d) The bearing of B from O.	A girl walks 2 km due east from a fixed point O to A, and then 3 km due south from A to B. Find: a) the total distance travelled b) the position vector of B relative to O c) $\|\overrightarrow{OB}\|$ d) The bearing of B from O. a) $5\ km$ b) $(2\boldsymbol{i} - 3\boldsymbol{j})\ km$ c) $3.61\ km$ (3 sf) d) $146°$ (3 sf)

Worked example	Your turn
In an orienteering exercise, a cadet leaves the starting point O and walks 30 km on a bearing of $150°$ to reach A, the first checkpoint. From A she walks 18 km on a bearing of $210°$ to the second checkpoint, at B. From B she returns directly to O. Find: a) the position vector of A relative to O b) $\lvert\overrightarrow{OB}\rvert$ c) the bearing of B from O d) the position vector of B relative O.	In an orienteering exercise, a cadet leaves the starting point O and walks 15 km on a bearing of $120°$ to reach A, the first checkpoint. From A he walks 9 km on a bearing of $240°$ to the second checkpoint, at B. From B he returns directly to O. Find: a) the position vector of A relative to O b) $\lvert\overrightarrow{OB}\rvert$ c) the bearing of B from O d) the position vector of B relative O. a) $(13.0\boldsymbol{i} - 7.5\boldsymbol{j})\ km$ (1 dp) b) $13.1\ km$ (3 sf) c) $157°$ (3 sf) d) $(5.2\boldsymbol{i} - 12.0\boldsymbol{j})\ km$